PHENOMENOLOGICAL ANALYSIS OF NEUTRAL PION ELECTROPRODUCTION AND THE γNN^* FORM FACTOR

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We have analyzed recent data on neutral pion electroproduction in the region of the $N^*(1236)$ resonance phenomenologically to determine the magnitude of the dominant magnetic dipole amplitude and its interference with the E_1^+ electric quadrupole amplitude. Evidence is found for the existence of significant scalar multipoles. The four-momentum-transfer dependence of the magnetic dipole amplitude is interpreted in terms of the γNN^* transition form factor between 0.05 and 0.4 $(\mathrm{BeV}/c)^2$.

In the preceding Letter[1] (hereafter referred to as I) extensive new data have been presented on neutral-pion electroproduction in the region of the first pion-nucleon resonance. Pion angular distributions were measured for several pion-nucleon center-of-mass energies at four-momentum transfers of 0.05, 0.13, 0.25, and 0.4 $(\mathrm{BeV}/c)^2$.

The present Letter describes a phenomenological analysis of these data which has provided three major results:

(1) The size of the electric quadrupole amplitude[2] E_1^+ is typically from 5 to 13 % of the resonant magnetic dipole amplitude M_1^+ in the reasonable agreement with photoproduction analyses.[3]

(2) There is a significant scalar-transverse term in the cross section at four-momentum transfers of 0.13 and 0.25 $(\mathrm{BeV}/c)^2$, which can be interpreted most simply as interference between the resonant S_1^+ and M_1^+ amplitudes.

(3) The γNN^* form factor is approximately proportional to the nucleon magnetic isovector form factor, in agreement with the measurements of Ash et al.[4] The exponential form-factor dependence obtained by Dufner and Tsai[5] from an analysis of noncoincidence electroproduction data differs from $G_{\mathrm{MV}}(q^2)$ by approximately 12 % below 0.4 $(\mathrm{BeV}/c)^2$ and is also consistent with the results.

The general form of the pion angular distribution is given in I. For neutral-pion electroproduction, which is insensitive to the pion pole term, photoproduction analyses[3] indicate that it is probably adequate to assume that the interaction only involves s and p pion-nucleon partial waves for center-of-mass energies less than 1350 MeV. In this case the most general form of the angular distribution is

$$\frac{1}{\Gamma_T}\left(\frac{d^3\sigma}{dE'd\omega_e d\Omega_\pi}\right)=\frac{d\sigma}{d\Omega_\pi}=A+B\cos\theta_\pi^*+C\cos^2\theta_\pi^*+(D+E\cos\theta_\pi^*)\sin\theta_\pi^*\cos\varphi_\pi+F\sin^2\theta_\pi^*\cos2\varphi_\pi, \quad (1)$$

where E' is the energy of the scattered electron, $\omega\theta$ is the laboratory solid angle for electron detection, Ω_π is the c.m. pion solid angle, θ_π^* is the c.m. pion polar angle, and φ_π is the pion azimuthal angle. The kinematic factor Γ_T is defined in I. It describes the major dependence of the cross section upon the electron scattering angle and ensures that the angular distribution $d\sigma/d\Omega\pi$ reduces to the photoproduction cross section in the limit of zero four-momentum transfer.

The six parameters A through F depend only upon the pion nucleon c.m. energy W, the square of the invariant four-momentum transfer q^2 (positive in the metric used here), and the polarization of the transverse components of the virtual photon ϵ. The parameters A and F can be expanded in terms of the contributing multipole amplitudes as follows[6]:

$$A=\frac{\pi W}{KM}\Big[|E_0^+|^2+\tfrac{5}{2}|M_1^+|^2+\tfrac{9}{2}|E_1^+|^2+|M_1^-|^2-\mathrm{Re}(M_1^-)(M_1^+)^*$$
$$-3\,\mathrm{Re}(E_1^+)(M_1^+-M_1^-)^*+\frac{\epsilon q^2}{q_0^{*2}}\{|S_1^-|^2+|S_1^+|^2-2\,\mathrm{Re}(S_1^-)(S_1^+)^*+|S_0^+|^2\}\Big],$$

$$B=\frac{2\pi W}{KM}\left[\mathrm{Re}\left\{(E_0^+)(M_1^+)^*+3(E_1^+)(E_0^+)^*-(E_0^+)(M_1^-)^*+\frac{\epsilon q^2}{q_0^{*2}}[S_0^+(2S_1^++S_1^-)^*]\right\}\right],$$

$$C=\frac{\bar{\pi}W}{KM}\Big[-\tfrac{3}{2}|M_1^+|^2+\tfrac{9}{2}|E_1^+|^2+3\,\mathrm{Re}\{(M_1^+)(3E_1^+-M_1^-)^* -3(M_1^-)(E_1^+)^*\}+\frac{3\epsilon q^2}{q_0^{*2}}\{|S_1^+|^2+2\,\mathrm{Re}(S_1^+)(S_1^-)^*\}\Big],$$

$$D=\frac{-2\bar{\pi}W}{KM}\left(\frac{q^2}{q_0^{*2}}\right)^{1/2}\mathrm{Re}[(E_0^+)(S_1^- - S_1^+)^* + (3E_1^+ + M_1^- - M_1^+)(S_0^+)^*]\{\tfrac{1}{2}\epsilon(\epsilon+1)\}^{1/2},$$

$$E=\frac{-6\bar{\pi}W}{KM}\left(\frac{q^2}{q_0^{*2}}\right)^{1/2}\mathrm{Re}[(E_1^+)(2S_1^- + S_1^+)^* + (M_1^- - M_1^+)(S_1^+)^*]\{\tfrac{1}{2}\epsilon(\epsilon+1)\}^{1/2},$$

$$F=\frac{\bar{\pi}W}{KM}\epsilon[\tfrac{9}{2}|E_1^+|^2-\tfrac{3}{2}|M_1^+|^2-3\,\mathrm{Re}\{(E_1^+)(M_1^+)^* + (M_1^-)(M_1^+ - E_1^+)^*\}], \qquad (2)$$

where q_0^* is the c.m. photon energy, $\bar{\pi}$ is the c.m. pion momentum, M is the proton mass, and $K=(W^2-M^2)/2M$.

Since the interaction is known to be dominated by the magnetic dipole amplitude M_1^+, examination of the expressions for the angular coefficients shows that A, C, and F will be the dominant terms in the cross section. Furthermore, if the other multipole amplitudes are sufficiently small, the three coefficients will be related by $-3A/5=C=F/\epsilon$.

For each data set at fixed W, q^2, and ϵ, the results have been fitted with an expression of the above form with A through F as free parameters. For five of the 14 data sets it was possible to obtain three-parameter fits in A, C, and F which represented the data well and which could not be improved significantly by adding extra free parameters. However, in no case was the relation $-3A/5=C=F/\epsilon$ obeyed within the errors, indicating the presence of appreciable interference terms in the cross section.

For the remaining data sets the fits were greatly improved by the inclusion of a $\cos\varphi_\pi$ term corresponding to the parameters D and E. However, because of the limited statistical precision of the data and the restricted range of pion polar angles, D and E had almost identical effects on the fit. Therefore, in order to decide whether the s-wave S_0^+ or the scalar quadrupole S_1^+ amplitude was responsible for the $\cos\varphi_\pi$ behavior, it was necessary to investigate the energy dependence of the interference term. Since the S_0^+ phase is expected to be small, its interference with the M_1^+ should change sign near resonance. On the other hand, the S_1^+, M_1^+ interference might be expected to peak at the resonance.

Figure 1 shows the behavior of the fitted parameters at $q^2\simeq 0.25$ (BeV/c)^2 as a function of W. The fact that the coefficient of the $\cos\varphi_\pi$ term (labeled $-E$ in the figure) remains positive across the resonance indicates that although the situation may be more complicated, the S_1^+, M_1^+ interference term is probably the dominant contribution. If this hypothesis is correct, the magnitude of the S_1^+ amplitude is approximately as large as that predicted by its threshold relation with the E_1^+ amplitude.

Although the unambiguous identification of the leading scalar multipoles is difficult, the values of A, C, and F did not depend on the inclusion of D or E, so that a fairly precise determination of the M_1^+ multipole and its interference with the electric quadrupole amplitude was possible.

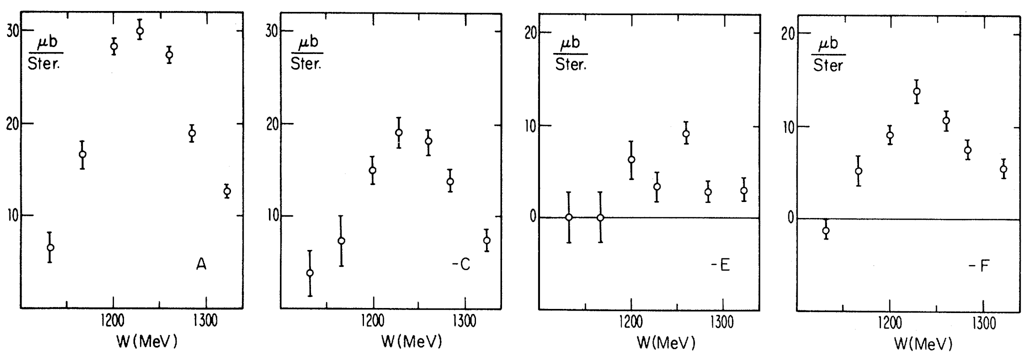

FIG. 1. Best-fit parameters at $q^2=0.25$ (BeV/c)^2.

The scalar contribution to the coefficient C can be no larger than 5 %, but the S_0^+ and S_1^- multipoles could conceivably make large contributions to A even though their interference with the M_1^+ is suppressed by phase considerations. Because of this possibility we have used C and F to extract the transverse multipole information.

The assumption that $\mathrm{Re}(E_1^+)(M_1^+)^* \gg \mathrm{Re}(E_1^+) \times (M_1^-)^*$ leads to the relation $\mathrm{Re}(E_1^+)(M_1^+)^* = \frac{1}{12}(C-F/\epsilon)$. The additional assumption that $|M_1^+|^2 \gg 3|E_1^+|^2$ leads to the expression

$$|M_1^+|^2 = -\frac{1}{6}\left(C+\frac{3F}{\epsilon}\right)\Bigg/\left(1+\frac{2\,\mathrm{Re}(M_1^-)(M_1^+)^*}{|M_1^+|^2}\right).$$

Although the M_1^-, M_1^+ interference term is expected to be negligible compared with $|M_1^+|^2$ in the vicinity of the resonance, the photoproduction analysis of Berends, Donnachie, and Weaver[3] indicates that it increases rapidly as W varies away from resonance. We have therefore used the results of Berends, Donnachie, and Weaver to obtain an approximate value for the ratio $\mathrm{Re}(M_1^-)(M_1^+)^*/|M_1^+|^2$ and increased the errors of $|M_1^+|^2$ appropriately.[7] The resulting correction to $|M_1^+|^2$ was 0 % at resonance and, typically, 15 % elsewhere.

The best phenomenological fits to the data and the values obtained for $|M_1^+|^2$ and $\mathrm{Re}(E_1^+)(M_1^+)^*/|M_1^+|^2$ are shown in Table I. The errors in all cases are estimated standard deviations. The errors of A, C, E, and F include only contributions from the relative errors of the data. The errors of $|M_1^+|^2$ and $\mathrm{Re}(E_1^+)(M_1^+)^*/|M_1^+|^2$ contain contributions from all known sources of error.

The dependence of the M_1^+ amplitude upon the four-momentum transfer can be interpreted as the form factor of the γNN^* transition if the assumption is made that the N^* behaves like a real particle.[4] Several different definitions of this form factor appear in the literature. The definition adopted here is that of Ash et al.[4] in order to facilitate the comparison of their coincidence measurements of neutral-pion electroproduction with the data presented in I. In this notation, the form factor $G_M^*(q^2)$ contains the complete four-momentum-transfer dependence of the

Table I. Results of the phenomenological analysis of π^0 electroproduction.

q^2 F^{-2}	q^2 (Bev^2)	W (Bev)	ϵ	A (μb/st)	C (μb/st)	E (μb/st)	F (μb/st)	NO. of datum points	χ^2	$\frac{\bar{\pi} W}{KM}\lvert M_1^+\rvert^2$ (μb/st)	$\frac{\mathrm{Re}(E_1^+)(M_1^+)^*}{\lvert M_1^+\rvert^2}$
1.19	0.0462	1.223	0.974	49.3±2.6	-40.0±3.6	------	-20.4±2.8	32	22.3	17.4±2.4	-.09±.03
1.21	0.0471	1.197	0.978	34.4±2.3	-22.1±3.1	------	- 8.4±2.7	41	58.1	9.1±2.0	-.12±.05
3.27	0.127	1.270	0.982	25.2±2.3	-17.9±2.7	-9.7±2.1	-18.3±2.7	58	53.2	10.7±1.8	+.01±.04
3.34	0.130	1.226	0.984	29.3±2.1	-13.1±2.6	-7.8±2.1	-21.8±2.5	55	54.0	13.3±1.6	+.06±.03
3.40	0.132	1.182	0.987	10.6±0.6	- 0.1±1.1	------	- 7.0±0.7	49	77.7	4.6±1.3	+.13±.05
6.16	0.240	1.321	0.984	12.7±0.8	- 7.5±1.1	-3.0±1.2	- 5.5±1.0	63	63.7	3.2±1.0	-.05±.05
6.24	0.243	1.284	0.985	19.0±0.8	-13.8±1.2	-2.8±1.1	- 7.5±1.0	61	80.0	5.1±1.2	-.10±.04
6.29	0.245	1.259	0.986	27.4±0.9	-18.1±1.4	-9.2±1.2	-10.7±1.1	66	52.2	7.4±1.4	-.08±.03
6.35	0.247	1.228	0.987	30.0±1.0	-19.1±1.7	-3.4±1.7	-13.8±1.3	59	48.0	10.1±1.2	-.04±.02
6.41	0.250	1.200	0.988	28.4±0.9	-15.0±1.5	-6.5±1.8	- 9.1±1.0	60	47.7	8.0± 1.4	-.06±.02
6.47	0.252	1.166	0.989	16.6±1.5	- 7.3±2.8	------	- 5.3±1.5	35	25.1	5.9±2.1	-.03±.05
6.55	0.255	1.132	0.990	7.1±1.1	- 3.7±2.4	------	1.3±1.0	26	23.8	-------	--------
10.22	0.398	1.279	0.978	24.8±1.5	-17.8±2.0	-3.1±1.3	-11.2±1.8	31	24.3	7.4±1.6	-.07±.04
10.37	0.404	1.234	0.980	39.3±3.2	-26.9±4.3	2.6±2.1	-11.0±3.1	26	44.7	9.8±2.0	-.13±.05

magnetic dipole amplitude, except for a factor of $\bar{q}^*$ (the c.m. photon momentum), which expresses the threshold dependence of the amplitude. From Eqs. (1) and (2), and Eq. (4) of Ash et al.[4] the γNN^* form factor is defined to be

$$G_M^*(q^2) = 2M\left[\frac{3}{2\alpha}\,\frac{\pi\Gamma}{\sin^2\delta}\,\frac{|M_1^+|^2}{|\bar{q}^*|^2}\right]^{1/2},$$

where $\Gamma(=120\text{ MeV})$ is the width of the resonance and δ is the $P_{3/2,3/2}$ phase shift.[8] This definition of the form factor is related to the matrix element μ^* of Dalitz and Sutherland[9] by the equation

$$G_M^*(0) = (M/W)^{1/2}\mu^*.$$

From an analysis of photoproduction data, Dalitz and Sutherland obtained $\mu^* = (1.28\pm0.02)2\times(\frac{2}{3})^{1/2}\mu_p$, where $\mu_p = 2.79$. Using this result, $G_M^*(0) = 2.93\pm0.05$. Ash et al. obtained $G_M^*(0) = 3.00\pm0.01$ by fitting the photoproduction data of Fischer et al.[10] These results are to be compared with the prediction of current algebra and SU(6) symmetry, $G_M^*(0) = 2.3$,[11] and the result of a recent current-algebra calculation by Barnes and Willimas,[12] $G_M^*(0) = 3.5$. The latter value is expected to be an overestimate of $G_M^*(0)$.

The form factor C_3 defined by Dufner and Tsai is related to G_M^* by the equation

$$G_M^*(q^2) = \left(\frac{2}{3}\right)^{1/2}\frac{M(M+W)}{W}\times\left[1+\frac{q^2}{(M+W)^2}\right]^{1/2}C_3(q^2).$$

The additional four-momentum-transfer dependence implied by the factor in parentheses differs from unity by less than 4.5 % at four-momentum transfers below 0.4 $(\text{BeV}/c)^2$.

The values for $G_M^*(q^2)$ have been determined by averaging over the resonance. Since the M_1^-, M_1^+ interference term changes sign at resonance, its effect on G_M^* was negligible at $q^2 = 0.13$ and 0.25 $(\text{BeV}/c)^2$. The correction due to this term raised the value of G_M^* by 4 % at $q^2 = 0.05$ $(\text{BeV}/c)^2$ and lowered it by 3 % at 0.4 $(\text{BeV}/c)^2$.

The values obtained for G_M^* are compared with the measurements of Ash et al. in Fig. 2. The agreement is generally good except for the lowest four-momentum-transfer point which seems rather high compared with the more precise photoproduction data. Also shown in the figure are the form-factor dependence obtained from an analysis of noncoincidence electroproduction data by Dufner and Tsai,[5] the prediction of the static theory of Fubini, Nambu, and Wataghin,[13] and the phenomenological form factor implied by the fully relativistic dispersion theory of Zagury.[6] The static-theory prediction that $G_M^*(q^2)$ is proportional to the nucleon magnetic isovector form factor is approximately correct in this region of four-momentum transfer, as concluded by Ash et al., but the data are also consistent with the exponential form-factor dependence suggested by Dufner and Tsai.

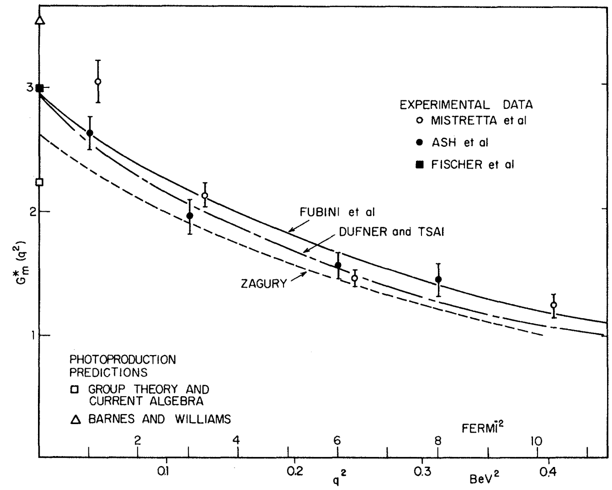

FIG. 2. The γNN^* transition form factor.

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[1]C. Mistretta et al., preceding Letter [Phys. Rev. Letters 20, 1074 (1968)].

[2]The notation used for the multipole amplitudes is that of N. Zagury, Phys. Rev. 145, 1112 (1966), and 150, 1406 (1966). The curve of Fig. 2 is taken from W. W. Ash, K. Berkelman, C. A. Lichtenstein, A. Ramanauskas, and R. H. Siemann, Phys. Letters 24B, 165 (1967).

[3]See, for example, F. A. Berends, A. Donnachie, and D. L. Weaver, Nucl. Phys. B4, 1 (1967).

[4]Ash et al., Ref. 2.

[5]A. J. Dufner and Y. S. Tsai, Stanford Linear Accelerator Center Report No. SLAC-PUB-364, 1967 (unpublished).

[6]Zagury, Ref. 2.

[7]The ratio $\text{Re}(M_1^-)(M_1^+)^*/|M_1^+|^2$ at $q^2=0$ can be obtained from Ref. 3. At the highest and lowest four-momentum transfers the use of this value led to a varia-

tion of M_1^+ across the resonance which was consistant with the behavior expected from the $P_{3/2,3/2}$ phase shift. However, at the two interintermediate points a more consistent behavior of M_1^+ was obtained when no correction was made for this term. We have therefore used one-half the correction at all four-momentum transfers and increased the errors appropriately.

[8]L. D. Roper, R. M. Wright, and B. T. Feld, Phys. Rev. 190, B138 (1965).

[9]R. H. Dalitz and D. G. Sutherland, Phys. Rev. 146, 1180 (1966).

[10]G. Fischer, H. Fischer, H. J. Kampgen, G. Knop, P. Schultz, and H. Wessels, in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, 1966 (University of California Press, Berkeley, Calif., 1967).

[11]Quoted by K. J. Barnes and R. M. Williams, Nucl. Phys. B3, 424 (1967).

[12]Barnes and Williams, Ref. 11.

[13]S. Fubini, Y. Nambu, and V. Wataghin, Phys. Rev. 111, 329 (1958). The curve of Fig. 2 is 7% lower than the approximate calculation of Ash et al. (Ref. 2).

DOLEN-HORN-SCHMID DUALITY AND THE DECK EFFECT*

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An extension to multiperipheralism is made of the Dolen-Horn-Schmid duality argument relating Regge poles to low-energy resonances. The Deck model is thereby interpreted as predicting the existence of the A_1, rather than as undermining experimental evidence for this resonance. It is shown in general that Dolen-Horn-Schmid duality permits a vast simplification in the calculation of multiple-production processes.

A remark of profound import for strong-interaction theory has been made by Dolen, Horn, and Schmid[1] in connection with finite-energy sum rules. They have observed that high-energy Regge behavior is consistent with low-energy resonance behavior only if extrapolation of the smooth Regge representation down to low energy gives a certain semilocal average over the resonance peaks. In other words what is usually called the "peripheral" approximation to a reaction amplitude must, without containing energy poles, in a rough sense represent the resonances. (The converse presumably is also true.) We refer to this startling notion as "Dolen-Horn-Schmid duality." Its implication for bootstrap theory is being pursued vigorously by many authors[2]; our object here is to suggest relevance to what has been called the "Deck effect."[3] We argue that the Deck peripheral model for a reaction such as $\pi N \to \rho\pi N$, explaining a peak in the final $\pi\rho$ mass spectrum without explicit insertion therein of a resonance, fails to imply the absence of a resonance. On the contrary, Dolen-Horn-Schmid duality means that when peripheral models of this kind predict large cross sections at low subenergies (the term "subenergy" is used to mean energy of a subsystem), there probably are resonances present. Such reasoning leads to enormous simplification of multiperipheral calculations.

The step needed to relate Dolen-Horn-Schmid to Deck is the extension of single peripheralism to double peripheralism. Deck's model for the above reaction, for example, is depicted in Fig. 1, corresponding to a double Regge-pole representation,[4] a representation supposed to have validity when both the πN and $\pi\rho$ final subenergies are large.[5] The highest trajectory for the right-hand momentum transfer is the Pomeranchuk; the highest for the left-hand momentum transfer is not the π, but the small mass of the physical pion enhances the Regge residue so that this tra-

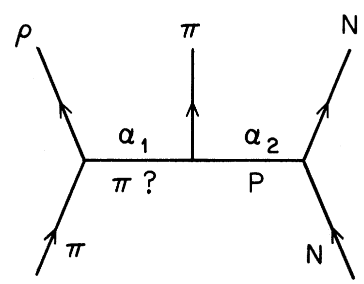

FIG. 1. Diagram representing the Deck doubly peripheral model for the reaction $\pi N \to \pi\rho N$.