Next, if $\omega << \omega_H$, which is valid except far out and for quite high frequencies, Eq. (4) becomes

$$\omega^2 - \omega[2\omega_s + (V^2\omega_p^{\ 2}/c^2\omega_H)] + \omega_s^{\ 2} = 0.$$

Hence

$$\omega = \omega_s + \frac{V^2\omega_p^{\ 2}}{2c^2\omega_H} \pm \left(\frac{V^4\omega_p^{\ 4}}{4c^4\omega_H^{\ 2}} + \frac{V^2\omega_p^{\ 2}\omega_s}{c^2\omega_H}\right)^{1/2}. \qquad (6)$$

In the special case where $\omega_s << V^2\omega_p^{\ 2}/2c^2\omega_H$, we find

$$\omega \doteq V^2\omega_p^{\ 2}/c^2\omega_H, \qquad (7)$$

which is identical to Gallet's equivalent result.

Thus it has been shown that Gallet's equations can be derived as limiting cases of a more general equation resulting from a quite different mechanism.

Several types of phenomena, such as the rising tones often associated with the "tails" of whistlers, are no doubt more easily understood in terms of the travelling-wave tube mechanism. However, it is likewise possible that the Doppler and proton-gyro concept can be made to cover cases not explainable by the former theory, and further work is progressing along these lines.

Evidently, then, the two processes are complementary rather than mutually exclusive, and the full explanation of all types of dawn chorus will entail a combination of the two theories.

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SEARCH FOR AN ELECTRIC DIPOLE MOMENT OF THE ELECTRON*

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It has been pointed out[1,2] that the detection of an electric dipole moment (EDM) in an elementary particle would constitute a violation of both time-reversal invariance and space-reflection invariance. Experimental upper limits on the size of an EDM for several elementary particles have been found.[3-7] A summary of these limits is given in Table I. Garwin and Lederman[8] have remarked that the method used in searching for the muon EDM could be extended to higher accuracy for the electron. The anomalous magnetic dipole moment (MDM) experiment of Schupp, Pidd, and Crane[9] is an experiment of this general type. The purpose of this note is to show that this experiment does yield a new upper limit on the size of an EDM of the electron and to describe an extension of the same experiment that is expected to have a considerably greater sensitivity for detecting an EDM of the electron.

Table I. Experimental upper limits of the electric dipole moments of elementary particles.

Particle	Electric dipole moment	Reference
Electron	$\lesssim 10^{-13}$ cm $\times e$	3, 4
Positron	$\lesssim 8 \times 10^{-13}$ cm $\times e$	4
Proton	$\lesssim 10^{-13}$ cm $\times e$	5
Neutron	$\lesssim 3 \times 10^{-20}$ cm $\times e$	6
Muon	$\lesssim 2 \times 10^{-15}$ cm $\times e$	7

In the experiment on the anomalous MDM an unpolarized beam of electrons of 100-kev energy is sent parallel to a magnetic field. A portion of the beam that has been polarized by Mott scattering from a thin gold foil at almost 90° is allowed to execute a helical path in the magnetic field (see Fig. 1). Application of a small trapping field allows the retention of the electrons in the nearly homogeneous magnetic field for times up to several hundred microseconds. The beam then strikes a second thin gold foil. Two Geiger counters, placed at a scattering angle of 80° and in orientations parallel and antiparallel to the magnetic field, are used to observe the Mott asymmetry as a function of the time the electrons are trapped in the field. The ratio of the counts in these two counters has a periodic variation with trapping time. The frequency of this variation, the so-called difference frequency, is the magnitude of the angular velocity of spin pre-

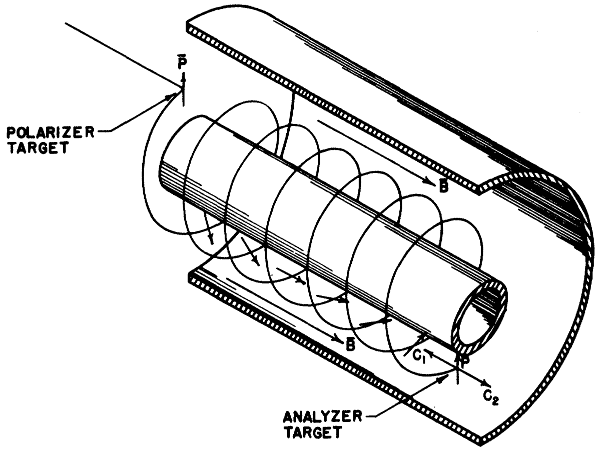

FIG. 1. Experimental geometry. The precession of the polarization vector $\vec{P}$ about the magnetic field $\vec{B}$ is indicated at several positions along the beam trajectory. The positions of the two counters are shown by C_1 and C_2.

cession relative to the velocity. In the absence of an EDM of the electron, this difference frequency is directly proportional to the anomalous part of the MDM. The existence of an EDM of the electron would increase this frequency by giving the electron spin an additional precession about the $\vec{v}\times\vec{B}$ direction. The contribution to this frequency of the EDM can, however, be distinguished by its energy dependence. Since in the anomalous MDM experiment the difference frequency was measured as a function of beam energy (the magnetic field being changed correspondingly to hold the beam at the same radius), a sensitive test of the existence of an EDM is contained in the result.

The general equation[10] governing the precession of the spin vector $\vec{\sigma}$ of an electron having an anomalous MDM and an EDM in both a magnetic field $\vec{B}$ and an electric field $\vec{E}$ (which vary only on a macroscopic scale[11]) is

$$d\vec{\sigma}/dt = \vec{\omega}_s \times \vec{\sigma}, \tag{1}$$

where

$$\vec{\omega}_s = -\frac{e}{m_0\gamma}\left[\vec{B} - \frac{\gamma}{\gamma+1}\frac{\vec{v}\times\vec{E}}{c^2}\right] - \frac{ea}{m_0}\left[\vec{B} - \frac{\gamma-1}{\gamma}\frac{\vec{B}\cdot\vec{v}}{v^2}\vec{v} - \frac{\vec{v}\times\vec{E}}{c^2}\right] - \frac{ef}{m_0 c}\left[\vec{E} - \frac{\gamma-1}{\gamma}\frac{\vec{E}\cdot\vec{v}}{v^2}\vec{v} + \vec{v}\times\vec{B}\right]. \tag{2}$$

The factors a and f are defined in terms of the MDM vector $\vec{\mu}$ and the EDM vector $\vec{p}$ by $\vec{\mu} = (1+a) \times e\hbar\vec{\sigma}/2m_0$, and $\vec{p} = fe\hbar\vec{\sigma}/2m_0c$; e, m_0, and $\vec{v}$ are the charge, rest mass, and velocity of the electron; $\gamma = (1-\beta^2)^{-1/2}$ and $\beta = v/c$. The difference frequency is defined as

$$\omega_D = |\vec{\omega}_s - \vec{\omega}_c|, \tag{3}$$

where $\vec{\omega}_c$ is the angular velocity of cyclotron rotation. For the case of $\vec{E}=0$ and $\vec{v}\cdot\vec{B}=0$

$$\omega_D = \omega_0(a^2+f^2\beta^2)^{\frac{1}{2}}, \tag{4}$$

where $\omega_0 = eB/m_0$. (The error introduced by assuming $\vec{v}\cdot\vec{B}=0$ is negligible.) It is seen that ω_D increases with increasing energy and that at infinite energy $\omega_D = (a^2+f^2)^{1/2}$. In the experiment on the anomalous MDM, $\omega_D^{\exp} > a$ when extrapolated to infinite energy, but $\omega_D^{\exp}$ decreases with increasing energy. (See Fig. 2.) The most likely

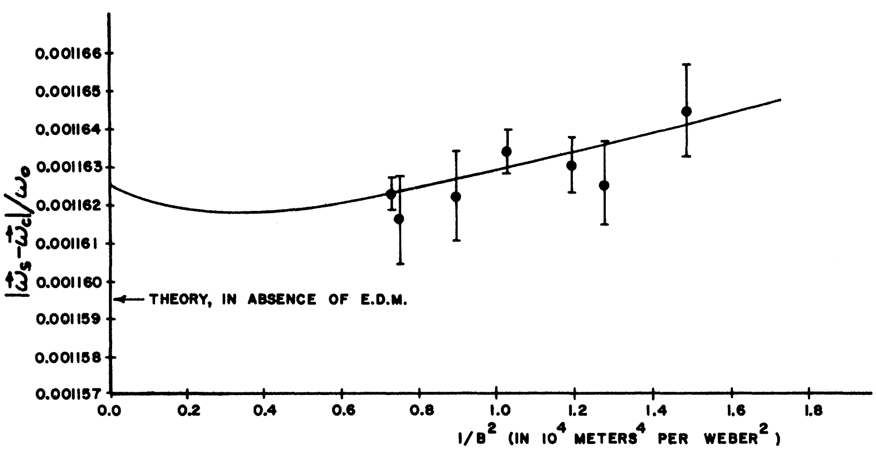

FIG. 2. Plot of difference frequency data. The experimental values of the difference frequency divided by eB/m_0 are extrapolated to infinite magnetic field (infinite energy). From the intercept $(a^2+f^2)^{1/2}$ the factor f (the EDM in units of $e\hbar/2m_0c$) can be found assuming the theoretical value of the g-factor anomaly a.

cause of this reversed slope is the presence of a small stray radial electric field. An electric field in any other direction, large enough to explain the slope, would have to be about 100 times larger than one that could reasonably be expected to exist in the apparatus. If one represents the stray electric field as a small radial electric field E, constant in magnitude at the orbit radius, ω_D is given by

$$\omega_D = \omega_0 \left\{ \left[a + \frac{E}{cB}\left(\frac{\beta^2 + a\beta^2 - 1}{\beta} \right) \right]^2 + f^2 \left[\beta + \frac{E}{cB} \right]^2 \right\}^{\frac{1}{2}}. \quad (5)$$

If a two-parameter (f and E) fit of Eq. (5) is made to the data, the theoretical value of a being assumed, the results are

$$|p| = (1.6 \pm 0.6) \times 10^{-15} \text{ cm} \times e,$$

$$E = -(0.048 \pm 0.013) \text{ v/cm}. \quad (6)$$

It is reasonable that a stray electric field of this size existed in the apparatus. The errors quoted are standard deviations of statistical error. Systematic errors are estimated to be of the order of 0.15 % in each measurement of ω_D/ω_0. When these errors are included, the results are consistent with the nonexistence of an EDM of the electron. It is concluded that

$$|p| \lesssim 3 \times 10^{-15} \text{ cm} \times e. \quad (7)$$

It appears that the smallest EDM detectable by this method is about 3×10^{-16} cm $\times e$. This would require more complete elimination of stray electric fields (and measurement of them, if possible), improvement in the accuracy of the measurement of ω_D/ω_0, and extension of the measurements of ω_D/ω_0 to higher energy (~600 kev).

The fundamental difficulty of this method of searching for an EDM is that the precession of the spin produced by the EDM is small compared with that produced by the anomalous part of the MDM. This difficulty can be removed if the magnetic field in the system of the electron can be reduced enough so that the spin precession frequency due to the whole MDM equals the cyclotron rotation frequency. This can be done by providing a small radial electric field (~20 v/cm). In the absence of an EDM the effect of the radial electric field is to fix the spin vector relative to the velocity vector. If there is an EDM there will be a precession of the spin caused by the EDM around the radial direction ($\vec{v} \times \vec{B}$), and there will not be any anomalous MDM precession present to obscure it.

For the radial electric field scheme to work, the polarization vector of the beam cannot initially be in the radial direction since there would then be no torque on the EDM. Since this is the initial orientation of the polarization vector in the experiment on the anomalous MDM, a modification of this setup is necessary. The modification is to allow the polarization vector to precess, due to the anomalous MDM, to an orientation parallel to the velocity before switching on the radial electric field. The only further modification needed is the re-orientation of the counters in the radial direction. The ratio of the counts in the two counters will then vary with trapping time according to the frequency of the precession produced by the EDM, given by

$$\omega_D = \omega_0 f(\beta + E/cB), \quad (8)$$

where the electric field is determined by

$$a + (E/cB)[(\beta^2 + a\beta^2 - 1)/\beta] = 0. \quad (9)$$

The amplitude of this variation will be the same as that observed in the anomalous MDM experiment. A second pair of counters, oriented like those in the anomalous MDM experiment, will be used to determine experimentally the proper adjustment of the radial electric field. The absence of a variation in the ratio of counts in these two counters when the trapping time is changed will indicate that the electric field is properly adjusted.

The practical limit of sensitivity of this method is imposed by the depolarization arising from the energy width of the beam in conjunction with the variation of the electric field with radius. Electrons of different energy will occupy orbits of different radii and so experience different electric fields. At only one radius will the electric field be such as to eliminate the anomalous MDM precession exactly. At other radii the anomalous MDM will cause additional precession which will cause a depolarization. If a $1/r$ falloff of the electric field and an energy width of 100 ev at 100-kev beam energy (caused by ionization losses in the first scattering foil) are assumed, a useful trapping time of 3×10^{-3} sec results. Since the phase of the EDM precession can be changed at will by changing the time the radial electric field is switched on, a tenth of a period of the EDM precession can be distinguished from spurious effects. An upper limit on the EDM of the electron of about 4×10^{-18} cm $\times e$ would result. When working to this accuracy, effects due to the component of the velocity parallel to the magnetic field, to stray electric fields, and to radial magnetic fields in the trapping region can be made negligibly small.

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[11]These equations refer to expectation values of the various operators over a wave packet whose dimensions are small compared with the lengths in which the fields vary appreciably.

THEORY OF THE LATTICE VIBRATIONS OF GERMANIUM

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The relation between frequency ω and wave vector $\vec{q}$ (the dispersion relation) for the normal modes of germanium has been investigated experimentally by Brockhouse and Iyengar[1] (see also Brockhouse[2]) and by Ghose, Palevsky, Hughes, Pelah, and Eisenhauer.[3] The results do not agree with prior calculations made by Hsieh,[4] on the assumption that only nearest neighbor atoms interact and using the Born-von Kármán theory of lattice dynamics. Nor is it sufficient to introduce second neighbor interaction[1]; indeed Herman[5] has shown that force constants between atoms out to at least fifth neighbors are required to account for the experimental results. The theory then involves numerous parameters which have no clear physical significance. Lax[6] proposes to fit the data to a force model involving one parameter to represent electrostatic interaction between quadrupoles generated by the lattice vibrations, and as many near neighbor parameters as proves necessary.

We have extended the Born-von Kármán theory to apply to a model of the germanium crystal in which each atom is treated as a charged core coupled isotropically to an oppositely charged shell of negligible mass. This atom model has had some success in accounting for the dielectric properties[7,8] and lattice dynamics[9] of alkali halide crystals. Its use introduces two long-range forces, firstly the electrostatic interaction between dipoles formed by the relative displacement of cores and shells, and secondly a force which originates from the massless character of the shells. The shell model therefore simulates two effects which must occur in practice, electrostatic interaction and relatively widespread redistribution of valence electron density when the nuclei are displaced. The condition that the crystal as a whole can have no dipole moment[6] is automatically satisfied, so that the electrostatic interaction need not be treated as quadrupole-quadrupole.

In the Born-von Kármán theory the dispersion relation is determined by the condition for solubility of the equations

$$\sum_{k'y}[M_{xy}(kk') - \omega^2 m_k \delta_{kk'}]U_y(k') = 0, \qquad (1)$$

where $\vec{U}(k)\exp i[\vec{q}\cdot\vec{r}(lk) - \omega t]$ is the displacement from $\vec{r}(lk)$ of the kth atom in the lth cell. We define $M_{xy}(kk')$ by writing

$$M_{xy}(kk') = B_{xy}(kk') + C_{xy}(kk'),$$

where $B_{xy}(kk')$ is related to the bonding force constant $\Phi_{xy}(lk, l'k')$ between atoms (lk) and $(l'k')$ by the equation

$$B_{xy}(kk') = -\sum_{l'}\Phi_{xy}(lk, l'k') \times \exp i\vec{q}\cdot[\vec{r}(l'k') - \vec{r}(lk)], \qquad (2)$$

and $C_{xy}(kk')$ is related to the electrostatic interaction between charged atoms by a similar expression.[10] For a diamond-type crystal with $\vec{q}$ along