in terms of spectral representations given by G. Källén in Proceedings of the CERN Symposium on High-Energy Accelerators and Pion Physics, Geneva, 1956 (European Organization of Nuclear Research, Geneva, 1956), Vol. II, p. 187.

[10]Källén remarked in his paper that a divergence of $\lim_{-Q^2\to\infty}\bar{R}^{\mathrm{reg}}(Q^2)$ as well as its vanishing will lead to infinite renormalization constants, but he overlooked the only possible counterexample.

[11]S. Drell and F. Zachariasen, Phys. Rev. 111, 1727 (1958); M. Gell-Mann and J. Mathews (to be published).

[12]See reference 9. The infrared divergence in quantum electrodynamics implies a nonuniform behavior of this type for the series in Eq. (3). According to arguments of Bloch-Nordsieck type, the matrix element for producing an electron pair plus any finite number of photons should vanish. Nevertheless, the sum over all numbers of photons should yield a finite result.

ERRATUM

PRECISE DETERMINATION OF THE MUON MAGNETIC MOMENT. R. L. Garwin, D. P. Hutchinson, S. Penman, and G. Shapiro [Phys. Rev. Letters 2, 213 (1959)].

It has recently been[1] established that the theoretical uncertainties in the lower limit of the muon mass from mesonic α-rays[2] are much smaller than previously supposed.[3] We therefore undertook a more careful search into possible systematic errors in our moment measurement. We have found and corrected one serious source of error. A method of testing was then designed which simulated the experimental situation and permitted the establishment of an upper limit on the remaining systematic errors in the electronic equipment of 1×10^{-5} or almost an order of magnitude less than our stated accuracy.

The circuit design used in this experiment has been intended to be completely aperiodic, i.e., should introduce no systematic errors in a frequency measurement (as opposed to a random error in measuring each particle due to the finite time resolution). Preceding the analysis equipment, however, is the "zero-crossing detector" which does the fast timing of both the muon and decay electron pulses. To circumvent the difficulties of long-term transit-time variations in phototubes, a single counter and timing circuit was used for both muon and electron pulses. This raises the possibility that the recovery of the phototube or fast timing circuit after the passage of a muon pulse will alter the apparent time of the electron. This effect is important only if it varies during the measurement interval. In particular, if electrons immediately following the muon pulse are delayed more than those coming later in the gate interval, the net effect is to make the apparent muon precession frequency appear larger than it actually is. Unfortunately, we observed on our circuit diagram a one-microsecond time-constant at the grid following the distributed zero-crossing amplifier. Had this time-constant been 50 millimicroseconds or >20 microseconds, no error would have been incurred, but we estimated the maximum error caused by it as $\sim4\times10^{-4}$.

Since there is available to us no measuring equipment that approaches the experimental apparatus in resolving ability, the existence of the postulated effect was established by inserting two pulses with fixed cable delay of 2 microseconds into the equipment. The first was analyzed by the circuitry as a muon pulse and the other as an electron pulse. The equipment output was displayed on a pulse-height analyzer as previously described. A third pulse which had been traveling in a long cable was then inserted between the "muon" and "electron" pulses. Its retarding effect on the electron timing could then be seen clearly. The fast timing circuit was then altered (by changing the offending time-constant) so that the effect was no longer apparent.

After modifying the circuitry, the following rigorous test was made to establish that there remained no other such sources of error. We built a source of random pulses whose probability of occurrence above a fixed threshold oscillated in time at a known rate. This consisted of a 6810A photomultiplier viewing a plastic scintillator which was irradiated with β rays. The focusing voltage of the tube was modulated at or near the frequency of the reference oscillator of the measuring apparatus, (86.2 Mc/sec). A 30-volt peak-to-peak signal was sufficient to achieve nearly 100% modulation of the output pulses. The slow coincidence circuits select the first pulse above a threshold after a dead time as a μ pulse and the next pulse as an electron. Since the probability of occurrence of these two pulses is strongly correlated at the reference oscillator frequency, the pulse-height analyzer display of the electron phase modulo the reference oscillator period will show a structure very similar to that for a precessing muon [although not restricted to the form $(1+a\cos\phi)$]. The actual experiment consists of comparing the phase distribution of electrons that come soon after a muon stop with those that come later. In the test, two groups of modulated random pulses were displayed, those that corresponded to "early" electrons and those that corresponded to "late" ones.

Figure 1 shows the actual pulse-height analyzer display. On the left is the distribution in phase of the early "electrons" and on the right that of the late "electrons." Ideally, the distributions will be identical in shape. A treatment of the test data by the same Fourier analysis used in the experiment shows no phase shift corresponding to a frequency error larger than 10^{-5}.

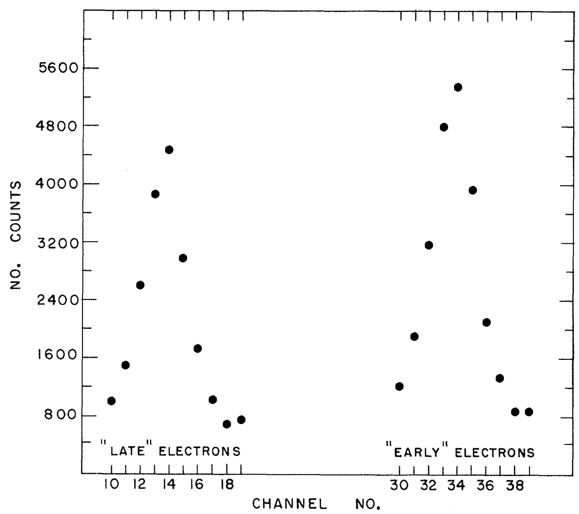

FIG. 1. Pulse-height analyzer display of the modulated counter test.

The measurement of the muon moment was then repeated using a smaller bromoform target, with the equipment modified and tested as above.

Expressing the final result in terms of the ratio of the resonance frequency of the muon to that of a proton in the same magnetic field, we obtained $f_\mu/f_p = 3.1834 \pm 0.0002$. Combining this value for the moment with the value for the lower limit on the mass, as discussed in reference 1, we obtain for the g factor $g_\mu > 2(1.00122 \pm 0.00008)$. This limit is seen to be in agreement with the theoretical prediction $g = 2(1.00116)$. The experiment to determine the absorption coefficient of lead for the $3D$ - $2P$ transition in μ-mesonic phosphorus is being repeated. Parratt[4] has pointed out that the width of the lead K edge should enable one to obtain either a more accurate mass value or a considerably different lower limit.

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[1]A. Petermann and Y. Yamaguchi, Phys. Rev. Letters 2, 359 (1959).

[2]Koslov, Fitch, and Rainwater, Phys. Rev. 95, 291 (1954).

[3]Cohen, Crowe, and DuMond, Fundamental Constants of Physics (Interscience Publishers, New York, 1957).

[4]L. G. Parratt (private communication).