## ELECTRICAL RESISTIVITY OF NICKEL AT LOW TEMPERATURES\*

F. C. Schwerer<sup>†</sup> and J. Silcox<sup> $\ddagger$ </sup> Department of Engineering Physics, Cornell University, Ithaca, New York (Received 27 November 1967)

Measurements of the temperature dependence of the electrical magnetoresistivity of a nickel polycrystal are analyzed to find the resistivity at zero magnetic induction as a function of temperature. It is seen that quantitative differences occur between these data and data taken at zero applied field.

In a recent Letter,<sup>1</sup> White and Tainsh have reported measurements of the temperature dependence of the electrical and thermal resistivities of high-purity nickel which have been used to deduce the dominant, thermally excited, electron-scattering processes.<sup>2</sup> There is ample experimental evidence<sup>3</sup> that the appropriate field acting on the conduction electrons is the magnetic induction  $B(=H+4\pi M_s)$ and not the applied field  $H$ . Accordingly experimental electrical resistivity data taken at H  $= 0$  include a term due to the normal magnetoresistance effect of the spontaneous magnetization<sup>4</sup>  $4\pi M_s$  (= 6.4 kG in nickel<sup>5</sup>). Therefore for comparison with theory the data should be analyzed to find the resistivity at  $B=0$ , i.e.,  $\rho(B=0, T)$ . We have developed an analysis of magnetoresistance data which enables us to find  $\rho(B=0)$  unambiguously. In this Letter, we apply the analysis to the thermal component of the electrical resistivity of a nickel sample and find that the coefficient of the  $T^2$  term in  $\rho(B=0, T)$  is significantly altered from the corresponding value in  $\rho(H=0, T)$ . Similar effects in other electronic transport processes, such as thermal conductivity, may reasonably be expected.

The effect of a magnetic field on the electrical resistivity can often be described by Kohler's rule<sup>6</sup>  $[\rho(B)-\rho_0]/\rho_0 = F(B/\rho_0)$ , where  $\rho_0$  $= \rho(B = 0, T)$  in the present experiments is the resistivity due to the thermal scattering processes. Difficulty arises because  $\rho(B, T)$  can be measured only for  $(9 \text{ kG})/\rho_0 < B/\rho_0 < (24.5$  $kG)/\rho_0$ , where the upper limit is instrumental and the lower limit is the field needed to remove the ferromagnetic domain structure. If  $F(B/\rho_0)$  is known, then  $\rho_0$  could be found by adjusting  $B/\rho_0$  until the magnetoresistance matches the appropriate section of  $F$ . Usually  $F$ is not known and we illustrate the analysis in that case with experimental data on the temperature dependence of the magnetoresistance of a nickel sample of relatively high purity.

Kohler's rule is rewritten in the form  $\kappa(T)\rho(B,$  $T = \rho^* \left[1 + F(\kappa B/\rho^*)\right]$ , where  $\kappa(T) = \rho^* / \rho_0$  now characterizes the scattering-process resistivity of the sample. We note that  $\kappa \rho \rightarrow \rho^*$  as  $\kappa B$  $\div$  0. For data taken at T=17°K,  $\kappa$  is chosen arbitrarily as 1.00. Data taken at 20K are also shown in Fig. 1 for values of  $\kappa = 1.00$  and 0.726. The second value of  $\kappa$  was chosen to make any one point of the 20'K data lie on the same curve as the 17°K data. If Kohler's rule is obeyed then not only will the points match but so will the slopes. The 20'K data will form a smooth continuous curve with the  $17^{\circ}$ K data, as is indeed observed, and the value of  $\kappa$  obtained will be unique. Provided there is sufficient overlap between data at neighboring temperatures, this analysis can be extended to map out the function  $p^*(1+F)$  over as large a range of  $\kappa B$  as proves feasible. In Fig. 1,  $\rho^*(1+F)$  is continuous over two decades of  $\ln \kappa B$ corresponding to  $17^{\circ}$ K < T < 64 $^{\circ}$ K. At the hightemperature, high-resistance end,  $\kappa \rho$  tends asymptotically to a constant value identified as  $\rho^*$ . From this value, good to a few percent, and the values of  $\kappa(T)$ , values of  $\rho(B = 0, T)$ can be obtained. Smooth curves are obtained only when  $B(=H+4\pi M_{S})\pm500$  G is used. This has proved to be true over a wide range of experiments.<sup>8,9</sup> and is considered as further evidence that  $B$ , not  $H$ , is the field acting on the conduction electrons. Success in obtaining this curve implies that Kohler's rule is obeyed experimentally for thermal scattering in nickel and that the corresponding values of  $\rho(B=0, T)$ can be determined.

Data were taken over a range of temperatures,  $1.4\textdegree$ K <  $T$  < 150 $\textdegree$ K, and the procedure outlined above was found to break down below 17'K and above  $64^{\circ}$ K. Below  $17^{\circ}$ K, the shape and slope of the magnetoresistance changed markedly and, at the lowest temperatures, became much less temperature dependent suggesting that the resistance was becoming impurity dominated. One criterion for the validity of Kohl-



FIG. 1. Transverse magnetoresistivity of nickel in the form of  $\kappa \rho$  as a function of  $\kappa B$  (see text). The bars denote the range over which data could be taken at each of the following temperatures: open squares, 1.4°K; hexagons, 12'K; closed circles, 17'K; open circles, 20'K; triangles, 23.6'K; and closed squares, 64'K. The long bar designates the data for eight temperatures between 24 and  $64^{\circ}\text{K}$ . For each temperature,  $\kappa$  has been chosen to adjust the data to the continuous curve. In addition, for pedagogical reasons, the 29°K data are shown with  $\kappa = 1.00$ . The dashed curve is a similar plot obtained by varying the concentration of chromium in nickel.

er's rule, suggested by Chambers,<sup>10</sup> is that the scattering should be dominated by one relaxation time, i.e., by one type of scattering center. This suggests that from 17 to  $2^\circ K$ , the impurity and thermal scattering are of comparable magnitudes, that Kohler's rule breaks down, and that it is impossible to find  $\rho(B = 0)$ , T). At  $1.4\textdegree K$ , the thermal scattering has diminished sufficiently that the impurity scattering now dominates. The problem of impurity scattering can evidently be treated by experiments similar to these described here. Such experiments at 4.2°K have been carried out<sup>9</sup> on nickel doped with iron, chromium, manganese, cobalt, and carbon. In analysis of these data,  $\kappa(c)$  and  $\rho_0(c)$  are now functions of the concentration of impurities  $c$  but in all other respects the analysis is as given above. Each impurity gives rise to a Kohler function  $F$  which is characteristic and unique for a given impurity, demonstrating that in nickel Chambers' criterion ' is a necessary one. We have compared the impurity magnetoresistance, i.e.,  $\rho(B, 1.4 \text{°K})$ , of the specimen discussed here with the Kohler functions corresponding to these particular impurities and find that the specimen corresponds closest to the chromium-doped series with a residual resistance  $\rho_i(B=0)$  of  $2.4\;{\rm n}\Omega\;{\rm cm},\;{\rm i.e.,}\;$  a residual resistance ratio  $\rho_{273}/\rho_{4.2} \sim 2600$ . The equivalent  $H=0$  ratio is 1400. Reasonable agreement was also found with the carbon-doped series but not with the others. These comparisons serve to give a

precision for  $\rho_i(B=0)$  of ~5%. The curve for the chromium-doped series together with the magnetoresistance of the present specimen at  $1.4\textdegree$ K is shown in Fig. 1 and illustrates the quality of the agreement. The deviations from the continuous curve which occur above 65'K are due to the increased importance of what are due to the increased importance of what<br>is probably spin-disorder scattering.<sup>11</sup> This is manifest as a decrease in resistivity with increasing field corresponding to spin ordering by the applied field. The source for scattering, i.e.,  $\rho_0$  is now a function of  $H$  and the field dependence of the total resistivity does not obey Kohler's rule.

Both  $\rho(B = 0, T)$  and  $\rho(H = 0, T)$  were analyzed in terms of the customary  $\rho(T) = \rho(1.4\text{°K}) + aT^{\hat{p}}$ +  $bT^q$ . Good fits were obtained for  $16^\circ K < T$  $<50^{\circ}$ K for  $p=2$ ,  $q=4$  as illustrated in Fig. 2. For  $\rho(H=0, T)$  the coefficients were  $a_H=14$  $\times 10^{-3}$  n $\Omega$  cm  $({}^{\circ}\text{K})^{-2}$  and  $b_H$  = 1.6 $\times 10^{-5}$  n $\Omega$  cm  $({}^\circ{\rm K})^{-4}$  in close agreement with Greig and Harrison<sup>12</sup> ( $a_H$ =15.5×10<sup>-3</sup>,  $b_H$ =1.7×10<sup>-5</sup>). However,  $\rho(B=0, T)$  was best given by  $a_B = 9.5 \times 10^{-3}$ and  $b_{\mathbf{B}} = 1.7 \times 10^{-5}$ . Thus analyzing for the normal magnetoresistance effect results in a significant change (~30%) in the coefficient of  $T^2$ . As pointed out by Harrison,<sup>13</sup> our data are consistent with his in predicting a substantial contribution from the  $T^4$  term even at 15 $\mathrm{K}$ . Good fits were also obtained with  $p = \frac{3}{2}, q = 4$ . The theoretical justification for using expressions of this nature to fit the data rests on identifying the  $T^{\rho}$  and  $T^{\rho}$  terms with different scat-



FIG. 2. The resistivity of nickel at zero applied field and zero induction shown as the function  $\rho(T)$  $= \rho (1.4) + aT^2 + bT^4$ ,

tering mechanisms, e.g., with electron-electron or electron-magnon scattering and electron-phonon scattering.<sup>14</sup> Thus we might expect two scattering processes and have no right to expect Kohler's rule to hold. The fact that it does hold suggests that possibly one single, thermally excited scattering mechanism with a complicated temperature dependence is dominant. Alternatively, both scattering processes are similar functions of the electron wave vectors.

The transition region from impurity to thermally dominated scattering  $\sim$  2 to 17<sup>°</sup>K in this study) is a temperature span in which the scattering-process resistivity cannot accurately be obtained. Similar studies on a substantially higher purity sample would enable the analysis to be made at lower temperatures with more sensitivity in differentiating between linear,  $\frac{3}{2}$ , and quadratic dependences. A glance at Fig. 2 reveals that the discrepancy between  $B=0$  and  $H=0$  data increases as the temperature decreases. Significant quantitative data will therefore only be obtained if care is taken to eliminate the magnetoresistance affects. Great care should also be taken to eliminate any similar effects in estimating the electronic contributions to other transport processes such as the thermal conductivity.

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)Present address; Edgar C. Bain Laboratory for Fundamental Research, United States Steel Corporation, Monroeville, Pa.

f.John Simon Guggenheim Memorial Fellow on leave at Service de Physique des Solides, Faculté des Sciences d'Orsay, 91, Orsay, France.

 ${}^{1}$ G. K. White and R. J. Tainsh, Phys. Rev. Letters 19, 165 (1967).

 $\overline{12}$ C. Herring, Phys. Rev. Letters 19, 167 (1967).  ${}^{3}$ See, e.g., J. R. Anderson and A. V. Gold, Phys. Rev. Letters 10, 227 (1963).

 ${}^{4}$ L. Berger and A. R. deVroomen, J. Appl. Phys. 36, <sup>2777</sup> (1965); A. I. Schindler and B. C. LaRoy, J. Appl. Phys. 37, 3610 (1966).

<sup>5</sup>R. M. Bozorth, Ferromagnetism (D. Van Nostrand Company, Inc., Princeton, N. J., 1951), p. 270. See also L. Hodges, D. R. Stone, and A. V. Gold, Phys. Rev. Letters 19, 655 (1967).

<sup>6</sup>See, e.g., J. M. Ziman, Electrons and Phonons (Clarendon Press, Oxford, England, 1960), p. 490.

To be precise, a nickel specimen at  $H=0$  contains ferromagnetic domains in an arrangement depending on magnetic history. Since the resistance depends on the orientation between  $4\pi\tilde{M}_{\rm s}$  and the electric current  $\overline{J}$ , the measured resistance at  $H=0$  is a complicated function of the domain arrangement. We report here only data pertaining to a saturated specimen for which  $\tilde{H} \parallel \tilde{B}$  and for which  $\tilde{J} \perp \tilde{H}$ .

<sup>8</sup>F. C. Schwerer and J. Silcox, Cornell Materials Science Center Report No. 739 (to be published).

 ${}^{9}F.$  C. Schwerer and J. Silcox, unpublished.

 $^{10}$ R. G. Chambers, Proc. Roy. Soc., Ser. A 238, 344 (1956).

<sup>11</sup>See, e.g., S. Chikazumi, Physics of Magnetism

(John Wiley & Sons, Inc., New York, 1964), p. 419.  $^{12}$ D. Greig and J. P. Harrison, Phil. Mag. 12, 71 (1965).

 $^{13}$ J. P. Harrison, private communication.

<sup>14</sup>See, e.g., Ref. 6, pp. 376 and 415.

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