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SPONTANEOUSLY GROWING TRANSVERSE WAVES IN A PLASMA DUE TO AN ANISOTROPIC VELOCITY DISTRIBUTION

Erich S. Weibel
Space Technology Laboratories, Incorporated,
Los Angeles, California
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Recently self-excited longitudinal plasma oscillations[1] and self-excited Alfvén waves[2] were found theoretically in plasmas having velocity distributions which deviate from the Gaussian. Upon examining the theories of these phenomena it was found that there exist also self-excited transverse electromagnetic waves, which involve only the electrons of a plasma, provided that their velocity distribution is sufficiently anisotropic. Their existence and rate of growth can be derived from the Boltzmann transport equation, neglecting the collision term and retaining only linear terms of the perturbation:

$$\frac{\partial f}{\partial t} + \vec{v}\cdot\frac{\partial f}{\partial \vec{r}} + \frac{e}{m}\left[\vec{v}\times\vec{B}_0\right]\cdot\frac{\partial f}{\partial \vec{v}} = -\frac{e}{m}\left[\vec{E} + \vec{v}\times\vec{B}\right]\cdot\frac{\partial f_0}{\partial \vec{v}}, \quad (1)$$

where $f_0(\vec{v})$ is a nonisotropic distribution which is stationary in absence of collisions, $\vec{B}_0$ is a constant magnetic field, f is the perturbation of the distribution function, while $\vec{E}$ and $\vec{B}$ represent the perturbation of the electromagnetic field. Assuming that the first order quantities $f(\vec{v}, \vec{r}, t)$, $\vec{E}(\vec{r}, t)$, and $\vec{B}(\vec{r}, t)$ depend on $\vec{r}$ and t only through the factor $\exp(i\omega t + i\vec{k}\cdot\vec{r})$, one obtains

$$i(\omega + \vec{k}\cdot\vec{v})f - \frac{e\vec{B}_0}{m}\cdot\left[\vec{v}\times\frac{\partial f}{\partial \vec{v}}\right] = -\frac{e}{m\omega}\left\{\omega\vec{E}\cdot\frac{\partial f_0}{\partial \vec{v}} + \left[\vec{k}\times\vec{E}\right]\cdot\left[\vec{v}\times\frac{\partial f_0}{\partial \vec{v}}\right]\right\}, \quad (2)$$

where the two homogeneous Maxwell equations were used to eliminate $\vec{B}$. It can be seen from (2) that the effects of the anisotropy are contained in the last term of the right-hand side. The magnetic field B_0 is not essential to the growth of disturbances; it is included since it does not complicate the analysis very much. The linear, first order, partial differential equation (2) for f was solved for the following special case:

$$f_0(\vec{v}) = F(v_0, v_z), \quad v_0^2 = v_x^2 + v_y^2,$$

with both $\vec{B}_0$ and $\vec{k}$ parallel to the preferred z-direction. $\vec{E}$ is chosen normal to the vector $\vec{k}$. Using the method of characteristics one obtains after some manipulation

$$f = \frac{e\left\{i(\omega + kv_z)(v_xE_x + v_yE_y) + (eB_0/m)(v_xE_y - v_yE_x)\right\}}{m\omega v_0\{(eB_0/m)^2 - (\omega + kv_z)^2\}} \times \left\{kv_0\frac{\partial f_0}{\partial v_z} - (\omega + kv_z)\frac{\partial f_0}{\partial v_0}\right\}.$$

Using the two inhomogeneous Maxwell equations together with the relations

$$\delta = e\int f(\vec{v}, \vec{r}, t)d^3v, \quad \vec{j} = e\int \vec{v}f(\vec{v}, \vec{r}, t)d^3v, \quad (3)$$

and eliminating the fields $\vec{E}$, $\vec{B}$, one obtains a relation between ω and k:

$$k^2 - \omega^2 = \frac{e^2}{m}\pi\int_{v_0=0}^{\infty}\int_{v_3=-\infty}^{\infty}\frac{(\omega + kv_3)\partial f_0/\partial v_0 - v_0k\,\partial f_0/\partial v_3}{(\omega + kv_3) \pm (eB_0/m)} \times v_0^2\,dv_0\,dv_3. \quad (4)$$

The contour of the v_3-integration must pass above the pole $v_3 = (1/k)(\omega \pm eB_0/m)$ in the v_3-plane.[3] If the distribution function has the form

$$f_0 = \frac{n}{u_0^2 u_3 (2\pi)^{3/2}}\exp\left[-\frac{v_0^2}{2u_0^2} - \frac{v_3^2}{2u_3^2}\right], \quad (5)$$

it is possible to evaluate the integral and one obtains

$$k^2 - \omega^2 = \omega_p^2\left\{A - \left(A\frac{\omega\pm\omega_c}{u_3k} + \frac{\omega}{u_3k}\right)\phi\left(\frac{\omega\pm\omega_c}{u_3k}\right)\right\}, \quad (6)$$

where

$$\omega_p^2 = ne^2/m, \quad \omega_c = eB_0/m, \quad A = (u_0/u_3)^2 - 1, \quad (7)$$

and

$$\phi(z) = \exp(-\tfrac{1}{2}z^2)\int_{-i\infty}^{z}\exp(\tfrac{1}{2}\xi^2)\,d\xi. \quad (8)$$

Equation (6) for certain real values of k admits of complex values of ω having negative imaginary parts. This can be seen most easily in the case $B_0 = 0$ and in the limit of large ω/u_3k. In this case $\phi(z) = z^{-1} + z^{-3}$ and (6) becomes

$$\omega^4 - (\omega_p^2 + k^2)\,\omega^2 - u_0^2\,\omega_p^2\,k^2 = 0. \qquad (9)$$

The same approximation would have been obtained (in the limit $u_3 k/\omega \ll 1$) for an arbitrary f_0, as may be seen by expanding the denominator of (4) in powers of v_3. The four solutions of (9) are

$$\omega = \pm\{\tfrac{1}{2}[\omega_p^2 + k^2 \pm ((\omega_p^2 + k^2)^2 + 4u_0^2\,\omega_p^2\,k^2)^{1/2}]\}^{1/2}$$

The solution ω_4, obtained by taking minus signs in both choices, is negative imaginary, showing the existence and rate of growth of self-excited waves. This solution is valid only when $u_3 k/\omega \ll 1$ which implies $u_0 \gg u_3$, as becomes evident from the approximate expression

$$\omega_4 \cong -\,i u_0\,\omega_p k/(\omega_p^2 + k^2)^{1/2}.$$

A more detailed study of (6) shows that for large values of k one obtains damped waves.

The author is indebted to Dr. B. D. Fried for many valuable discussions of these matters.

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SOLID STATE INFRARED QUANTUM COUNTERS*

N. Bloembergen
Harvard University,
Cambridge, Massachusetts,
(Received December 29, 1958)

Since maser operation is based on stimulated emission of radiation, masers have an inherent limiting noise temperature of $h\nu/k$ due to spontaneous emission.[1,2] Weber[2] has called attention to the fact that it is possible to construct quantum-mechanical amplifiers without spontaneous emission noise. In fact, this is the usual state of affairs for x-ray or γ-counters. This note describes how a solid counter for infrared or millimeter wave quanta might be constructed in principle.

Consider a crystal containing ions which, among others, have the energy levels shown in Fig. 1. Salts of the rare earths and other transition group ions, which may be embedded as impurities in host lattices, offer examples of this situation.[3,4] The distance between the ground level and level E_2 is such that $h\nu_{12} \gg kT$. If, for example, $h\nu_{12} \sim 100$ cm^{-1} and $T \sim 2°$K, only the ground state is populated. Intense light at the optical frequency ν_{23} is not absorbed, because level E_2 is empty. Whenever an incident infrared quantum $h\nu_{12}$ is absorbed, the light will induce a second transition to E_3, provided its intensity produces transitions at a faster rate than the radiationless decay or spontaneous emission from level E_2 back to the ground state.

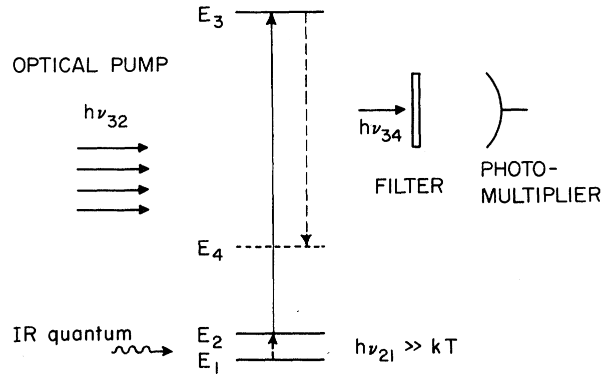

FIG. 1. Infrared quantum counter. Several ions of transition group elements have appropriate energy level diagrams: $h\nu_{21} = 1 - 5000$ cm^{-1}, $h\nu_{32} = 10^4 - 5\times 10^4$ cm^{-1}.

Spontaneous emission from E_3 to E_2 will produce resonance radiation. The system will be repumped, and several quanta $h\nu_{32}$ may be reemitted for each incident quantum $h\nu_{21}$. It will be difficult to detect these quanta $h\nu_{32}$ in the presence of the intense pumping flux, although one may use discrimination in polarization and direction of propagation. When radiation due to spontaneous emission from level E_3 to E_1 is able to leave the crystal, quanta $h\nu_{31}$ may be counted directly. If this radiation is self-absorbed, a fourth level will provide an effective discrimination in frequency. The fluorescent quanta $h\nu_{34}$ may be counted with a photomultiplier and a suitable filter.

A variation of this scheme is that E_1 is an occupied deep impurity level, E_2 is an empty impurity level, and E_3 represents the conduction band. The incident quantum $h\nu_{21}$ triggers a photoconductive avalanche in the semiconductor near absolute zero of temperature.

It is illuminating to point out the relationship with optical pumping methods proposed by