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MICROWAVE EMISSION FROM HIGH-TEMPERATURE PLASMAS*

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Recent investigations[1,2] have suggested alarmingly high energy losses from plasmas contemplated in fusion research due to cyclotron radiation. Details will be published elsewhere of a calculation we have made of this effect in which it is concluded that the loss is definitely negligible for plasma temperatures less than and most probably greater than 100 kev. The emission for the fundamental frequency (eH/m_ec) and the first few harmonics[3] is sufficient, however, to be detectable and holds promise of being a very useful plasma "thermometer" to measure the electron temperature.

If an electric field, $E = E_0 e^{i\omega t}$, (where ω is $> \omega_p$, the plasma frequency) is incident on a magnetically confined plasma, the equation of motion in the x direction for the electrons can be shown to result in the first-order nonrelativistic equation:

$$-m_e\omega^2\left[1+\tfrac{3}{2}\left(\frac{v}{c}\right)^2\right]\frac{P_{xr}}{eN(v)S_r(v)}$$

$$= eE_x - ig\omega\frac{P_{xr}}{eN(v)S_r(v)} + \frac{im_e\omega r\omega_0 P_{yr}}{eN(v)S_r(v)}, \qquad (1)$$

where m_e is the rest mass of the electron, (v/c) is its velocity relative to light velocity, e is its charge, ω is the frequency of the incident radiation, g is a collision damping term, $\omega_0 = (eH/m_ec)$ where H is the confining magnetic field, E_x is the amplitude of the incident wave, $N(v)$ is the Maxwell-Boltzmann distribution of the electron velocity, and $S_r(v)$ is the coupling strength of the rth harmonic frequency to the electron relative to the fundamental. The plasma polarization for the rth harmonic is

$$P_{xr}\,dv = S_r(v)\,N(v)\,\xi(v)\,dv, \qquad (2)$$

where $\xi(v)$ is the electron displacement along the x axis for an electron of velocity v. The total polarization is

$$P_x = \int_0^\infty \sum_{r=1}^{\infty} P_{xr}\,dv. \qquad (3)$$

Substitution of Maxwell's equations into Eq. (1) results in an elementary second-order differential equation from which the index of refraction and absorption coefficient for the incident radiation may be deduced. From the absorption integrated over the path of the reflected or transmitted wave (the index of refraction is shown to be large but slowly changing), the emission is inferred by use of Kirchhoff's relation. The principal effect of summing over all r and integrating over the velocity is to greatly reduce the immense height of the line (when velocity broadening is neglected) without appreciably widening the line. This is intuitively obvious, perhaps, if one considers that the number of electrons having precisely the correct energy and therefore relativistic mass to resonate at a given frequency is reduced as the temperature rises; therefore the height of the line depends inversely on temperature. The effect of electrons resonating below the resonant frequency for a given electron energy, however, cancels the effect of electrons of less energy resonating above the line; therefore the width of the line is essentially unchanged. This "cooperative" cancellation is, we believe, the source of the discrepancy with other work[1,2] which predicts very much greater emission. The calculation was carried out explicitly for a spatially varying magnetic field characteristic of mirror machines. Even for the fundamental frequency the emission is of the order of 10^{-4} of that for a black body for small laboratory plasmas of 50-kev temperature. The effect is markedly temperature dependent, however, going as $T^{-5/2}$, and rises to 10^{-2} for a 10-kev plasma. Although the spatial fluctuations can be neglected since we are only interested in the coherent transmitted radiation, the fluctuations in velocity space dominate the Doppler motion calculation above 10 kev for electron densities of 10^{14}/cc. We may neglect the fluctuations for higher temperature cases only where the Doppler effect is unimportant, namely for emission perpendicular to the magnetic field and for emission parallel to the field in a mirror machine of less than 20% magnetic field change; for perpendicular motion the fluctuations have a negligible effect below temperatures well in excess of 100 kev. The rth harmonic has an intensity only roughly $r(2kT/mc^2)^{r/2}$ as great as the fundamental, and when detectable furnishes a promising means of determining the plasma temperature by measuring the intensities of the harmonics relative to the fundamental.

*This work was undertaken primarily while the author was a consultant to the University of California Radiation Laboratory at Livermore during 1954-1956.

[1]B. A. Trubnikov and V. S. Kudryavtsev, Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958 (United Nations, Geneva, 1958), A/conf. 15/P/2213.

[2]Hayakawa, Hokkyo, Terashima, and Tsuneto, Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958 (United Nations, Geneva, 1958), A/conf. 15/P/1330.

[3]J. Schwinger, Phys. Rev. 75, 1912 (1949).

FAST ION HEATING*

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The remarkably fast ion heating observed in high-current "stabilized" gas discharges[1] may be due partly to an ion relaxation process described by Schluter.[2] The periodically varying magnetic field required by this process may arise from the transient excitation of magnetic compression waves[3] occasioned by the violent initial collapse of the discharge. If the axial current flows entirely in a thin surface layer of equilibrium radius r, and the exterior magnetic field is purely azimuthal, the frequencies of the compression modes are given by $\omega_N^2\mu_0\rho r^2 = \sigma^2B^2(1+N^2r^2/\sigma^2R^2)$, where ρ is the average density, B is the average axial (stabilizing) magnetic field, R is the major radius of the torus, and N is zero or an integer. The constant, σ, is a root of $\sigma J_0(\sigma)=J_1(\sigma)$. For compressions that are in phase in a cross section of a discharge, $\sigma=1.841$, the lowest root. In Zeta and Sceptre, $\omega_0/2\pi$ is typically about 0.7 Mc/sec, and the next few modes are only slightly higher in frequency. If we approximate the magnetic field inside a discharge by $B_z=B(1-A\cos\omega t)$, the total energy of the compression mode is of the order $W=\rho\pi^2r^4RA^2\omega^2/8$. In Zeta and Sceptre, under typical conditions, the total energy comes to about $1000A^2$ joules per mode. This is to be compared with the total ion energy, which at 100 ev ion temperature, is roughly 1000 joules in each case.

To get an idea of how quickly the compression mode energy can be given to the ions, we use Schluter's expression for the ion temperature e-folding time, which is $1/\lambda=9(\gamma^2+\omega^2)/A^2\omega^2\gamma$, where γ is the ion collision frequency.[4] We estimate e-folding times on the basis of only one mode, with $A=1$, and for typical operating conditions. In Zeta, for ion temperatures of 1, 10, and 100 ev, we get $1/\lambda$ values of 0.05, 0.005, and 0.1 millisecond. We get corresponding values for Sceptre of 0.3, 0.01, and 0.01 millisecond. In arriving at these figures, we used ion densities of 10^{14} and 9×10^{14} per cm^3, total currents of 1.6 and 1.0×10^5 amp, r values of 14 and 6 cm, and R values of 100 and 57 cm, for Zeta and Sceptre, respectively.

We conclude that this picture is in accord with the observed times and magnitudes of ion heating, with a not unreasonable choice of A values for several of the lower modes of a toroidal discharge.

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[1]See the group of papers beginning with P. Thonemann et al., Nature 181, 217 (1958).

[2]A. Schluter, Terzo Congresso Internazionale sui