for W that has been derived by several authors[2]:

$$W_{El}(r) = \pi[\langle \varphi_0 | V | \varphi_c u_p(r)\rangle^2]_{\text{Av}}\rho_{El}.$$

φ_c are states of the target particles i, φ_0 is the ground state, u_p are states of the extra nucleon, V is the nucleon-nucleus interaction $\sum_i v(\vec{r} - \vec{r}_i)$, the integration is over target particles, $[\ ]_{\text{Av}}$ denotes average over product states $\varphi_c u_p$ of angular momentum l near energy E, and ρ_{El} is the density of such states. For N near 50 and Z nonmagic, and an extra neutron, one expects ρ_{El} to have less than half the normal value. Provided that the matrix elements do not vary in a reciprocal fashion (and there is no reason to expect such perverse behavior), W will be less in proportion. Since the neutron orbit completing $N=50$ is a g-orbit, $W(r)$ is expected to be somewhat peaked at the nuclear surface. For Z near 50, and N nonmagic, ρ_{El} should have about half the usual value. There will also be surface peaking especially if neutrons have begun to fill the h-orbit.

To discuss the effect of such changes in W on the strength function, s, one may use[3]

$$s \sim \int W(r)\,|u(r)|^2\,dr,$$

where $u(r)$ is the nucleon wave-function in the complex potential. Between single-particle levels (i.e., near $A \sim 100$ for s-waves), not only is s decreased by the reduction in W, but it is further decreased if W is surface-peaked since $u(r)$ has a surface node. These two facts may thus explain the discrepancy in the observed values of s near $N, Z = 50$. Near the center of a single-particle level, $|u(r)|^2 \sim W^{-2}$ and $s \sim W^{-1}$, so s is increased by a reduction in W. This leads one to expect an especially large p-wave strength function near $A \sim 90$ and may help to explain why the capture cross section at 50 kev is so large in Nb.[4] One also expects a large s-wave strength function near $A \sim 50$ caused by a reduction in W due to magic number 28, and there is some weak evidence for this. Furthermore the observed[5] diminution in the width of the photonuclear peak near closed shells may be associated with a reduction in W.

---

*On leave of absence from Columbia University.

[1] A. E. S. Green and P. C. Sood, Phys. Rev. 111, 1147 (1958).

[2] J. M. C. Scott, Phil. Mag. 45, 1322 (1954); C. Bloch, Nuclear Phys. 3, 137 (1957); G. E. Brown and C. de Dominicis, Ann. Phys. (to be published).

[3] C. E. Porter, Phys. Rev. 100, 935 (1955).

[4] J. H. Gibbons (private communication).

[5] R. Nathans and J. Halpern, Phys. Rev. 92, 207 (1953).

POSSIBLE RESONANT STATE IN PION-HYPERON SCATTERING*

R. H. Dalitz and S. F. Tuan
Enrico Fermi Institute for Nuclear Studies and Department of Physics,
University of Chicago, Chicago, Illinois
(Received April 27, 1959)

With charge independence, it is convenient to describe the s-wave scattering processes of low-energy K^--proton collisions by two complex scattering lengths A_0 and A_1, one each for the $I=0$ and $I=1$ channels, related to the complex phase shifts δ_I by

$$k\cot\delta_I = 1/A_I(k), \tag{1}$$

where k denotes the center-of-mass momentum of the $K^- - p$ system. Since the $K^- - p$ interaction is expected to have short range ($\sim\hbar/m_K c$), Jackson et al.[1] have suggested that it is reasonable to neglect[2] the energy dependence of these amplitudes for c.m. energies below ~ 50 Mev. On this basis, an analysis[3] of the $K^- - p$ interaction data available from bubble-chamber investigations at low energies[4] has led to the following four solutions[5] for these amplitudes A_0 and A_1:

$$A_0 = (0.20 + 0.78i)\text{ f},\ A_1 = (1.62 + 0.39i)\text{ f}, \quad \text{(a+)}$$

$$A_0 = (1.88 + 0.82i)\text{ f},\ A_1 = (0.40 + 0.41i)\text{ f}, \quad \text{(b+)}$$

and the sets (a-), (b-) obtained from (a+), (b+) by reversing the signs of the real parts of both A_0 and A_1. As Jackson and Wyld[6] have recently pointed out, the "repulsive" interactions, that is amplitudes of the type (a-) and (b-), predict the lower elastic scattering cross sections at very low energies, owing to their destructive interference with the Coulomb scattering, and are in accord with the trend found for the cross sections at the lowest energies in emulsion studies.[7] It will be pointed out here that this situation makes it quite probable that there should exist a resonant state for pion-hyperon scattering at an energy of about 20 Mev below the $K^- - p$ (c.m.) threshold energy. In the present discussion, charge-dependent refinements due to the Coulomb interaction and the $K^- - \bar{K}^0$ mass difference will be neglected.

With Eq. (1), the $\bar{K} - N$ scattering amplitude for the s-wave of isotopic spin I takes the form

$$\langle \bar{K}N | T | \bar{K}N \rangle = (a_I + b_I)/\{1 - ik(a_I + ib_I)\}, \qquad (2)$$

where $a_I + ib_I = A_I$ and the total c.m. energy E equals $(m_p + m_K)(1 + k^2/2m_p m_K)$. In the neighborhood of the threshold $E_0 = m_p + m_K$, expression (2) may be analytically continued as a function of E from the real axis $E > E_0$ into the upper half of the complex E-plane and thus onto the real axis $E < E_0$. If a_I is large and negative, expression (2) has a pole P in this neighborhood, corresponding to $k = -i/(a_I + ib_I)$. This pole lies close to the real axis $E < E_0$, but on the (unphysical) lower half-plane reached by analytic continuation from the upper half-plane across the cut which must exist between $E = E_0 = m_p + m_K$ and the $\pi - \Sigma$ threshold $E = m_\Sigma + m_\pi$. With solution (a-), this particular pole occurs in the $T = 1$ amplitude; with solution (b-), it occurs in the $T = 0$ amplitude. As pointed out earlier,[3] this pole leads to a resonance-like energy dependence of $\mathrm{Im}\langle K^- p | T | K^- p \rangle$ in the unphysical region $E < E_0$ of interest for K-meson dispersion relations, with peak at c.m. momentum $ik = a_I/(a_I^2 + b_I^2)$. The effect of this pole on the pion-hyperon scattering in this energy region has now been investigated. For simplicity, our remarks here will be confined to the $T = 0$ state [relevant for the amplitude (b-)], since this concerns only the $\pi - \Sigma$ system. For the $T = 1$ state [relevant for the amplitude (a-)], the situation is quite similar, although complicated by the participation of both $\pi - \Lambda$ and $\pi - \Sigma$ systems in general.

The amplitude for $\pi - \Sigma$ scattering is related to the $K^- - p$ amplitude through the unitarity condition. This relationship may be made explicit by expressing each in terms of the K-matrix.[8] For $T = 0$ and $E > E_0$, the K-matrix has three real elements,[9] $\alpha = \langle \bar{K}N | K | \bar{K}N \rangle$, $\beta = \langle \bar{K}N | K | \pi\Sigma \rangle$, and $\gamma = \langle \pi\Sigma | K | \pi\Sigma \rangle$. The amplitude $A(k)$ is expressible in terms of these parameters as follows:

$$A(k) = a + ib = -\alpha + i(q/E)\beta^2/\{1 + i(q/E)\gamma\}, \qquad (3)$$

where q denotes the c.m. momentum of the $\pi - \Sigma$ system at energy E. For s-wave interactions,[10] the assumption that α, β, and γ are energy independent is appropriate in the neighborhood of $E = E_0$. This is equivalent to the zero-range approximation of Jackson et al.,[1] i.e., to the assumption of a constant amplitude A, provided the variation of q/E is also neglected, a reasonable approximation sufficiently close to $E = E_0$. After identifying A with the expression (3) at $E = E_0$, it is convenient to choose for the remaining parameter the $\pi - \Sigma$ scattering phase shift σ_0 at this threshold energy. The $\pi - \Sigma$ scattering phase shift σ_Σ at energy E is then given by

$$\frac{q}{q_0}\cot\sigma_\Sigma = \frac{1}{\lambda}\cot\sigma_0 \frac{1 + ik\lambda(-a - b\tan\sigma_0)}{1 + ik\lambda(-a + b\cot\sigma_0)}, \qquad (4)$$

where q_0 corresponds to the threshold energy E_0, and $\lambda = E_0/E$ will henceforth be replaced by unity.

For comparison with the expression (2), the further approximation $(q/q_0) \sim 1$ leads from Eq. (4) to the following expression for the $\pi - \Sigma$ scattering amplitude,

$$\langle \pi\Sigma | T | \pi\Sigma \rangle = \frac{(1 - ika)\sin\sigma_0 + ikb\cos\sigma_0}{1 - ik(a + ib)} e^{i\sigma_0}. \qquad (5)$$

This expression (5) also has a pole at $k = -i/(a + ib)$. The expressions which correspond to (3) and (5) without these approximations similarly have a complex pole in common.

To indicate the energy dependence of σ_Σ for $E < E_0$, Eq. (4) may be written[10]

$$(q/q_0)^{2l+1}\cot\sigma_\Sigma = \cot(\sigma_0 - \theta), \qquad (6)$$

where θ is the angle

$$\theta = \psi - \varphi = \arg\left(\frac{-i}{a + ib} - k\right) - \arg\left(\frac{-i}{a + ib}\right), \qquad (7)$$

shown in Fig. 1. When a is large and negative, the pole P lies close to (and to the left of) the positive imaginary k-axis. As k runs from 0 up the imaginary axis past P, the angle θ increases rapidly from zero to large values (at most 180°). If $b/a \ll 1$ and $-90° \leq \sigma_0 \leq 0°$ (or $90° \leq \sigma_0 \leq 180°$), then σ_Σ will definitely pass through $\pm 90°$ between energies E_0 and $E_0[1 - 1/(2a^2 m_p m_K)]$, an energy range over which the zero-range approximation appears well justified. However, the energy at which $\sigma_\Sigma = \pm 90°$ does not generally coincide with the peak of $\mathrm{Im}\langle \bar{K}N | T | \bar{K}N \rangle$; in fact, if σ_0 is positive and a little below 90°, it is quite possible that σ_Σ does not take the value $\pm 90°$ within the energy range for which the zero-range approxi-

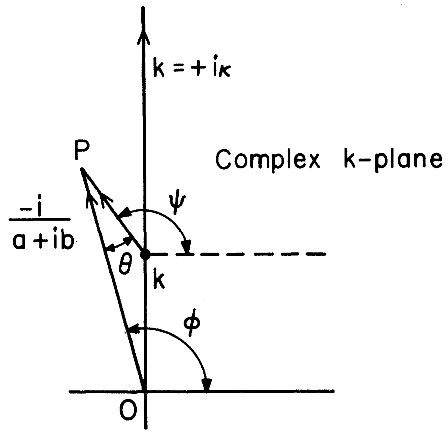

FIG. 1. The location of pole P on the complex k-plane.

mation is reasonable.

In Fig. 2, cross sections for $T=0$ π - Σ elastic scattering are plotted for solution (b-) as function of σ_0. As expected from general theorems,[11] these cross sections show prominent S-shaped or pointed cusps at the K^--p threshold. For $-90° \leq \sigma_0$ (mod 180°) $\leq 0°$, quite a narrow resonance (half-width $\lesssim 20$ Mev) appears in these cross sections just below this threshold. We may make the following remarks:

(*i*) The resonance will be still more pronounced if the K-meson is scalar. In this case, the resonant scattering takes place in a $p_{\frac{1}{2}}$ state, for

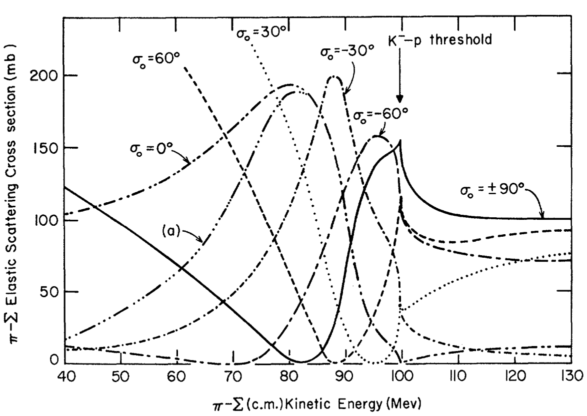

FIG. 2. The total cross section predicted for $T=0$ π - Σ elastic s-wave scattering is plotted as function of π - Σ c.m. energy in the neighborhood of the K^--p threshold for various values of σ_0, the π - Σ scattering phase shift at this threshold energy, assuming the K meson to be pseudoscalar. Curve (a) depicts the energy dependence of the $T=0$ π - Σ elastic scattering in the $p_{\frac{1}{2}}$ state, for the case $\sigma_0=0°$, assuming the K meson to be scalar.

which the cross section must vanish at the pion-hyperon threshold, and so falls off more rapidly than for an $s_{\frac{1}{2}}$ state as E decreases below the resonant energy. This is illustrated for the case $\sigma_0=0°$ by curve (a) of Fig. 2.

(*ii*) The $T=1$ resonances obtained for solution (a-), which gives the more satisfactory K^--p elastic cross sections at low energies,[3] will be sharper than those for solution (b-) and will persist for a wider range of σ_0, since the ratio b_1/a_1 for the former solution is smaller than b_0/a_0 for the latter.

That the K-baryon couplings contribute in an important way to the features of pion-hyperon elastic scattering must be emphasized here, since a number of authors[12] have attempted to discuss pion-hyperon scattering in terms of elementary pion-hyperon couplings alone. However, it is clear from Fig. 2 that the $K^-+p\rightarrow\Sigma+\pi$ reactions, which are due to the K-baryon couplings, have a strong effect on the π - Σ scattering near the K^--p threshold. A perturbation treatment of the effect of the K-baryon coupling on the pion-hyperon scattering is a very poor approximation. Similarly, the π - Σ resonance discussed here arises primarily from the K-interactions, being a consequence of the properties of low-energy $\bar{K}$ - N scattering; it therefore appears unrelated with the pion-hyperon resonances investigated recently by Landovitz and Margolis,[12] and Nauenberg[12] for particular pion-hyperon coupling schemes.

The existence and isotopic-spin character of this resonance will have to be established indirectly, for example:

(a) By examining the correlations between outgoing pions and hyperons in strange particle reactions. For example, in the reactions

$$K^-+p\rightarrow\begin{cases}\Sigma^{\pm}+\pi^{\mp}+\pi^0, & (8a)\\ \Lambda\ +\pi^{+}+\pi^{-}, & (8b)\end{cases}$$

the pion-hyperon Q-values may be expected to peak markedly at this resonance energy. A peak in the π - Λ Q-values could occur only if $I=1$ held for the resonant state. Other examples are

$$\pi^-+p\rightarrow\begin{cases}\Sigma^{\mp}+\pi^{\pm}+K^0, & (9a)\\ \Sigma^{-}+\pi^0+K^+, & (9b)\end{cases}$$

$$\pi^++p\rightarrow\ \Sigma^++\pi^0+K^+. \qquad (10)$$

Only an $I=1$ (or $I=2$) π - Σ resonance[13] can produce strong π - Σ correlations in reactions (9b) and (10), whereas either $I=0$ or $I=1$ (as well as $I=2$) resonances can contribute to (9a).

(b) By the study of inelastic hyperon-nucleon scattering $Y+p \to Y+\pi+p$, following the extrapolation procedures discussed recently by Chew and Low,[14] which allow determination of $\pi^0-\Lambda$ and $\pi^0-\Sigma^+$ (as well as $\pi^0-\Sigma^-$) cross sections from observations on the recoil protons.

(c) The distribution of $\pi-\Sigma$ Q-values in the capture reactions

$$K^- + d \to \begin{cases} \Sigma^\pm + \pi^\mp + n, & (11a) \\ \Sigma^- + \pi^0 + p, & (11b) \end{cases}$$

may be substantially affected[15] by a $\pi-\Sigma$ resonance just below E_0 since the final $\pi-\Sigma$ systems of these reactions have c.m. energy below E_0, owing to the deuteron binding energy and the neutron recoil energy. As discussed by Karplus and Rodberg,[16] however, this situation is complicated by the strong $\Sigma-N$ and $\pi-N$ interactions in the final state.

A detailed discussion of the derivation of the expressions quoted in this Letter is at present in preparation.

---

*Work carried out under the auspices of the U. S. Atomic Energy Commission.

[1]Jackson, Ravenhall and Wyld, Nuovo cimento 9, 834 (1958).

[2]The suggestion of P. T. Matthews and A. Salam [Phys. Rev. Lett. 2, 226 (1959)] that the K^--p system has a resonant state at $\epsilon_\gamma \sim 15$ Mev c.m. kinetic energy is equivalent to replacing this assumption by $k\cot\delta = [A(k)]^{-1} = [A(0)]^{-1} + \frac{1}{2}Rk^2$, where $A(0) = (\Gamma_K/k)(-\epsilon_\gamma - i\Gamma_\pi)^{-1}$ and $R = (k/\Gamma_K)(1/m_p + 1/m_K)$ are constant. If the $A(k)$ determined in reference 2 for $k = 113$ Mev/c are correct, a resonance is possible in the low-energy region only if the effective range R is taken unreasonably large ($\gtrsim 7/m_K$) relative to $1/m_K$. Also the resonance must be very broad ($\Gamma_K + \Gamma_\pi \gtrsim 120$ Mev at energy ϵ_γ). There is nothing compelling in the present data to support this resonance proposal; in fact, the smallness of the charge-exchange/elastic-scattering ratio (see reference 4) at ~40-Mev laboratory energy conflicts sharply with the notion of a dominant isotopic-spin resonance. As pointed out in reference 6, the simpler assumption of constant A_I is not in conflict with the existing data.

[3]R. H. Dalitz and S. F. Tuan, Ann. Phys. (to be published).

[4]Alvarez, Bradner, Falk-Variant, Gow, Rosenfeld, Solmitz, Tripp, and Watson, Phys. Rev. Lett. 2, 312 (1959); Nordin, Rosenfeld, Solmitz, Tripp, and Watson, Bull. Am. Phys. Soc. Ser. II, 4, 288 (1959); A. H. Rosenfeld (private communication).

[5]The unit is the fermi (f); 1 fermi $= 10^{-13}$ cm.

[6]J. D. Jackson and H. W. Wyld, Phys. Rev. Lett. 2, 355 (1959).

[7]Ascoli, Hill, and Yoon, Nuovo cimento 9, 313 (1958); Alles, Biswas, Ceccarelli, Gessaroli, Quareni, Going, Gottstein, Puschel, Tietge, Zorn, Crussard, Hennessy, Dascola, and Mora (to be published).

[8]B. Lippmann and J. Schwinger, Phys. Rev. 79, 469 (1950).

[9]This remark assumes time-reversal invariance (and parity conservation) for the strong interactions.

[10]Both $\bar{K}-N$ and $\pi-\Sigma$ systems will be s-wave only with negative parity for the K meson (relative to even parity for the Σ hyperon). For a scalar K meson, the final $\pi-\Sigma$ system will be $p\frac{1}{2}$, and an energy dependence $\beta = \beta_0(q/q_0)$, $\gamma = \gamma_0(q/q_0)^2$ is appropriate. With the latter situation, the phase shift σ_Σ is still given by Eq. (4) after replacement of (q/q_0) on the left by $(q/q_0)^3$.

[11]See the review article by A. M. Lane and R. G. Thomas, Revs. Modern Phys. 30, 257 (1958), for example.

[12]M. Gell-Mann, Phys. Rev. 106, 1296 (1957); M. Ross, Phys. Rev. 112, 986 (1958); L. F. Landovitz and B. Margolis, Phys. Rev. Lett. 2, 318 (1959); M. Nauenberg, Phys. Rev. Lett. 2, 351 (1959).

[13]The presence of an $I=2$ $\pi-\Sigma$ resonance in reactions (9) and (10) can be distinguished by examination of the competing $\Sigma^+ + \pi^+ + K^0$ final states in reaction (10).

[14]G. F. Chew and F. E. Low, Phys. Rev. (to be published).

[15]This was pointed out to us by Professor J. D. Jackson.

[16]R. Karplus and L. Rodberg, Phys Rev. (to be published).