Considerably more precise information concerning the density of antiparticles in the galaxy can be obtained by carrying a simple scintillation gamma-ray spectrometer some distance from the earth in a satellite or space probe to look specifically for 0.51-Mev photons.[7] A search could be made, also, for the high-energy (~170 Mev) gamma rays, although the spectrometer to be used for these would be larger and more complicated.

An explicit search for positrons in space also can be made quite simply, using an experimental arrangement such as that shown schematically in Fig. 1. With this equipment one would detect positrons arriving at the annihilation block by the resulting coincident 0.51-Mev light pulses in the two NaI crystals. Using the interplanetary cosmic-ray flux found by Van Allen[8] and assuming a resolving time of 10^{-7} sec for NaI crystals, I estimate that at distances from the earth greater than ~20 000 miles this type of instrument could detect positrons if their number density were $10^{-14}/\mathrm{cm}^3$ or greater.

Detection of positrons in space in the neighborhood of the earth's orbit would not necessarily be firm proof of the existence of antimatter on a cosmic (or even galactic) scale, because positrons from cosmic-ray showers or from other processes in the solar system might find their way to this region. Thus, the search for the characteristic annihilation gamma rays probably represents the better test for the widespread existence of antiparticles.

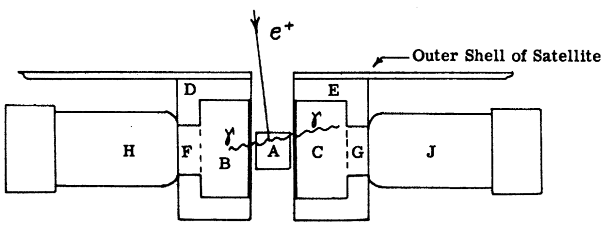

FIG. 1. Schematic representation of satellite instrumentation to detect positrons in space. A—annihilation block of low-Z material; B, C—NaI crystals shielded from sun by ~1 mg/cm² foil over surface; D, E—lead shields to reduce individual counting rates of the two NaI crystals; F, G—light pipes; H, J—photomultipliers. The system would include electronic circuits which would register only coincident 0.51-Mev pulses in crystals B and C.

*A division of General Dynamics Corporation.

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K-MESON NUCLEON INTERACTION*

Robert Karplus, Leroy Kerth, and Thaddeus Kycia
Department of Physics and Lawrence Radiation Laboratory,
University of California, Berkeley, California

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It is to be expected that the experimental study of K-meson production, scattering, and absorption processes will shed light on the fundamental interactions of the strange particles. The most promising method of analysis for determining the coupling constants and relative parities of the particles exploits the analyticity properties of various reaction amplitudes.[1,2] The recently measured angular distribution of the K^+-proton scattering cross section at various intermediate energies[3] and the K^--proton scattering parameters[4,5] enable us to make a more accurate study of the K-nucleon-hyperon coupling terms than has previously been possible. Unfortunately, the accuracy is not sufficient to lead to a definite conclusion about the K-meson parity, but we do conclude that the K^--proton force is attractive. We hope that the present discussion also has value because it will make clear some of the difficulties that must be faced.

The numerical work is carried out most easily if one uses the observations to calculate the function $f(\omega)$ of K-meson laboratory energy ω, ($\hbar=c=m_K=1$)

$$f(\omega)=\frac{1}{\pi}\int_1^4\frac{A_+(\omega')}{\omega'-\omega}d\omega'+\frac{1}{\pi}\int_{\omega_{\Lambda\pi}}^{4}\frac{A_-(\omega')}{\omega'+\omega}d\omega', \qquad (1)$$

where $A_\pm(\omega)$ are the imaginary parts of the $K^\pm$-proton forward scattering amplitudes and $\omega_{\Lambda\pi}$ is the (nonphysical) threshold for the reaction

$$K^-+p\rightarrow\Lambda+\pi. \qquad (2)$$

Since the integrals in Eq. (1) have been cut off, it is necessary to use a dispersion relation that does not weight the high-energy region. We have used this expression for the real parts $D_\pm(\omega)$ of the forward scattering amplitudes:

$$\begin{aligned}\omega_0D_+(\omega)&-\tfrac{1}{2}(\omega_0+\omega)D_+(\omega_0)-\tfrac{1}{2}(\omega_0-\omega)D_-(\omega_0)\\ &\cong\omega_0f(\omega)-\tfrac{1}{2}(\omega_0+\omega)f(\omega_0)-\tfrac{1}{2}(\omega_0-\omega)f(-\omega_0)\\ &\quad-\omega_0(\omega_0^2-\omega^2)\left\{\frac{p_\Lambda X_\Lambda}{(\omega_\Lambda^2-\omega_0^2)(\omega_\Lambda+\omega)}+\frac{p_\Sigma X_\Sigma}{(\omega_\Sigma^2-\omega_0^2)(\omega_\Sigma+\omega)}\right\}. \qquad (3)\end{aligned}$$

Here p_Λ (p_Σ) is the parity of $K^-\Lambda$ ($K^-\Sigma$) relative to the nucleon and X_Λ (X_Σ), proportional to the $K^-\Lambda N$ ($K^-\Sigma N$) coupling constant g_Λ^2 (g_Σ^2), is the magnitude of the residue at the pole $\omega=\omega_\Lambda$ (ω_Σ); these quantities are given by the expressions ($\lambda=\Lambda,\Sigma$)

$$\omega_\lambda=(m_\lambda^2-m_p^2-m_K^2)/2m_p,$$
$$X_\lambda=\frac{g_\lambda^2}{4\pi}\left|\frac{(m_\lambda+p_\lambda m_p)^2-m_K^2}{4m_pm_\lambda}\right|, \qquad (4)$$

in terms of the masses m_λ, m_p, and m_K of the hyperons, proton, and K meson, respectively.

The energy ω_0 in Eq. (3) is one at which experimental information is available. By using a value substantially different from the meson rest energy, one can suppress the importance of the nonphysical region and of the amplitude $D_-(\omega_0)$, which decreases with energy.

The optical theorem is used to obtain $A_\pm(\omega)$ from the measurements of total cross section in the physical region. The K^+-proton total cross sections were taken from the data obtained by the use of nuclear emulsions[6] for energies below 200 Mev, the results of our experiment,[3] and the data of Burrowes et al.[7] for the higher energies. A curve was fitted to the data within an estimated statistical error of 10%. The low-energy K^--proton total cross sections have been measured in a hydrogen bubble chamber.[8] At higher energies the values are $\sigma_-=52\pm9$ mb at $\omega=2.08$,[9] and $\sigma_-=60\pm20$ mb at $\omega=2.52$.[10] We assumed the value $\sigma_-=45$ at $\omega=4$, and passed a smooth curve through all the points. The statistical error of the fit was estimated at 15%.

The values of $A_-(\omega)$ in the nonphysical region were obtained by extrapolating with the parameters of Dalitz and Tuan.[4,5] The integral is very sensitive to the sign of $D_-(1)$, the real part of the scattering amplitude at zero energy.[4] The combination of the integral and the scattering amplitude that occurs in Eq. (3), however, is not sensitive to this sign as long as the zero-range approximation gives the correct sign and a reasonably accurate value for $D_-(\omega_0)$. This is the case because the zero-range formula gives an analytic expression for the scattering amplitude which then essentially satisfies a dispersion relation by itself.

The results for the integration are given in Table I. They were obtained with a positive choice of $D_-(1)$. In the fourth column are the measured values[3] of $D_+(\omega)$; their magnitudes are simply related to the total cross sections, because it was found that the scattering is isotropic at $\omega=1.46$ (225 Mev) and it was assumed that it is isotropic at the lower energies. In the last column is the value of D_- obtained from the zero-range parameters, which represent the data adequately.[4] An examination of the bubble chamber results[8] indicates that most of the elastic scattering is indeed a diffraction effect (i.e., the real part of the

Table I. Integrals and scattering amplitudes used in the dispersion relation Eq. (3). The K-meson energy is ω measured in units of the rest energy. The unit for the other quantities is the K-meson Compton wavelength.

ω	$f(\omega)$	$f(-\omega)$	$D_+(\omega)$	$D_-(\omega)$
1.00	3.0 ± 0.3	...	-1.25 ± 0.14	...
1.17	3.1 ± 0.3	5.2 ± 1.0	-1.24 ± 0.14	+0.56
1.285	3.1 ± 0.3	4.9 ± 0.8	-1.23 ± 0.14	+0.40
1.46	3.0 ± 0.3	...	-1.20 ± 0.08	...

scattering amplitude is small) and that there is accordingly some tendency toward forward scattering. Considering the experimental accuracy, one may estimate $|D_-(1.17)|<2$ and $|D_-(1.285)|<2$, consistent with Table I.

Since it is clearly not possible to solve for X_Λ and X_Σ separately, we have calculated the "average" coupling $\langle pX\rangle_{\text{Av}}$ at an "average" pole $\bar{\omega} = \frac{1}{2}(\omega_\Lambda + \omega_\Sigma)$,

$$\langle pX\rangle_{\text{Av}} = \tfrac{1}{2}(\bar{\omega}^2 - \omega_0^2)(\bar{\omega}+\omega)\left[\frac{p_\Lambda X_\Lambda}{(\omega_\Lambda^2 - \omega_0^2)(\omega_\Lambda + \omega)} + \frac{p_\Sigma X_\Sigma}{(\omega_\Sigma^2 - \omega_0^2)(\omega_\Sigma + \omega)}\right], \qquad (5)$$

for various choices of ω and ω_0 in Eq. (3). The results are listed in Table II. We conclude that this determination of $\langle pX\rangle_{\text{Av}}$ leads to the value

$$\langle pX\rangle_{\text{Av}} = 0.0 \pm 0.5, \qquad (6)$$

where we have included some provision for the uncertainty in $D_-(\omega_0)$.

Our conclusion, then, is that the K^+-proton interaction, at the present accuracy, is not sufficiently sensitive to the coupling parameters X_Λ and X_Σ to permit their evaluation from the present data. This is not too surprising, because Eq. (3) depends only on the difference of amplitudes at closely spaced energies that are quite distant from the poles ω_Λ and ω_Σ. We must also point out an important weakness of this type of analysis: only measurements on the $K^\pm$-neutron system can make possible a separation of the $K\Lambda P$ and $K\Sigma P$ coupling terms by their different isotopic properties. The determination of the K-meson parity from the angular dependence of associated $K\Lambda$ and $K\Sigma$ photoproduction[2] does not suffer from this defect and may, therefore, lead to a definitive result sooner.[11]

A few words are in order about the amplitude $D_-(\omega)$.[12] If it deviates substantially from its zero-range value at the energies of interest, then Eq. (6) will be modified somewhat [the right-hand side contains $D_-(\omega_0)$ with a coefficient -0.2]. The characteristic feature of the zero-range approximation, that the scattering is very strong in one of the two isotopic spin states,[4] depends essentially only on the fact that the large K^--p elastic cross section is accompanied by a substantial charge-exchange cross section at low energy. Each satisfactory set of parameters includes one scattering length whose real part exceeds 1.5 fermis in magnitude. We should like to add that only an extremely unlikely change in the experimental situation could reduce this value to 1 f. Since the elastic K^--proton interaction cannot have a range exceeding $\hbar/2m_\pi c \sim 0.7$ f, it follows that the interaction is attractive in the isotopic state with the large scattering length. If $D_-(1)$ is negative, then the attraction is so strong as to give rise to a "bound" state which will be unstable to decay into a pion-hyperon system. $D_-(\omega)$ can be expected to change sign near $\omega \sim 1.2$, a meson laboratory momentum of 300 Mev/c. If $D_-(1)$ is positive, then the attraction is not strong enough to give rise to a "bound" state. In this case $D_-(\omega)$ will decrease monotonically from its low-energy value; near $\omega \sim 1.2$, it will already be quite small. Since the forces are attractive and strong in both of these cases, one cannot expect to distinguish between them by a study of the nuclear "optical model" potential for K^- mesons, which is then not simply related to the low-energy scattering amplitude. A measurement of the Coulomb interference effects in the angular distribution of K^--proton scattering will, of course, decide the matter.

Table II. "Average" couplings determined from Eq. (3) by the use of data at the energies ω and ω_0. The results are independent, to the accuracy of this work, of the choice of sign for $D_-(\omega)$.

ω	ω_0	$\langle pX\rangle_{\text{Av}}$
1.46	1.285	-0.2 ± 0.6
1.46	1.17	-0.1 ± 0.4
1.00	1.285	$+0.1 \pm 0.4$
1.00	1.17	$+0.3 \pm 0.6$

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MAGNITUDE OF RENORMALIZATION CONSTANTS*

S. G. Gasiorowicz
Lawrence Radiation Laboratory, University of California, Berkeley, California

and

D. R. Yennie and H. Suura
University of Minnesota, Minneapolis, Minnesota
(Received May 11, 1959)

Five years ago Källén[1] published a proof that at least one of the renormalization constants in quantum electrodynamics is infinite. A subsequent more refined analysis[2] led to the conclusion that it is the electron wave function renormalization which diverges. Recently Johnson[3] raised the question whether a formulation of the theory in another gauge might not alter this result. We have re-examined this problem and arrive at the conclusion that (a) Källén's proof is not conclusive, (b) the renormalization constants could be finite under rather special circumstances, and (c) the question of gauge invariance is quite irrelevant to this problem. We begin with a brief review of Källén's work on this problem.[4] It can be shown that the charge renormalization $(1-L)^{-1/2}$ can be expressed in the form

$$(1-L)^{-1} = 1 + \bar{\Pi}(0), \tag{1}$$

where by definition

$$\bar{\Pi}(Q^2) = P\int_0^\infty da\, \frac{\Pi(-a)}{a+Q^2}, \tag{2}$$

and

$$\Pi(Q^2) = -\frac{V}{3Q^2}\sum_{p^{(n)}=Q} \langle 0|j_\mu|n\rangle\langle n|j_\mu|0\rangle; \tag{3}$$

P denotes the principal value. Källén shows that in spite of the indefinite metric associated with quantum electrodynamics, $\Pi(-a)$ is positive for positive a, and that therefore a lower limit to the integral in Eq. (1) can be obtained by considering any subset of eigenstates of the Hamiltonian. The simplest of these are the states consisting of an incoming electron-positron pair. Our task therefore is to study the high-energy limit of the matrix element $\langle 0|j_\mu|p,p'\rangle$.

By the use of reduction formulas[1,5] one may obtain the following compact expression for the matrix element in question:

$$\langle 0|j_\mu|p,p'\rangle = \langle 0|j_\mu^{(0)}|p,p'\rangle[1-\bar{\Pi}(Q^2)+\bar{\Pi}(0) - i\pi\Pi(Q^2) + F_1(Q^2) - F_1(0)] + iQ_\nu\langle 0|m_{\mu\nu}^{(0)}|p,p'\rangle F_2(Q^2), \tag{4}$$

$$Q = p + p', \tag{4a}$$

where

$$N^2\bar{u}(p')\{\iint d^4x\,d^4y\, e^{i(p'x+py)}[\theta(-x)\theta(-y)\langle 0|\{[j_\mu(0),\bar{f}(x)],f(y)\}|0\rangle - \theta(-y)\theta(y-x)\langle 0|\{[j_\mu(0),f(y)],\bar{f}(x)\}|0\rangle]\}v(p)$$
$$= F_1(Q^2)\langle 0|j_\mu^{(0)}|p,p'\rangle + iF_2(Q^2)Q_\nu\langle 0|m_{\mu\nu}^{(0)}|p,p'\rangle. \tag{5}$$