versal by about 16°C, in contrast to the Te-Se alloy mentioned above, for which the reversal temperature was apparently lowered about 5°C. The impurity concentration of the two samples of Fig. 1, as determined from the Hall coefficient R at 77°K and the approximate formula $R=1/pe$ (p = carrier density, $e=1.6\times10^{-19}$ coulomb), is 7.2×10^{14} carriers/cm^3 for sample 1 and 2.2×10^{15} carriers/cm^3 for sample 2. In addition, the upper reversal temperature for a third sample with $p=7.2\times10^{16}$ was found to occur at 498°K and this demonstrates the shift of the upper reversal temperature with impurity concentration. Long[13] has computed that the hydrostatic pressure used in this experiment causes the energy gap to decrease by 0.032 ev, and this in turn is responsible both for the decrease in the lower reversal temperature which he reported and for the decrease in Hall coefficient with pressure in the region below the "crossover" point at about 217°C on both pairs of curves in the present experiment. However, the cause of the upper reversal shift still remains unknown.

[1]P. I. Wold, Phys. Rev. 7, 169 (1916).
[2]Fukuroi, Tanuma, and Tobisawa, Science Repts. Research Insts., Tôhoku Univ. A1, 373 (1949).
[3]V. E. Bottom, Science 115, 570 (1952).
[4]A. Nussbaum, Ph. D. thesis, University of Pennsylvania, 1954 (unpublished).
[5]H. Fritzsche, Science 115, 571 (1952).
[6]H. B. Callen, J. Chem. Phys. 22, 518 (1954).
[7]R. Gáspár, Acta Phys. Acad. Sci. Hung. 7, 289 and 313 (1956).
[8]G. Dresselhaus, Phys. Rev. 105, 135 (1957).
[9]A. Nussbaum, Phys. Rev. 94, 337 (1954).
[10]T. J. Davies, J. Appl. Phys. 28, 1217 (1957).
[11]H. Roth, as reported at the International Conference on Semiconductors, Rochester, August, 1958 (unpublished).
[12]D. Long, Phys. Rev. 99, 388 (1955).
[13]D. Long, Phys. Rev. 101, 1256 (1956).

NEUTRON-PROTON MASS DIFFERENCE BY DISPERSION THEORY

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An attempt to explain the neutron-proton mass difference by introducing in the divergent integrals of the second-order electromagnetic self-mass a fundamental length has been made by Feynman and Speisman.[1] They show that the proton can in fact turn out lighter than the neutron, in spite of its electrostatic energy, if the anomalous moments of the nucleons are introduced in the interaction and the integrals are cut off at sufficiently high energy. Since then it has been realized that the electromagnetic self-masses of the nucleons might be finite even in a microscopic causal theory if the electromagnetic form factors vanish sufficiently rapidly at high momenta, as indicated by certain consistency requirements.[2] With this assumption Wick and Sorensen[3] have attempted a calculation of the nucleon mass difference on the basis of a formalism developed by Low.[4] Their calculation yields a negative result, giving a proton heavier than the neutron. No direct comparison can, however, be made between the two calculations. In FS the relativistic Born approximation to the self-energy is used, while WS are essentially led to a Born approximation formula with form factors, in which only positive-energy states are kept.

For this reason we have re-examined the problem by using as a basis the expression of the Compton scattering amplitude derived with a dispersion relation approach. The single-nucleon contribution is in our case simply the FS expression with their arbitrary cutoffs replaced by the electromagnetic form factors of the nucleons. We find that the mass difference that one obtains by extrapolating at high momenta the experimental form factors is wrong in sign and in magnitude. The reason is that the radii of the experimental distributions correspond to cutoffs considerably lower than those used in FS. We also find that the correct mass difference can be obtained, from the single-nucleon contribution and without contradicting the Stanford data, with a rather pathological neutron charge distribution, concentrated at small distances.[5] It may well be, on the other hand, that the main effect comes from the many-particle intermediate states.

We start, as in FS, with the expression:

$$\langle p'|S_2|p\rangle = -i\delta m(2\pi)^4\,\delta(p-p')\bar{u}(p')u(p) = \tfrac{1}{2}\delta(p-p')\int\frac{d^4k}{k^2-i\epsilon}\,\phi(k), \quad (1)$$

where

$$\phi(k) = i\int d^4x\, e^{ikx}\langle p|T(j_\mu(\tfrac{1}{2}x), j_\mu(-\tfrac{1}{2}x))|p\rangle, \quad (2)$$

with $j_\mu(x)$ the Heisenberg current operator. Instead of inserting directly in (2) a set of ingoing states in order to evaluate the current-ordered product, we consider the retarded amplitude for Compton scattering of virtual photons given by

$$M_{\mu\nu}=i\int d^4x\, e^{ikx}\langle p'|[j_\mu(\tfrac{1}{2}x),j_\nu(-\tfrac{1}{2}x)]|p\rangle\eta(-x), \quad (3)$$

from which $\phi(k)$ can be easily obtained. It is known from the work on Compton scattering of real photons[6] that one can write

$$M_{\mu\nu}=\sum_i \bar{u}(p')I_{\mu\nu}^{(i)}u(p)M^{(i)}(\nu,\Delta^2), \quad (4)$$

where $I_{\mu\nu}^{(i)}$ are a set of gauge-invariant tensors formed with the γ matrices and three of the four momenta involved, and Δ^2 the conventional invariant energy and momentum transfer.[7] From causality one has for the scalar amplitudes:

$$\mathrm{Re}\,M^{(i)}(\nu,\Delta^2)=\frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{\mathrm{Im}\,M^{(i)}(\nu',\Delta^2)}{\nu-\nu'}d\nu'+P_n(\nu), \quad (5)$$

with $P_n(\nu)$ a polynomial in ν. There is one set $I_{\mu\nu}^{(i)}$ such that the single-nucleon contribution to $\mathrm{Im}M^{(i)}$ gives for $M^{(i)}$ the relativistic Born approximation for Compton scattering from nucleons with the experimental magnetic moments. For this choice there are reasons to believe[8] that the polynomial $P_n(\nu)$ is absent in (5).

We take the same attitude for virtual photons and use relations (5) without polynomial also in our case. Then, even without writing the invariants explicitly, we can immediately write the one-nucleon contribution M^N:

$$M_{\mu\nu}^N=\sum\left[\frac{\langle p'|j_\mu|P-k\rangle\langle P-k|j_\nu|p\rangle}{2M\nu+k^2-i\epsilon}+\frac{\langle p'|j_\nu|P+k\rangle\langle P+k|j_\mu|p\rangle}{-2M\nu+k^2+i\epsilon}\right], \quad (6)$$

where the sum is on the spins of the intermediate nucleon state. By using the well-known expressions for the matrix elements of the current,[9] one obtains

$$\phi^N(k)=\bar{u}(p)\left\{[eF_1(k^2)\gamma_\mu+i\mu F_2(k^2)\sigma_{\mu\nu}k_\nu]\frac{-i\gamma(p-k)+M}{-2pk+k^2}[eF_1(k^2)\gamma_\mu-i\mu F_2(k^2)\sigma_{\mu\nu}k_\nu]+(k\to-k)\right\}u(p). \quad (7)$$

Equation (7) gives for the self energy exactly the FS expression with their arbitrary cutoff functions replaced by the form factors $F_1(k^2)$, $F_2(k^2)$, while the calculation in (3) obtained for $\phi^N(k)$ an expression in which the invariant denominators of Eq. (7) were replaced by those corresponding to positive-energy intermediate states only.

If one uses for the form factors the exponentials suggested by the experimental data of Hofstadter,[10,11]

$$F_1^{(p)}(k^2)=F_2^{(p)}(k^2)=F_2^{(n)}(k^2)=\alpha^4/(\alpha^2+k^2)^2,$$
$$F_1^{(n)}(k^2)=0, \quad (8)$$

with $\alpha^2=37.5\mu^2$, the mass difference turns out to be

$$\Delta M=\delta m^{(n)}-\delta m^{(p)}=-0.30\,(1/137\pi)\,M=-0.66\ \text{Mev},$$

instead of the experimental value $M_{\text{exp}}=+1.2$ Mev. No possibility seems to exist of obtaining even the right sign of ΔM if one uses extrapolations to high values of k^2 with one-parameter functions determined from the experimental data at low values of k^2, even allowing for the experimental errors in the mean square radii.

If the bulk of the effect is to be explained by the single-nucleon term one must introduce form factors with important contributions from high mass values in the spectral function. Inserting the spectral representation for the form factors, one finds for the electromagnetic self-mass of a nucleon, neutron or proton (in units of the nucleon mass)

$$\delta m=\frac{1}{137\pi}[L_{11}-\tfrac{1}{2}\mu L_{12}-(\tfrac{1}{2}\mu)^2L_{22}], \quad (9)$$

where

$$L_{ij}=\pi^{-2}\int dm^2dm'^2(m^2-m'^2)^{-1}\times\rho_i(m^2)\rho_j(m'^2)[f_{ij}(m^2)-f_{ij}(m'^2)]. \quad (10)$$

μ is the nucleon anomalous magnetic moment and $\rho_1(m^2)$, $\rho_2(m^2)$ are the spectral functions for the charge and magnetic moment, respectively, as defined by Chew et al.,[12] and

$$f_{11}(m^2)=-\tfrac{1}{2}[\tfrac{1}{4}m^2\ln m^2+(1+\tfrac{1}{2}m^2)(4-m^2)w(m^2)],$$
$$f_{12}(m^2)=\tfrac{3}{2}[\tfrac{1}{2}m^2\ln m^2+m^2(4-m^2)\,w(m^2)],$$
$$f_{22}(m^2)=\tfrac{1}{4}[-\tfrac{1}{2}m^2+\tfrac{1}{2}m^2(3+\tfrac{1}{2}m^2)\ln m^2+(4+\tfrac{1}{2}m^2)m^2(4-m^2)w(m^2)],$$

with

$$w(m^2) = [m^2(4-m^2)]^{-1/2} \arctan[m^{-2}(4-m^2)]^{1/2} \quad \text{for } m^2<4$$

$$= [m^2(m^2-4)]^{-1/2} \ln \tfrac{1}{2}[m + (m^2-4)^{1/2}] \quad \text{for } m^2>4.$$

We have tried a model in which only the neutron charge spectral function has contributions from high mass values.[13] With $F_1^{(p)}$, $F_2^{(p)}$, and $F_2^{(n)}$ as in (8) and $F_1^{(n)}$ given by

$$F_1^{(n)}(k^2) = \epsilon\omega^4 k^2/(k^2+\omega^2)^2,$$

which corresponds to a charge distribution of the form $e^{-\omega r}/r - \frac{1}{2}\omega e^{-\omega r}$ and which is consistent with the Stanford data for $\epsilon \lesssim 2.2\times10^{-2}M^{-2}$, we find, for ϵ equal to its upper limit, the right ΔM for $\omega \simeq 10M$. The high value of ω raises, however, the question of the importance of the many-particle states. Conversely, if many-particle states are important, it seems that they should also be important in the spectral representations of the form factors and again give rise to distributions highly concentrated at small distances.[14]

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[1]R. P. Feynman and G. Speisman, Phys. Rev. 94, 500 (1954) (referred to as FS in the following). See also A. Petermann, Helv. Phys. Acta 27, 441 (1954); K. Huang, Phys. Rev. 101, 1173 (1956).

[2]Lehmann, Symanzik, and Zimmermann, Nuovo cimento 2, 425 (1955).

[3]G. C. Wick, Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics, 1957 (Interscience Publishers, New York, 1957), Chap. I, p. 41; R. A. Sorensen, Atomic Energy Commission Report NYO-7119 (unpublished). (Both papers are referred to as WS.)

[4]F. Low, Phys. Rev. 97, 1992 (1955).

[5]A calculation of the mass difference of the pions has been done by Riazzuddin (Phys. Rev. Lett. to be published). In his case the electromagnetic structure of the pions is not known and he derives a mean square radius from the mass difference. If, however, a strong pion-pion interaction exists, for which there seems to be some indication, the actual mean square radius will be larger than the one found from the mass difference and the situation would be quite similar to the nucleon case.

[6]N. N. Bogoliubov and D. V. Shirkov, Doklady Akad. Nauk S.S.S.R. 113, 529 (1957); R. E. Prange, Phys. Rev. 110, 240 (1958).

[7]$\nu = -(Pk)/M$; $P = \frac{1}{2}(p+p')$; $\Delta^2 = (p-p')^2$.

[8]M. Gell-Mann and J. Mathews, in 1958 Annual International Conference on High-Energy Physics at CERN, edited by B. Ferretti (CERN, Geneva, 1958), p. 98.

[9]G. Salzman, Phys. Rev. 99, 973 (1953).

[10]R. Hofstadter, in 1958 Annual International Conference on High-Energy Physics at CERN, edited by B. Ferretti (CERN, Geneva, 1958), p. 11 ff.

[11]In our calculation also time-like k^2 occurs while the experimental results refer only to space-like k^2. The usual Feynman technique of integration takes care of this.

[12]Chew, Gasiorowicz, Karplus, and Zachariasen, Phys. Rev. 110, 265 (1958); Federbush, Goldberger, and Treiman, Phys. Rev. 112, 642 (1958).

[13]L. I. Schiff [1958 Annual International Conference on High-Energy Physics at CERN, edited by B. Ferretti (CERN, Geneva, 1958), p. 14] has proposed for $F_1^{(n)}$ a function different from zero at large k^2 consistent with all the low-momenta data; since, however, it does not vanish for $k^2\to\infty$ it cannot give a finite electromagnetic self-mass.

[14]As to the importance of the one-meson contribution it may be relevant to note that by extrapolating the approximate expression for the amplitude of meson electroproduction given by Fubini, Nambu, and Wataghin [Phys. Rev. 111, 329 (1958)], stated to be reliable up to $k^2 \approx 0.3M^2$, the effect of the $\frac{3}{2}$, $\frac{3}{2}$ resonant state drops out, and the contribution will be negligible.

ELECTRON-NEUTRINO ANGULAR CORRELATION IN THE BETA DECAY OF NEON-23*

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The electron-neutrino angular correlation coefficient for the beta decay of Ne^{23} as measured by Ridley[1] represents the most serious experimental disagreement with the presently accepted vector and axial-vector interpretation of beta decay. On the assumption that the decay of Ne^{23} is a Gamow-Teller transition, the angular correlation coefficient λ will be $+\frac{1}{3}$ or $-\frac{1}{3}$ corresponding, respectively, to the tensor or axial-vector interactions. Ridley obtained the value $\lambda = -0.05\pm0.10$ which implies a mixture of T and A in comparable strength. We have re-evaluated the angular correlation coefficient in the decay of Ne^{23} using the same technique previously employed[2-4] to measure the angular correlations of Ne^{19}, A^{35}, and He^6. Our experimental result, $\lambda = -0.37\pm0.04$, indicates that the axial-vector in-