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IMAGE OF THE FERMI SURFACE IN THE VIBRATION SPECTRUM OF A METAL*

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The lattice vibrations of the ions in a metal are partly screened by the conduction electrons. We shall see that this screening changes rather rapidly on certain surfaces in the space of phonon $\vec{q}$-vectors and that therefore on these surfaces the frequencies ω vary abruptly with $\vec{q}$. The calculations we have done give the result that $\omega(\vec{q})$ is a continuous function of $\vec{q}$ but that on the surfaces in question

$$|\mathrm{grad}_{\vec{q}}\, \omega(\vec{q})| = \infty . \tag{1}$$

The location of these surfaces is entirely determined by the shape of the electronic Fermi surface, using a simple geometrical construction.

To explain the physical origin of this effect let us first describe the conduction electrons by a free electron gas, with Fermi wave number k_F. One then finds that an embedded charge distribution,

$$\rho_{\mathrm{ext}}(\vec{r}) = \rho_0 e^{i\vec{q}\cdot\vec{r}}, \tag{2}$$

induces an electronic charge density

$$\rho_{\mathrm{el}}(\vec{r}) = -F(q)\rho_0 e^{i\vec{q}\cdot\vec{r}}, \tag{3}$$

where

$$F(q) = \frac{1}{\pi a_0 q^2}\left[1 + \frac{k_F}{q}\left(1 - \frac{q^2}{4k_F^2}\right)\ln\left|\frac{q+2k_F}{q-2k_F}\right|\right]; \tag{4}$$

here a_0 is the Bohr radius. Note that near $q = 2k_F$,

$$F(q) = \frac{1}{2\pi a_0 k_F}\left(1 + \frac{1}{2k_F}(q - 2k_F)\ln|q - 2k_F|\right), \tag{5}$$

and

$$\frac{dF(q)}{dq} = \frac{1}{4\pi a_0 k_F^2}\ln|q - 2k_F| \approx -\infty . \tag{6}$$

The last equation shows an abrupt decrease of the ability of the electrons to screen the embedded charge distribution as soon as q exceeds $2k_F$. This is due to the fact that as long as $q < 2k_F$, $\rho_{\mathrm{ext}}(\vec{r})$ causes virtual excitations of some electrons with conservation of energy while when $q > 2k_F$ such excitations are no longer possible (see Fig. 1). Now a lattice vibration of wave vec-

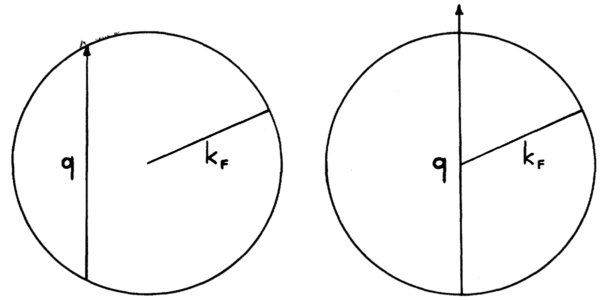

FIG. 1. Virtual excitations for $q < 2k_F$ and $q > 2k_F$.

tor $\vec{q}$ produces a change of ionic charge density of the form

$$\rho_{\text{ion}}(\vec{r}) = \sum_\nu A_\nu \exp[i(\vec{q}+\vec{K}_\nu)\cdot\vec{r}], \tag{7}$$

where $\vec{K}_\nu$ are the reciprocal lattice vectors. Therefore we expect an abrupt change of the restoring force whenever $\vec{q}$ is such that, for some reciprocal lattice vector $\vec{K}_\nu$,

$$|\vec{q}+\vec{K}_\nu| = 2k_F. \tag{8}$$

On the surfaces in $\vec{q}$-space defined by (8), one finds the singularity (1) as a consequence of (6).

Next we consider noninteracting Bloch electrons with a Fermi surface given by

$$E(\vec{k}) = \zeta. \tag{9}$$

Again one finds singularities of $\text{grad}_{\vec{q}}\,\omega$, whose locus is determined by the following construction (Fig. 2): Let Π_1 and Π_2 be two parallel planes in $\vec{k}$-space touching the Fermi surface at $\vec{k}_1$ and $\vec{k}_2$. Then there exists exactly one reciprocal lattice vector $\vec{K}_\nu$ such that the vector $\vec{q}$, defined by

$$\vec{q} = \vec{k}_2 - \vec{k}_1 + \vec{K}_\nu, \tag{10}$$

lies in the fundamental Brillouin zone. At this point $\vec{q}$, Eq. (1) holds. The totality of pairs of planes Π_1 and Π_2 generate the required locus of singularities of the vibration spectrum by means of Eq. (10).

Finally the question arises how the Coulomb interaction between the conduction electrons affects our conclusions. We have partly allowed for this interaction by regarding it as a perturbation and summing certain important terms in the resulting perturbation expansion of $\omega(\vec{q})$. These terms

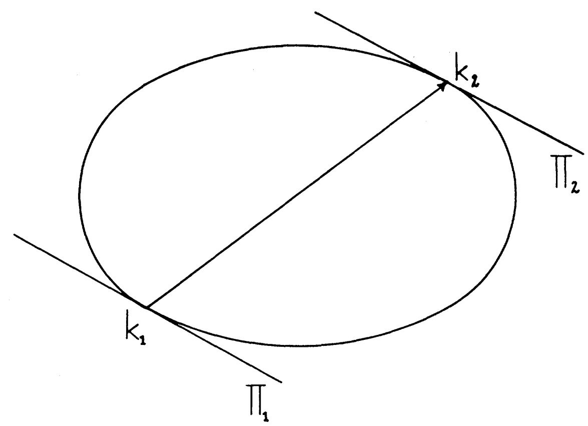

FIG. 2. Construction for the case of Bloch electrons.

do not affect the nature of the singularities, but only their magnitude. It therefore appears unlikely that the Coulomb interactions can obliterate the effect we have discussed, which basically reflects the sharpness of the Fermi surface.

The magnitude of the effect may be quite large (very roughly of the order of percent), and its observation in lattice vibration spectra would give rather direct information about the shape of the Fermi surface.

Similar "images" of the Fermi surface may be expected in spin wave spectra, when the interaction between localized spins is brought about by exchange with conduction electrons.

A detailed report is in preparation.

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ROLE OF SINGLE PHONON EMISSION IN LOW-FIELD BREAKDOWN OF SEMICONDUCTORS AT LOW TEMPERATURES

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Electrical breakdown at low temperatures and at low field strengths[1] has been observed in both n- and p-Ge,[2-4] but has never been seen in Si.[5] It is the purpose of this note to correlate these observations with the known energy-band structure and phonon spectra of Ge and Si and thereby to suggest a simple, necessary condition for LFB (low-field breakdown). Difficulties in establishing a sufficient condition for LFB are discussed, and some comments are made concerning breakdown data for Ge doped with deep-lying acceptors. Finally, several experiments are proposed.

The LFB observed in Ge at liquid He temperatures is produced through impact ionization of shallow impurities by free carriers heated by the applied field. It has been suspected by those working on this problem that the absence of LFB