ANALYSIS OF DEUTERON STRIPPING EXPERIMENTS*

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Deuteron stripping experiments analyzed according to the theory of Butler[1] are a nearly unique source of information on the orbital angular momentum and single-particle widths of nuclear bound states. Although a vast amount of such information has been obtained in recent years, a number of problems in the understanding of the Butler theory remain. In particular the orbital angular moment of the captured particle can normally be extracted unambiguously from the experimental data, but the reduced width cannot. Furthermore the reasons for the success of the Butler theory or for its failures, at large angles for example, are not clearly understood.

We wish to point out that recent work by Chew and Low in a different context cast light on these questions.[2] They show that in reactions in which there is a contribution from the exchange of a single particle, there can appear isolated poles in the renormalized Born approximation to the cross section, usually for unphysical scattering angle, and that the residue at these poles can be simply related to quantities of physical interest.[3] Stripping is such a reaction and the Butler theory is the renormalized Born approximation. To see this let us write the Butler theory for the process $A(d,p)B$. The Feynman graph corresponding to the lowest order Born approximation is shown in Fig. 1. The cross section in the center of mass corresponding to this diagram with the vertices and propagators treated exactly, that is renormalized, is[1]

$$\frac{d\sigma}{d\Omega}=\frac{(2J_B+1)}{(2J_A+1)}\frac{3Rk_p}{k_d}\left(\frac{2M_A}{M_B}\right)^3\frac{\alpha}{1-\alpha r_t}\frac{1}{(q^2+k_n^{\,2})^2}\frac{\Theta^2}{(2l+1)^2}$$
$$\times\left\{q[lj_{l-1}(qR)-(l+1)j_{l+1}(qR)]-ik_nj_l(qR)\frac{lh_{l-1}(ik_nR)-(l+1)h_{l+1}(ik_nR)}{h_l(ik_nR)}\right\}^2, \tag{1}$$

where R is the nuclear reaction radius, r_t the triplet effective range, and α is related to the deuteron binding energy, B_d: $\alpha^2\hbar^2/m=B_d$, where m is the nucleon mass. k_ni is the wave number of the captured neutron, captured into a state with orbital angular momentum l. Θ is related to the reduced width γ by $\gamma^2=3\hbar^2\Theta^2/2mR$. M_A and M_B are the masses of the target and daughter nuclei. The term in brackets contains the usual spherical Bessel functions. All wave numbers are appropriate to center-of-mass motion.

The terms in Eq. (1) are easily interpreted in terms of Fig. 1. The factor $\alpha/(1-\alpha r_t)$ is related to the normalization of the asymptotic state of the deuteron and hence is the probability for dissociation corresponding to vertex 1.[4] The energy denominator or propagator for the neutron intermediate state is $1/(q^2+k_n^{\,2})$. At vertex 2 a neutron with orbital angular momentum l must be captured at radius R. The probability that this partial wave and its derivative are in the neutron plane wave with momentum $\vec{q}$ is given by the term in brackets, and the probability that this wave will be captured is just the reduced width, Θ. The other factors are kinematical.

Equation (1) has a second order pole at the unphysical momentum transfer $q^2=-k_n^{\,2}$. Using energy conservation, we see that this corresponds to a scattering angle with $\cos\theta>1$, and hence to an angle beyond zero, but for large incident deuteron energy, not very far beyond. At the pole all other contributions to the stripping process, for example compound nucleus, etc., are finite. Thus at the pole the Butler theory is exact, since it is infinitely larger than all other contributions, and the residue at the pole is exactly given by Eq. (1). If one measures the cross section, the only unknown in Eq. (1) is the reduced width. This may be found by finding the residue at the pole, that is by dividing the experimental cross section by the theoretical one, except for the reduced width, and then extrapolating to the pole. Doing this, we obtain

$$(2J_B+1)\Theta^2 = \frac{d\sigma}{d\Omega}\frac{k_d}{k_p}\frac{(2J_A+1)}{3R}\left(\frac{M_B}{2M_A}\right)^3 (2l+1)^2 \frac{1-\alpha r_t}{\alpha}\frac{(q^2+k_n^{\ 2})^2}{\{\quad\}^2}\Bigg|_{q^2=-k_n^{\ 2}}, \tag{2}$$

where now $d\sigma/d\Omega$ is the experimental cross section. Figure 2 presents the results of such an analysis for the reaction $C^{12}(d,p)C^{13}$.[5] We see that the data allow an unambiguous extrapolation to the pole.

In our analysis we exploit the dependence of the Born term on momentum transfer through the energy denominator $1/(q^2+k_n^{\ 2})$. Usually it is the Bessel function term that is of interest since it has a strong dependence on l. For extracting the reduced width, this term can be a hindrance rather than a help once l is determined since it can vanish, or be small at small angles, and thus can give trouble when we divide through by it and try to extrapolate. Figure 2 represents a particularly favorable case, but it is by no means unique. A further complication can be caused at very small angles by the Coulomb scattering. This is partly avoided by using high incident deuteron energies and low-Z targets, but even for higher Z it should be possible to make the extrapolation if one uses stripping theory with Coulomb corrections, so long as the Coulomb field can be treated adiabatically,[6] since then the particles may be thought to "ride up" the Coulomb barrier and "slide down" the other side without polarization of the internal deuteron motion. So long as this is so the Born approximation, with Coulomb wave functions,[7] will still be very large at $q^2=-k_n^{\ 2}$ and the extrapolation may be used.

From our point of view the validity of the Butler theory rests in the near vanishing of the energy denominator at small angles. Thus the Butler theory, or Born approximation in general, works not only when interactions are weak, but also when energy denominators are small. In fact if the energy denominator is small, the Born

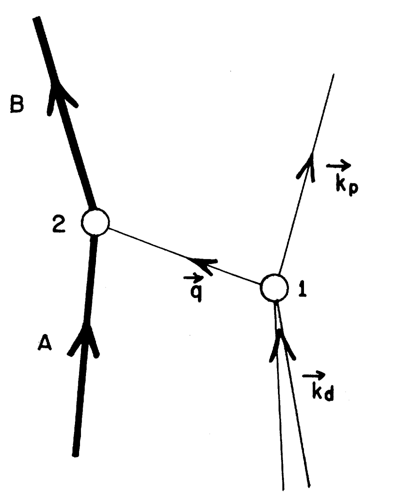

FIG. 1. Lowest order Feynman graph for the reaction $A(d,p)B$ with incoming deuteron wave number $\vec{k}_d$, outgoing proton $\vec{k}_p$, and momentum transfer $\vec{q}$.

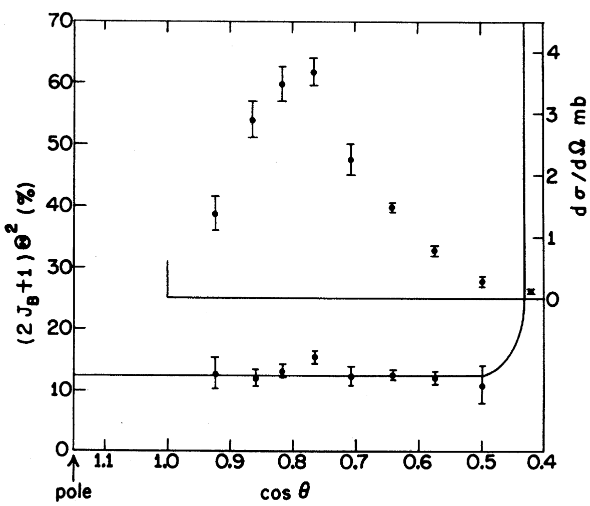

FIG. 2. The function of Eq. (2) plotted against the cosine of the center-of-mass scattering angle for the reaction $C^{12}(d,p)C^{13}$, C^{13} being left in the 3.08-Mev excited state,[5] with $l=0$ and $R=4.2$ f. Extrapolation to the pole at $\cos\theta=1.15$ gives $(2J_B+1)\Theta^2=12.5\%$. The experimental differential cross section is also shown plotted on the displaced right-hand scale.

approximation will get better the larger the interaction matrix elements. At larger angles the Butler theory fails because it is no longer "riding the shoulder" of the pole.

It is clear that methods such as these may be used in the analysis of a large number of direct interactions in which a single particle is exchanged.

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[1]A comprehensive list of references may be found in S. T. Butler, Nuclear Stripping Reactions (John Wiley and Sons, New York, 1957).

[2]G. F. Chew and F. E. Low, Phys. Rev. (to be published).

[3]The analytic properties of the scattering amplitudes have not been investigated rigorously. We shall assume that the extrapolation to the pole is permitted.

[4]J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics (John Wiley and Sons, New York, 1952), p. 611.

[5]Freemantle, Gibson, and Rotblat, Phil. Mag. 45, 1200 (1954).

[6]S. T. Butler, reference 1, p. 62.

[7]See W. Tobocman and M. H. Kalos, Phys. Rev. 97, 132 (1955).

EFFECTS OF ATOMIC ELECTRONS ON p-p AND n-p SCATTERING. II*

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It has been pointed out[1] that Coulomb excitation of atomic electrons by the incident proton and the excitation of atomic electrons caused by the acceleration of the target proton in a proton or neutron collision may be expected to take place with appreciable probability. It was also brought out that in classical mechanics these effects do not matter because of the smallness of the energy changes and that in a quantum treatment the classical phenomenon of the disappearance of an electron with original energy and its reappearance at nearly the same angle with slightly changed energy is replaced by the effect of cross product terms on coherent scattering with the incident wave. For special channels these effects did not compensate[1] the incoherent scattering. It appeared possible, therefore, that corrections for atomic electron effects to low-energy p-p scattering comparable with those for vacuum polarization come under consideration. On the other hand, the existence of compensations at high energies due to the smallness of the proton wavelength in comparison with atomic dimensions was realized.[1] Calculations by de Wit, Fischer, and Zickendraht[2] have since then shown in the cases of monopole, dipole, and quadrupole excitation exact compensation of coherent scattering interference effects by inelastic scattering provided contributions of all channels for different orbital angular momenta L of the proton wave are combined and the following approximations are made: (a) the asymptotic phase $kr - \frac{1}{2}L\pi - \eta\ln(2kr) + \arg\Gamma(L+1+i\eta)$ of the regular Coulomb function is used and a corresponding replacement of the true by the asymptotic phase is made for the s-wave scattering anomaly, (b) in the evaluation of radial integrals all high-frequency parts involving $2kr$ in the phase are neglected. This compensation was interpreted as an indication of the smallness of effects of electron Coulomb excitation on observed scattering. It proved difficult, however, to extend the method of summation over L to the evaluation of the residual effect, improvements on the asymptotic phase leading to difficult sums. On the other hand, a general reason for the compensation becomes apparent as previously expected[1] for high-energy scattering, the nucleon wavelength being short in comparison with atomic dimensions even at $\frac{1}{4}$ Mev. The present note is based on calculations employing this viewpoint. Its formal aspects had been previously carried out for Coulomb excitation of nuclei[3] employing classical action functions S_g and S_n for the ground and nth excited states. These S give solutions of the wave equation by an extension of the $\exp(iS/\hbar)$ representation of the wave function for the case of one S. Classical mechanics for relative motion with atomic quantum excitation is obtained by keeping the first two terms in the expansion of each S in powers of $\hbar$. This approximation, for which the compensation effect is practically exact, is justifiable provided the dimensions of the region occupied by the perturbing energy are large in comparison with the nucleon wavelength, a condition satisfied for the hydrogen molecule as well as the atom. Within wide limits the equivalence to classical mechanics holds independently of the