Swihart, Bull. Am. Phys. Soc. 10, 7 (1965)] and was in agreement with Nam's calculation for σ_{S1}.

[3]D. C. Mattis and J. Bardeen, Phys. Rev. 111, 412 (1958).

[4]R. A. Ferrell and R. E. Glover, III, Phys. Rev. 109, 1398 (1958).

[5]S. B. Nam, unpublished. (Work referred to in Ref. 1.)

[6]W. L. McMillan and J. M. Rowell, Phys. Rev. Letters 14, 108 (1965).

[7]D. J. Scalapino, J. R. Schrieffer, and J. W. Wilkins, Phys. Rev. 148, 263 (1966).

[8]W. L. McMillan and J. M. Rowell have given the result of their complex $\Delta(\omega)$ from the same α^2F in their article in a "Treatise on Superconductivity," edited by R. D. Parks (to be published), and the agreement of the two is quite good.

[9]J. M. Rowell and W. L. McMillan, in Proceedings of the Tenth International Conference on Low Temperature Physics, Moscow, U. S. S. R., 31 August-6 September 1966 (to be published). See also Ref. 8.

LINEAR-$\vec{k}$ VALENCE-BAND SPLITTING IN InSb

C. R. Pidgeon

Francis Bitter National Magnet Laboratory,* Massachusetts Institute of Technology, Cambridge, Massachusetts

and

S. H. Groves†

Lincoln Laboratory,‡ Massachusetts Institute of Technology, Lexington, Massachusetts

(Recevied 11 March 1968)

In this Letter we report the observation of an interband magneto-optical transition in InSb caused by the linear-$\vec{k}$ term in the valence-band energy $E(\vec{k})$. An unambiguous assignment is made by a study of the anisotropy of the spectra [with the magnetic field $\vec{H}$ in the $(1\bar{1}0)$ crystal plane] using left- and right-circularly polarized light. From the relative strength and position of this transition we determine the size of the linear-$\vec{k}$ term. A similar procedure is used to measure the warping of the InSb valence band, a quantity over which there is currently some disagreement.

The theory of the inversion asymmetry terms in zinc-blende crystals has been known for some time.[1,2] These terms—the lowest order are k and k^3 terms—result from the antisymmetric potential or inversion asymmetry of the zinc-blende lattice. Combined with the spin-orbit interaction, they split the twofold energy degeneracy at a given $\vec{k}$ value. Because the splittings are small, they have been difficult to observe experimentally. Recently, it has been suggested that a beat frequency in the Shubnikov-de-Haas effect in n-type HgSe[3] gives the conduction-band inversion-asymmetry splitting,[4] and the same effect is apparently seen in n-type GaSb.[5] This has not been useful for determining the size of the linear-$\vec{k}$ term which splits the valence band.[6] Evidence of the linear-$\vec{k}$ splitting has been given recently by Robinson[7] from microwave cyclotron-resonance experiments in p-type InSb. However, difficulties in the interpretation of this type of experiment resulting from poor resolution of the lines ($\omega_c\tau \sim 0.9$) and the sensitivity to strain effects make it desirable to have independent determinations. We find the linear-$\vec{k}$ term to be about three times smaller than suggested in Ref. 7.

The physical explanation of "extra" transitions due to the linear-$\vec{k}$ and warping interactions is not difficult to understand and we discuss this before going into the theoretical formalism. The calculation of magnetic energy levels in InSb has been carried out by Pidgeon and Brown,[8] by an extension of the method used by Luttinger and Kohn for the Ge valence band.[9,10] Energy levels calculated by the method of Ref. 8 (to be discussed more fully below) are shown in Fig. 1 for the top of the valence band and the bottom of the conduction band. To diagonalize the magnetic Hamiltonian appropriate to this problem it is necessary to omit inversion-asymmetry terms and some of the valence-band warping.[11] The solid lines in Fig. 1 show the optical transitions which can take place in the Faraday configuration ($\vec{E} \perp \vec{H}$): Electrons can be excited from the two "a"-set valence-band ladders into the $m_J = \frac{1}{2}$ conduction sub-band, or from the two "b"-set ladders into the $m_J = -\frac{1}{2}$ conduction sub-band. Usually the inversion-asymmetry and warping terms omitted from the magnetic Hamiltonian cause negligible error. However, if two levels which interact through the omitted terms are nearly degenerate in energy, considerable admixing of wave functions can take place. This in turn can cause observable extra transitions. In

this Letter we are concerned with extra transitions to the lowest conduction-band levels caused by the linear-$\vec{k}$ and warping interactions represented by C and $(\gamma_3-\gamma_2)$, respectively, in Fig. 1.

The full 4×4 effective-mass matrix Hamiltonian to order k^2 for the light- and heavy-hole valence bands of a zinc-blende semiconductor can be written as[12]

$$D = D^+ + D^-, \tag{1}$$

where D^+ is the even part given by Luttinger,[10]

$$D^+ = -m^{-1}[(\gamma_1+\tfrac{5}{2}\gamma_2)\tfrac{1}{2}k^2-\gamma_2(k_x^2J_x^2+k_y^2J_y^2+k_z^2J_z^2)-2\gamma_3(\{k_x,k_y\}\{J_x,J_y\}+\text{cyclic permutations})$$
$$+(e/c)\kappa\vec{J}\cdot\vec{H}+(e/c)q(J_x^3H_x+J_y^3H_y+J_z^3H_z)], \tag{2}$$

and D^- is the odd part (zero for materials with inversion symmetry)

$$D^- = -(2/\sqrt{3})C[k_x\{J_x,V_x\}+k_y\{J_y,V_y\}+k_z\{J_z,V_z\}]. \tag{3}$$

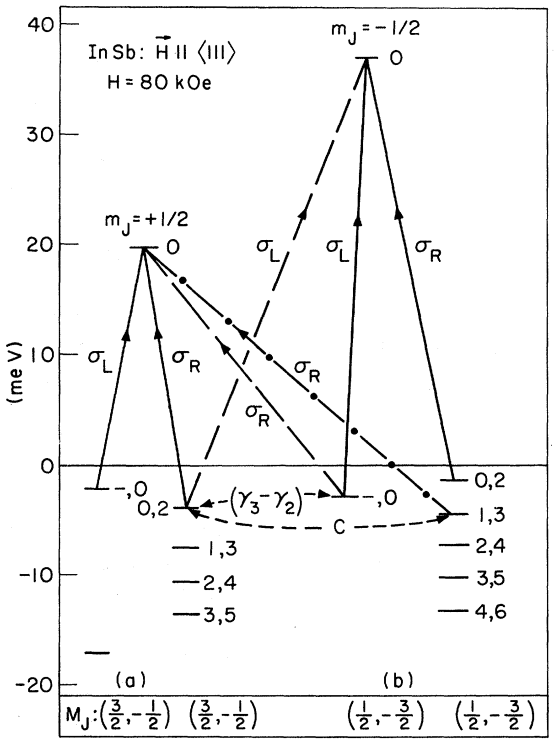

FIG. 1. Lowest magnetic energy levels for the valence and conduction bands of InSb. The numbers by the levels in the a and b valence ladders are the Landau quantum numbers n for the two component states. The corresponding total angular-momentum quantum numbers M_J are given below. Allowed and extra transitions for the σ_L ($\Delta M_J=+1$) and σ_R ($\Delta M_J=-1$) spectra are shown: solid line with arrow, allowed; dashed line with arrow, warping induced; dot-dashed line with arrow, inversion-asymmetry induced.

The notation is defined in Refs. 10 and 12. γ_1, γ_2, γ_3, κ, and q are the effective-mass parameters for a Ge-type semiconductor; C is the additional constant describing the linear-$\vec{k}$ interaction. The J_i are 4×4 angular-momentum matrices for $J=\frac{3}{2}$. $\vec{k}$ is the kinetic momentum operator $(\vec{p}+e\vec{A}/c)$, where $\vec{A}$ is the vector potential of the external magnetic field. The quantity $\{a,b\}\equiv\frac{1}{2}(ab+ba)$, $V_x\equiv J_y^2-J_z^2$.

We consider first the effective-mass equation corresponding to the even part, D^+, of Eq. (1). We put $k_3=0$ [where $\vec{H}$ is along the "3" direction in the (1, 2, 3) coordinate system], because it is the $k_3=0$ states which are important for the onset of interband optical transitions. The effective mass equation is then exactly soluble only for $\vec{H}\parallel\langle111\rangle$. In Ref. 8, D^+ is split into two parts for $\vec{H}$ in the $(1\bar{1}1)$ crystal plane:

$$D^+ = D_0 + D_1. \tag{4}$$

The principal anisotropy is included in D_0 which is treated exactly. The general 4×4 equation for D_0 decouples into two 2×2 equations, giving rise to two ladders (a and b) of light- and heavy-hole levels. The lowest levels from the solution of D_0 in the a and b set of the valence band, and the first spin-up and spin-down levels of the conduction band, are those shown in Fig. 1, for $\vec{H}\parallel\langle111\rangle$. As mentioned earlier, allowed transitions are shown for the Faraday configuration (left- and right-circularly polarized light, σ_L and σ_R).

The explicit form of D_1 is given in Ref. 10, Eq. (87); in particular, we are interested in the term r_3. This is proportional to the warping parameter $(\gamma_3-\gamma_2)$ and causes an interaction between certain closely spaced levels of the a and b ladders for $\vec{H}\parallel\langle111\rangle$ and $\langle211\rangle$ but not for $\vec{H}\parallel\langle100\rangle$ or $\langle110\rangle$. This interaction between two nearly degenerate states is shown in Fig. 1.

The resulting admixing of wave function causes an extra transition to each of the two lowest conduction-band levels, shown by the dashed lines in Fig. 1 and labeled w on the experimental $\langle 111\rangle$ magnetoreflection trace in Fig. 2. We have also verified that these transitions are seen for $\vec{H} \parallel \langle 211\rangle$ and are absent for $\vec{H} \parallel \langle 110\rangle$.

With the source of these transitions recognized, it is desirable to go to the eigenvalue problem for $\vec{H} \parallel \langle 111\rangle$ where the warping can be treated exactly. The solution to the effective-mass equation $D^+\langle 111\rangle f = \epsilon f$ is

$$f_n^{\,s} = \begin{pmatrix} A_{3,n}{}^{s}\Phi_n \\ A_{5,n}{}^{s}\Phi_{n+2} \\ A_{6,n}{}^{s}\Phi_{n-2} \\ A_{4,n}{}^{s}\Phi_n \end{pmatrix} \quad \text{for } n \geq 2, 3, \cdots, \tag{5}$$

where s runs over the four ladders and Φ_n is the harmonic-oscillator function of quantum number n. The subscripts on the eigenvectors A are written to conform with the notation of Ref. 8. They refer to the associated band-edge cell periodic functions in the (J, M_J) representation: $u_3^{3/2}, u_5^{-1/2}, u_6^{1/2}, u_4^{-3/2}$. For $n = -1, -2$, $A_3 = A_6 = A_4 = 0$; for $n = 0, 1$, $A_6 = 0$. Using the band parameters given in Ref. 8 we have diagonalized $D^+\langle 111\rangle$ numerically, making fine adjustments on the higher band parameters [and in particular the warping term $(\gamma_3 - \gamma_2)$] to achieve a good fit to both the relative positions and strengths of the allowed and warping-induced transitions found experimentally.

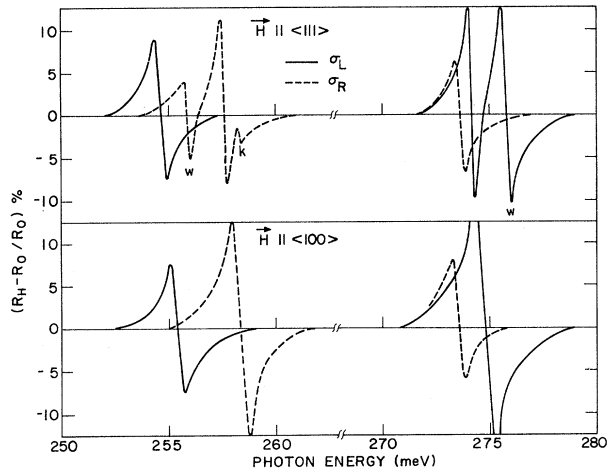

FIG. 2. Magnetoreflection spectra for $\vec{H} \parallel \langle 111\rangle$ and $\vec{H} \parallel \langle 100\rangle$ in the Faraday configuration, with $H = 84$ kOe and $T = 1.5$°K. Transitions to the $m_J = \frac{1}{2}$, $n = 0$ conduction-band level occur to the left-hand side of the break in the energy scale and those to the $m_J = \frac{1}{2}$, $n = 0$ conduction-band level occur to the right-hand side. w labels warping-induced transitions and k labels the linear-$\vec{k}$-induced transition.

The explicit form of D^- from Eq. (3) has been given for $\vec{H} \parallel \langle 001\rangle$.[1] For $\vec{H}$ in any direction in the $(1\bar{1}0)$ crystal plane we have made the coordinate transformations given in Ref. 10. The details of this are given elsewhere.[13] It is found that for $\vec{H} \parallel \langle 111\rangle$ and $\langle 211\rangle$ a strong interaction is possible between nearly degenerate states, shown by C in Fig. 1. This gives rise to an additional σ_R transition to the $m_J = \frac{1}{2}, n = 0$ conduction-band level (shown as a dot-dashed line in Fig. 1 and labeled as k in Fig. 2). The interaction states are the lowest heavy-hole level in the a set (now a three-component state as a result of the warping interaction),

$$\psi_2 = A_{3,0}{}^{-}\Phi_0 u_3 + A_{5,2}{}^{-}\Phi_2 u_5 + A_{4,0}{}^{+}\Phi_0 u_4, \tag{6}$$

and the second heavy-hole level in the b set,

$$\psi_3 = A_{6,1}{}^{-}\Phi_1 u_6 + A_{4,3}{}^{-}\Phi_3 u_4. \tag{7}$$

Since no other interactions with these levels are present, we may treat this 2×2 problem exactly. The interaction matrix element is then

$$Q_{23} = (2eH/3c)^{1/2}[A_{3,0}{}^{-}A_{6,1}{}^{-} - 6^{1/2}A_{5,2}{}^{-}A_{6,1}{}^{-} + \sqrt{3}A_{5,2}{}^{-}A_{4,3}{}^{-} - (i/\sqrt{2})A_{4,0}{}^{+}A_{6,1}{}^{-}]C. \tag{8}$$

It may be shown[13] that Q_{23} is given by

$$Q_{23}{}^2 = \delta^2\left[1 - \left(\frac{1-x}{1+x}\right)^2\right], \tag{9}$$

where 2δ is the energy separation between, and x is the ratio of the strengths of, the experimentally determined forbidden and allowed transitions. Thus, from Eq. (9), the experimental results give C directly. Inversion-asymmetry transitions for $\vec{H} \parallel \langle 100\rangle$ and $\langle 110\rangle$ may also occur, but these are about five times weaker than the above, and have not been observed in the present work.

The magnetoreflection spectra of Fig. 2 were taken on pure InSb samples ($N \sim 10^{14}$ cm^{-3}) at 1.5 °K. Because of the large conduction-band spin splitting, the transitions to the spin-down and spin-up conduction sub-bands are widely separated and permit unambiguous assignments of transitions. Following Johnson[14] we identify the sharp symmetric lines with exciton ground states.

We make the assumption that the binding energies for all transitions to the same $n=0$ conduction-band level are equal; so the differences in energy between transitions are given in terms of the Landau-level theory. This is reasonable since all the valence levels involved have about the same mass (~20 times greater than the conduction mass).

Relative to the associated allowed transition in the σ_R spectrum, the lowest warping-induced transition in Fig. 2 has a strength of about $\frac{1}{2}$, and the linear-$\vec{k}$ transition a strength of about 1/15. Using this and the energy separations we obtain the following values for the warping and linear-$\vec{k}$ parameters: $(\gamma_3-\gamma_2)=1.0$; $C=3.0\times10^{-3}$ a.u. or 4.2×10^{-10} eV cm. This gives a linear-$\vec{k}$ upbending of the heavy-hole band of about 1.0×10^{-4} eV, as shown in Fig. 3. We estimate the maximum error in $\gamma_3-\gamma_2$ to be ±15%, and that in C to be ±30%. The result for $\gamma_3-\gamma_2$ is in good agreement with the cyclotron resonance work of Bagguley, Robinson, and Stradling[15] and not with the more recent work of Tohver and Ascarelli.[16] The value for C is about three times smaller than that given in Ref. 7, but is in good agreement with the theoretical estimate of Kane.[17]

At low magnetic fields the linear-$\vec{k}$ terms will have a large perturbing effect on the magnetic energy levels, and the perturbation approach discussed here will break down. In this case an energy-level calculation such as that described in Ref. 11 becomes necessary. However, the small size of the linear-$\vec{k}$ transition observed experimentally justifies our approach. We have seen this transition in the region from 30 to 100 kOe, where the small perturbation method is found to be valid. Finally, it is worth summarizing the steps leading to the assignment of the linear-$\vec{k}$ transition and warping-induced transitions:

(1) The basic energy-band level scheme, and identification of the allowed transitions, are known from Ref. 8.

(2) From the relative energies of the allowed and extra transitions in the ⟨111⟩ case, we identify the valence-band levels of origin.

(3) Equation (1) shows that the only interactions present between the pairs of levels involved (Fig. 1), which will also give rise to transitions to the $n=0$ conduction-band levels in the polarizations found experimentally, are the linear-$\vec{k}$ and warping interactions.

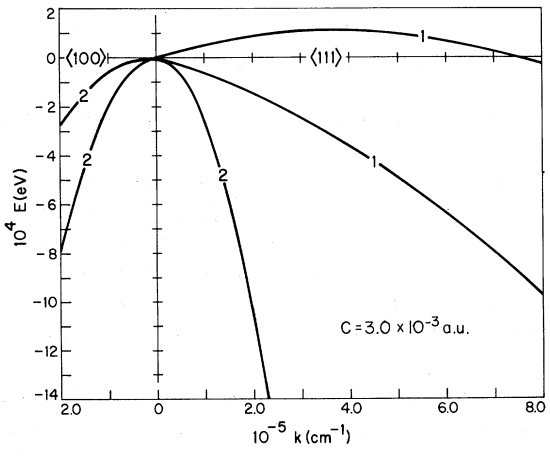

FIG. 3. Valence-band dispersion relations for $\vec{k}$ along ⟨100⟩ and ⟨111⟩ directions which result from the parameters determined here. The numbers indicate the degeneracy of the band. In all other directions both light- and heavy-mass bands are split. The maximum splitting occurs for the ⟨111⟩ heavy-mass band.

*Supported by the U. S. Air Force Office of Scientific Research.

†Visiting scientist, Francis Bitter National Magnet Laboratory, Massachusetts Institute of Technology, Cambridge, Mass.

‡Operated with support from the U. S. Air Force.

[1]G. Dresselhaus, Phys. Rev. 100, 580 (1955).

[2]R. H. Parmenter, Phys. Rev. 100, 573 (1955).

[3]C. R. Whitsett, Phys. Rev. 138, 829 (1965).

[4]L. M. Roth, S. H. Groves, and P. W. Wyatt, Phys. Rev. Letters 19, 576 (1967); also, L. M. Roth, to be published.

[5]D. G. Seiler and W. M. Becker, Phys. Letters 26A, 96 (1967).

[6]Only the k^3 term contributes to the splitting of the Γ_6 conduction band of GaSb. The conduction band of HgSe has Γ_8 symmetry, with both k and k^3 terms. The splitting is measured at large $\vec{k}$ values, however, and it has not been possible to separate the contributions of each term.

[7]M. L. Robinson, Phys. Rev. Letters 17, 963 (1966).

[8]C. R. Pidgeon and R. N. Brown, Phys. Rev. 146, 575 (1966).

[9]J. M. Luttinger and W. Kohn, Phys. Rev. 97, 869 (1955).

[10]J. M. Luttinger, Phys. Rev. 102, 1030 (1956).

[11]R. L. Bell and K. T. Rogers [Phys. Rev. 152, 746 (1966)] have numerically solved the full Hamiltonian, truncated to order 240×240, for one assumed set of band parameters, with $H\parallel\langle100\rangle$.

[12]G. Dresselhaus and M. S. Dresselhaus, in The Optical Properties of Solids, edited by J. Tauc (Academic Press, Inc., New York, 1966), p. 198. k^3 terms are also present in the valence band, but depend upon the presence of the admixed s-like conduction band. This admixing is not present for the valence band levels involved in the transitions considered here; so we have

not included these terms in D^-.
[13]C. R. Pidgeon and S. H. Groves, to be published.
[14]E. J. Johnson, Phys. Rev. Letters 19, 352 (1967).
[15]D. M. S. Bagguley, M. L. Robinson, and R. A. Stradling, Phys. Letters 6, 143 (1963).
[16]H. T. Tohver and G. Ascarelli, Bull. Am. Phys. Soc. 11, 94 (1968).
[17]E. O. Kane, J. Phys. Chem. Solids 1, 249 (1957).

SPIN-INDEPENDENT OSCILLATIONS OF A DEGENERATE ELECTRON LIQUID*

S. C. Ying† and J. J. Quinn
Department of Physics, Brown University, Providence, Rhode Island 02912
(Received 19 March 1968)

We derive a dispersion relation valid for arbitrary wavelength of the spin-independent oscillations of a electron fluid. Analytic expressions for the long-wavelength limit can be obtained quite easily. In addition, results for all wavelengths can be obtained numerically for the situation in which the Fermi-liquid interaction is approximated by a finite series of spherical harmonics.

In his classic paper on the oscillations of a degenerate electron fluid in the presence of a dc magnetic field, Silin[1] considers both spin waves and spin-independent disturbances. For the spin-independent disturbances he proceeds by using Maxwell's equations to relate the electric field to the charge and current densities, and then expresses these in terms of the distribution function $f(\theta,\varphi)$. The kinetic equation then becomes a homogenous integral equation for the function $f(\theta,\varphi)$. Silin then expands $f(\theta,\varphi)$ in the form of a series of spherical harmonics, and notes that in the limit of infinite wavelength, the different harmonics are independent. For $n \geq |m| > 1$, he obtains the eigenfrequencies[2] $\omega_{nm} = n\omega_c(1+A_n)+O(q^2)$, where ω_c is the cyclotron frequency and q is the wave vector. Recently, Mermin and Cheng[3] have extended Silin's analysis to shorter wavelength by evaluating the term of order q^2 for propagation perpendicular to the dc magnetic field. Essentially, these authors note that the n,m spherical harmonic is connected by a term linear in q to the $n\pm1, m\pm1$ spherical harmonics. They then use perturbation theory to evaluate the q^2 term.

In this note we present a method of analysis which is valid for arbitrary wavelength. The long-wavelength limit of our result reproduces, in a rather simple fashion, the results of Mermin and Cheng. In addition, if the Fermi-liquid interaction is approximated by a finite number of terms in the usual spherical-harmonic expansion, we need only solve a finite-size determinantal equation for any value of qr_c, where r_c is the cyclotron radius. If all the coefficients A_n are small compared with unity, then a calculation linear in the A_n should be adequate for values of $qr_c > 1$, and the determinantal equation becomes rather simple. In particular, if all the A_n are set equal to zero for $n \geq 2$, the secular equation reduces to the well-known dispersion relations $\sigma_{zz}{}^0 \approx 0$ and $\sigma_{xx}{}^0\sigma_{yy}{}^0 + \sigma_{xy}{}^{0^2} \approx 0$ for polarizations parallel and perpendicular to the dc field, respectively,[4] where σ^0 is the conductivity tensor in the absence of Fermi-liquid effects.

The spin-independent kinetic equation for a collisionless electron liquid is

$$(-i\omega + iq_x v + \omega_c \partial/\partial\varphi) f(\theta,\varphi) + (iqv_x + \omega_c \partial/\partial\varphi)\delta\epsilon_1(\theta,\varphi) + e\vec{E}\cdot\vec{v} = 0. \quad (1)$$

Here θ,φ are polar coordinates in $\vec{k}$ space and $f(\theta,\varphi)$ is defined by

$$\delta f(\vec{k}) = (-\partial f_0/\partial\epsilon) f(\theta,\varphi), \quad (2)$$

where $\delta f(\vec{k})$ is half the trace with respect to spin of the deviation from thermal equilibrium of the density matrix caused by the electric field $\vec{E}$. We have assumed space-time dependence of the form $\exp(-i\omega t + iq\cdot x)$, and taken the dc magnetic field to define the z direction. The function $\delta\epsilon_1$ is given by

$$\delta\epsilon_1(\vec{k}) = \frac{2}{(2\pi)^3}\int d^3k'\, \Phi(\vec{k},\vec{k}')\delta f(\vec{k}'), \quad (3)$$

where $\Phi(\vec{k},\vec{k}')$ is the spin-independent part of the interaction function. We introduce $\vec{R}(\theta,\varphi)$, the periodic part of the position vector in real space of an electron on the Fermi surface, and note that

$$e^{i\vec{q}\cdot\vec{R}(\theta,\varphi)} = \sum_{m=-\infty}^{\infty} J_m(X\sin\theta)e^{im\varphi}. \quad (4)$$

Here $X = q_x v_F/\omega_c$ and J_m is the mth-order Bessel function. We define the Fourier coefficients