FACTORIZATION, ω CROSSOVER, POLARIZATION, AND FINITE-ENERGY SUM RULES FOR KAON-NUCLEON SCATTERING

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KN finite-energy sum rules, with phase-shift analyses as input, are evaluated to determine the t dependence of the relevant Regge-exchange amplitudes. The spin nonflip contributions are similar to those deduced from high-energy Regge fits; however, the spin-flip contributions of the high-energy fits are inconsistent with the sum-rule results. The implications for polarization and K^+n charge exchange are discussed.

We have used phase-shift analyses of K^+p and K^-p elastic scattering to evaluate a set of finite-energy sum rules (FESR) and so determine the t dependence of the relevant Regge-exchange amplitudes. Assuming the ρ contribution to be known, our conclusions are the following: (i) For ω, there is some evidence of a zero in both the nonflip and flip amplitudes ImA' and ImB at $-t \sim 0.15$ (units: GeV = 1), as the usual crossover mechanism requires[1,2]; there is no evidence of a wrong-signature nonsense zero in B for $-t < 0.8$; $\nu B/A' \sim +1$ to $+3$ for $-t \lesssim 0.6$ in contradiction to high-energy fits which have taken this ratio to be either zero[1] or negative.[3] (ii) For R, $\nu B/A' \approx +10$, which is of opposite sign to that previously used in some high-energy fits.[1] (iii) For P and P', $\nu B/A' \sim +1$,[4] which is again of opposite sign to that in high-energy fits[1,3]; there is also evidence of a no-compensation mechanism for P'.[5] Our results support exchange degeneracy of ρ with R and P' with ω for the ratio of residues $\nu B/A'$, although the trajectories are not found to be degenerate.

The sum rules are generated by considering amplitudes $a(\nu, t, m)$, defined by

$$a(\nu, t, m) = (M/4\pi^2)(\nu_0^2 - \nu^2)^{\frac{1}{2}m} F(\nu, t), \qquad (1)$$

where $\nu = (s-u)/4M$ and $\nu_0 = \mu + t/4M$. $F(\nu, t)$ may be any amplitude from among $A'^{(-)}$, $\nu B^{(-)}$, $\nu A'^{(+)}$, and $B^{(+)}$, where the superscript denotes that the amplitude is half the sum (+) or half the difference (−) of the K^-p and K^+p amplitudes. We parametrize the energy dependence of F in the high-energy region with expressions of the form $\nu(\nu^2 - \nu_0^2)^{\frac{1}{2}[\alpha(t) - \delta]}$, where $\alpha(t)$ is the Regge trajectory and $\delta = 1$, 1, 0, and 2 for the above four amplitudes, respectively. Using analyticity to match the amplitudes evaluated below $\nu = \nu_1$ with the Regge parametrizations evaluated above ν_1, the set of generalized FESR takes the form[4,6]

$$\int_0^{\nu_1} d\nu \, \mathrm{Im}a(\nu, t, m) = \sum_i \frac{\mathrm{Im}a_i(\nu_1, t, m)}{\alpha_i(t) + m + 2 - \delta} \left(\frac{\nu_1^2 - \nu_0^2}{\nu_1} \right), \qquad (2)$$

where the sum is over all the relevant Regge-pole contributions, i.e., $P + P' + R$ for the (+) amplitudes and $\rho + \omega$ for the (−) amplitudes. We evaluate Eq. (2) for integral m from −2 to 3 at $t = 0$ and from 0 to 3 for $t \neq 0$. For even m, the sum requires $(-1)^{\frac{1}{2}m}$ ImF from the $\Lambda\pi$ threshold to ν_1 together with the Λ and Σ pole terms; for odd m, $(-1)^{\frac{1}{2}(m+1)}$ ReF is required in the region above the KN threshold, but ImF and the Λ and Σ poles are required in the region below this threshold. These sum rules have the advantage that by varying m, one may study, in addition, the phase of the asymptotic amplitude.

To determine the amplitudes as functions of t, we used phase-shift analyses for K^-p and K^+p scattering up to a matching energy $\nu_1 = 1.53$ GeV ($\sqrt{s} = 2$, $p_{\mathrm{lab}} = 1.46$). For K^+p scattering, Lea, Martin, and Oades[7] have found several solutions in this region; we used a solution of type I which suggests an inelastic P_{11} resonance and also a nonresonant solution of type IV (solutions of type II gave amplitudes very similar to type I, while type III is not favored by the authors[7]). For K^-p scattering, Kim[8] has analyzed the data from threshold to 550 MeV/c using a K-matrix effective-range parametrization for the partial-wave amplitudes. For the range 780-1220 MeV/c, Armenteros et al.[9] have a preliminary analysis using a background-plus-resonance parametrization. Lacking any better procedure, we extrapolated their energy-dependent fits to the region 550-1460 MeV/c and confirmed that they still reproduced the K^-p total cross sections. This extrapolation is invalid for each partial wave separately since some exceed the unitarity limit; but for the total amplitudes, which are resonance dominated, we believe it to be a fair approximation. Furthermore, this solution agrees approximately with the K^-p polarization from 1100 to 1350 MeV/c of Cox et al.[10] For the unphysical region in K^-p scattering, we use the extrapolation of Kim's solution although we allow the $Y_1^*(1385)$ coupling to have its broken-SU(3) value[11] as well as the almost negligible value found by Kim. For the Λ and Σ couplings we use Kim's values of 13.5 and 0.3 or, alternatively, Zovko's of 5.7 and 1.7.[12]

The K^-n analysis of Armenteros et al.[9] is less reliable since it does not reproduce the total cross sections very well, while the only K^+n analysis is from 0 to 813 MeV/c, so that we cannot separate the isospin contributions further by appealing to the Kn system. However, within the approximation of resonance saturation, one may separate the four classes of Regge poles, and Igi and Matsuda have done this for R and ρ.[13] Resonance approximation implies that $\mathrm{Im}(K^+p) = \mathrm{Im}(K^+n)$; so $\mathrm{Im}\,''(\rho-R)'' = 0$ and hence ρ and R contributions become confused with each other. For the ω, the resonance approximation assumes that the average of K^+p and K^+n amplitudes is the same as the average K^-p and K^-n resonance backgrounds, which approximation we believe to be more reliable. Using all the known Y^* resonances of mass less than 2.2 GeV/c^2, we have evaluated the sum rules for $A'\omega$, $\nu B\omega$, $\nu^2 A'\omega$, and $\nu^3 B\omega$.

A selection of the results of evaluation of the sum rules are shown in the figures. At $t=0$, we plot the results in Fig. 1 of evaluating the left-hand side of Eq. (2) against m, which provides a clear insight into the relative importance of different moments m and into the phase. The smooth curves represent extrapolations to 1.53 GeV of Regge high-energy fits to the 6- to 20-GeV data. For $\nu A'^{(+)}$ and $A'^{(-)}$ the Phillips and Rarita parameters[1] show good agreement, if one remembers that a confrontation of data below 1.53 GeV and above 6 GeV is being presented. For even m, these sum rules could be evaluated directly by using total cross sections when a higher matching energy might be employed.[14] $A'^{(-)}$ for $m=2$ is the forward dispersion relation evaluated by Kim, and it is much more sensitive to low-energy data and coupling constants than to the Regge contribution. Indeed, we regard the sum rules for $m=-2$ and -1 as only providing a consistency check on our data set. For $\nu B^{(-)}$, the Phillips and Rarita solutions have $\nu B/A' = 0$ for the ω and +11 for the ρ, and these fit with our results approximately. Reasonable agreement for $B^{(+)}$ is obtained only by using the more recent result from $\bar{K}N$ charge-exchange fits that $\nu B/A' = +8.3$ for the R^{15} (rather than approximately -9),[1] and also the πN FESR result[4] that $\nu B/A' = +1$ for P and P'. Figure 1 also shows that there is some inconsistency in our data set, since one can see that our $B^{(+)}$ and $\nu B^{(-)}$ results for $m=-2$ cannot be attributed to lower lying Regge contributions—but imply some error in the P waves near threshold or in the unphysical region, etc. Subject to this uncertainty, these sum rules, as well as those at $t \neq 0$ to be described below, allow us to conclude that the most favored data set is with Kim's coupling constants and negligible $Y_1^*(1385)$ coupling, together with

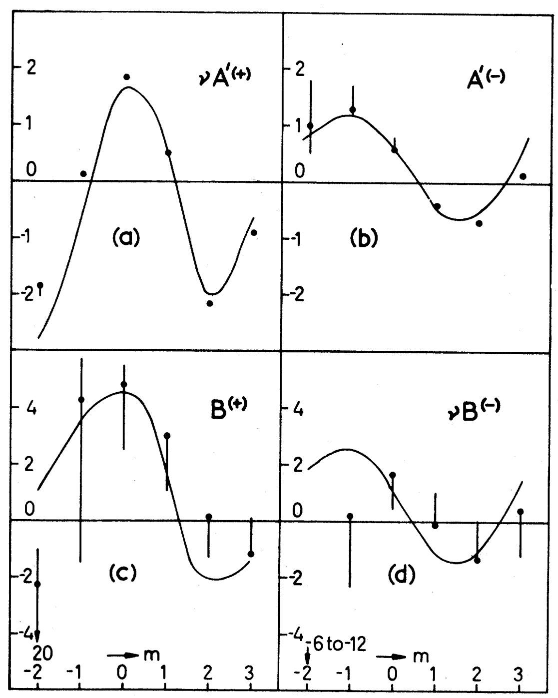

FIG. 1. (a)-(d) Evaluation of Eq. (2) (in units GeV $=1$) at $t=0$ for different moments m and amplitudes F. The points are for Kim's coupling constants and unphysical region plus a nonresonant K^+p solution, with the error bars showing the extent of the values obtained using the other choices discussed in the text. The continuous curves represent the extrapolations to 1.53 GeV of high-energy fits as described in the text.

the nonresonant K^+p solution. We would emphasize that one should consider several independent sum rules in determining the coupling constants because, for instance, the $A'^{(-)}$ forward dispersion relation is equally well satisfied by Zovko's coupling constants plus an SU(3) $Y_1^*(1385)$ coupling.

For the t-dependent sum rules we use the difference between different data sets to estimate the errors, although the results of our favored set are also indicated in Fig. 2. We pay attention to the t dependence of only those sum rules which agree reasonably at $t=0$. For $\nu A'^{(+)}$ and $m=0$ we find slight evidence of a dip at $-t\approx 0.5$, and this agrees with the conclusions of Barger and Phillips using πN sum rules.[4] The P' trajectory is less strongly coupled in KN scattering than in πN and, indeed, we find a less pronounced dip, so that this is consistent with its interpretation as a double zero in the P' contribution due to the no-compensation mechanism.[5] The R trajectory is masked by the P and P' except in $B^{(+)}$ for $m=1$, where real parts are involved. In this case, we find no evidence of a zero for $-t \leqslant 0.8$, so that a Chew-mechanism nonsense zero is excluded. A Gell-Mann nonsense-choosing zero, or else no zero of $\alpha(t)$ in this range, are both possible. Exchange degeneracy with the ρ would suggest that the R trajectory passed through zero for $-t\sim 0.5$, while fits to η production[1] tend to favor solutions with a flatter trajectory.

For the $(-)$ amplitudes we assume that the ρ contribution is given correctly by the K^-p charge-exchange fits of Derem and Smadja,[15] who incorporate factorization constraints. Then we may subtract this contribution as shown in Figs. 2(c) and 2(d) to investigate the ω. Turning to $A'^{(-)}$, which should be dominated by ω, we find solutions using Kim's coupling constants in which the imaginary part changes sign for $-t\approx 0.2$ as is needed to explain the crossover phenomena. However, there is some dilution by lower lying Regge poles or else the data are not sufficiently reliable, since we find no corresponding zero either in Re$A'^{(-)}$ for $m=+1$ which is quite constant with t, or in the $m=2$ moment of $A'^{(-)}$. The resonance approximation shown in Fig. 2(e) agrees well with the crossover phenomenon, as does the higher moment relation for $\nu^2 A'^{(-)}$. For Im$B^{(-)}$ one has a similar behavior after subtracting the ρ contribution with use of the K^+p nonresonant solution, and this is again confirmed by the resonance result of Fig. 2(f) and by its higher moment. The $m=1$ sum rule for Re$B^{(-)}$ is very

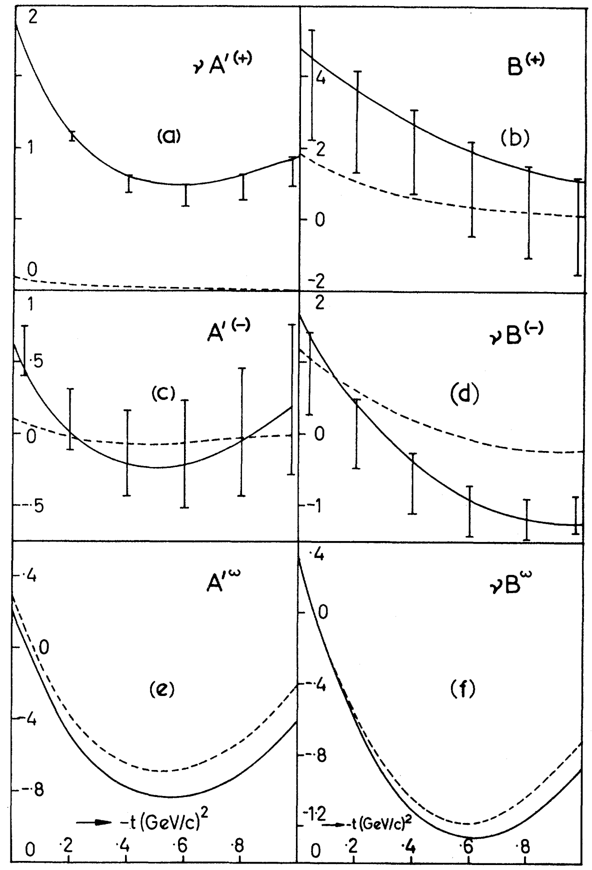

FIG. 2. (a)-(d) Evaluation of Eq. (2) (in units GeV $=1$), with moment $m=0$, for different momentum transfers t and amplitudes F. The continuous curve is for our favored data set with the error bars showing the extent of the values with the other choices. For the (+) and (−) amplitudes, respectively, the R and ρ contributions from Derem and Smadja, extrapolated to 1.53 GeV, are shown by the dotted lines. (e)-(f) Evaluation of the ω part of the sum rules shown in (c) and (d) using resonance saturation with both SU(3) (solid line; $f=0.36$) and Zovko (dotted-line) couplings for the pole terms.

sensitive to the data and is inconclusive. If it were not for the lack of a zero in Re$A'^{(-)}$ as determined by the sum rules, we should be unreserved about our confirmation of the usual ω crossover mechanism of one pole with all residues passing through zero at $t=-0.13$ because of factorization.[2,3] We find no evidence in the sum rules of any additional zero (sense-nonsense zero) in $B\omega$ for $-t\leqslant 1$, which is inconsistent with $\alpha_\omega = 0.45+9t$, found by Contogouris et al.[16] from an analysis of $\pi N\to\rho N$; our results tend to favor a flatter trajectory. For our ω contribution, $\nu B/A'\approx +1$–$+3$ for $0<-t\lesssim 0.7$, and this is in qualitative agreement with ω dominance of the isoscalar nucleon form factor which leads to $\nu B/A' \approx 0.5$ at $t = m_\omega^2$. In their fit to NN data, Rarita et al.[3] here used $\nu B/A' \approx -6$ for the ω as the predominant spin-flip contribution to fit the pp polarization data. This model is in conflict with the $\bar{p}p$ polarization data of Daum et al.[17] at 2-3 GeV/c, which lends support to the conclusion that additional important spin-flip contributions must be included in NN scattering.

Our determination of the signs and t dependence of the Regge-pole spin-flip contributions in KN scattering allows us to predict $K^\pm p$ polarization with some confidence. We find that the K^-p polarization should be larger than K^+p polarization for $-t \gtrsim 0.3$ and smaller below, both being positive at least up to $-t \approx 1$. The K^-p polarization data of Daum et al.[17] from 1.4 to 2.3 GeV/c agree with the sign of our predictions and also with the magnitude, being about 40% at $-t \sim 0.3$ and about 100% at $-t \sim 0.6$, whereas high-energy fits have tended to predict a negative polarization. Another source of difficulty in the intermediate energy region has been the K^+n charge-exchange data at 2.3 GeV/c discussed by Rarita and Schwarzschild,[18] who found that conventional Regge fits gave only half the differential cross section needed in the peak region. This process is spin-flip dominated with ρ and R contributing; the sign change of the R spin flip is enough to increase the predictions by about 50% for $-t \sim 0.2$ without the need to introduce a ρ' contribution.

Our analysis shows that finite-energy sum rules provide a very useful and reliable insight into Regge-pole parameters. With more accurate phase-shift analysis, one would be able to investigate the properties of lower lying Regge poles also. Further details of our analysis will be published elsewhere. We are grateful to Dr. R. J. N. Phillips for his interest and discussions.

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