

significant that the most prominent feature of the forward-peaked alpha particles is their relatively large number at about 40 Mev in the laboratory system. Alpha particles of 40 Mev have the same velocity as that of the incident oxygen ions.

The symmetric parts of the energy spectra in the center-of-mass system will be compared to nuclear evaporation calculations.<sup>1</sup> Similar measurements on other reactions of this type are in progress. A detailed description of the

apparatus used will be published elsewhere.

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<sup>1</sup>Dostrovsky, Rabinowitz, and Bivins, Phys. Rev. **111**, 1659 (1958).

<sup>2</sup>L. Wolfenstein, Phys. Rev. **82**, 690 (1951).

<sup>3</sup>T. Ericson and V. Strutinski, Nuclear Phys. **8**, 184 (1958).

### X-RAY YIELDS IN $\mu$ -MESONIC ATOMIC TRANSITIONS

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In a series of very careful experiments Stearns and Stearns<sup>1</sup> have shown that the total  $K$  x-ray yield per stopped muon for muons stopping in various light materials is smaller by a considerable amount than would be predicted on the grounds of any simple theory of the effect. In the actual experimental arrangement used by Stearns and Stearns the incident  $\mu$ -meson flux is known and the outgoing x-ray energy is measured. The data are interpreted as follows: On the basis of a plausible picture of the capture process the  $\mu$  mesons, in the main, pass through the  $2P$   $\mu$ -mesonic atomic state. In this state the fate of the  $\mu$  meson is decided in a competition between the radiative  $2P-1S$  transition and a  $2P-1S$  transition in which the energy released is taken up by an Auger electron ejected into the continuum. Other nonradiative processes such as direct muon capture by the nucleus contribute negligible branching ratios. Since the Auger probability is roughly independent of  $Z$  and since the radiative transition goes like  $Z^4$ , we might expect, on this picture, that the x-ray yield  $Y$  would go like

$$Y = \text{const } Z^4 / (C_1 + Z^4 \text{const}),$$

where  $C_1$  gives the Auger probability. Indeed Stearns and Stearns find that empirically the yield as a function of  $Z$  has this form, but that the constant  $C_1$  determined from experiment would require an Auger probability some 300 times larger than that computed by standard perturbation-theory methods.<sup>2</sup> In an attempt to account for this large discrepancy, Day and Morrison<sup>3</sup> have invoked collision mechanisms involving the  $\mu$ -mesonic atom and its neighbors in the

dense material. Since this explanation in terms of collisional de-excitation has gained considerable acceptance,<sup>4</sup> we felt that it would be worthwhile to indicate why, in our belief, it cannot be correct.

According to their argument the  $\mu$ -mesonic atom may undergo a collision with a neighboring atom whose electrons take up the  $2P-1S$  de-excitation energy. Day and Morrison estimate that the cross section for this,  $\sigma(2P-1S) \approx \pi a_0^2$ , where  $a_0$  is the electron Bohr radius. The fact that the collision cross section emerging from their calculation depends on  $a_0$  and not on  $a_\mu$ , the muon Bohr radius, is what gives them the factor of 300 needed to explain the experimental results of Stearns and Stearns. In this discussion the usual perturbation theory calculation of the Auger effect is not questioned. Now the collision de-excitation process of Day and Morrison may be looked at as an "external" Auger effect, i.e., one taking place during the time of collision  $t$ . It can readily be seen that the cross section  $\pi a_0^2$  implies a transition probability of the order  $10^{14}/\text{sec}$  for this "external" Auger effect; whereas the usual perturbation theory gives the value  $\sim 10^{11}/\text{sec}$  for the same Auger transition,  $2P-1S$ , in a free atom. It appears to us that it is a priori unlikely that the Auger process which is inefficient for taking up the  $2P-1S$  non-radiative transition energy in an isolated atom could be made so much more efficient by collision with another atom. More exactly speaking, we asked why should an electron in a neighboring atom be ejected by an Auger process  $\sim 300$  times more efficiently than an electron in the given atom itself? In fact, if we estimate the cross

section of the de-excitation of the  $\mu$ -mesonic atom from the  $2P$  to the  $1S$  state by a colliding

electron, the Born approximation gives

$$\sigma(2P - 1S) = 2\pi \int \sigma(\theta) d \cos \theta, \quad (1)$$

where

$$\sigma(\theta) = \frac{K}{K_0} \left( \frac{m}{\mu} \right)^2 a_{\mu}^2 \frac{2^3 Z^8}{[(K^2 + K_0^2 - 2KK_0 \cos \theta)^2 a_{\mu}^2 + (9/4) Z^2]^3}, \quad (2)$$

$K_0$  and  $K$  being the initial and final momenta of the electron. Thus

$$\sigma < \pi a_{\mu}^2.$$

Because of the factor of  $a_{\mu}^2$  which appears, this collision de-excitation cross section is much too small to play any role in explaining the discrepancy. In fact, any collision mechanism<sup>5</sup> involving the transfer of the large energy of the  $2P \rightarrow 1S$  transition of the  $\mu$ -mesonic atom to an electron will have a very small cross section on account of the great disparity in the masses of the meson and the electron. Hence, in our opinion, the experiment of Stearns and Stearns remains unexplained. We do not know of a mechanism which will account for the data. However, it is our belief that the usual perturbation-theory calculation of the Auger effect cannot be trusted. The reason is that the Auger perturbation and the other potentials in the Hamiltonian are, especially for light elements, of the same order of magnitude and any separation of the Hamiltonian is arbitrary. One of us<sup>6</sup> has pointed out that widths of atomic levels subject to the Auger effect are also very poorly estimated by the perturbation theory.

In order to tie the discrepancy down more sharply, it would seem very important to do an experiment in which the Auger rate is measured directly. A possibility<sup>7</sup> is to do a coincidence measurement between  $3D \rightarrow 2P$  or  $3S \rightarrow 2P$  photons produced by the cascading muon and the subsequent Auger electrons. This would measure the combined "external" and "internal" Auger rates. A material like carbon would seem particularly suitable. In the experiments of Stearns and Stearns, the Auger electrons were not actually

seen.

Then, in summary, the discrepancy between the experiment of Stearns and Stearns and the theory of x-ray yields seems to us to be unresolved.

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<sup>1</sup>M. B. Stearns and M. Stearns, *Phys. Rev.* **89**, 1573 (1957).

<sup>2</sup>G. R. Burbidge and A. H. deBorde, *Phys. Rev.* **89**, 189 (1953).

<sup>3</sup>T. B. Day and P. Morrison, *Phys. Rev.* **107**, 912 (1957). We are grateful to Dr. Day for a communication in which he says he is now in substantial agreement with the results of this Letter. He also points out that T. M. Helliwell had arrived at similar conclusions (unpublished) as has L. Madansky (private communication).

<sup>4</sup>See, for example, D. West, in *Reports on Progress in Physics* (The Physical Society, London, 1958), Vol. XXI, p. 271.

<sup>5</sup>An example of another mechanism is an "exchange" Auger effect in which the muon makes a transition to a neighboring atom. In evaluating this process, there will be a factor corresponding to the electron efficiency in taking up the muon energy loss and a factor corresponding to the probability of muon transfer. The muon can only make a transfer when the two atoms are within a distance  $a_{\mu}/Z$  from each other, and hence this exchange process will have a cross section of at most  $\pi a_{\mu}^2$ .

<sup>6</sup>T. Y. Wu, *Phys. Rev.* **66**, 291 (1944); *Can. J. Research* **A28**, 542 (1950).

<sup>7</sup>One of us (J. B.) would like to thank Professor L. Madansky for a discussion of this possibility.