served values of the charge and magnetic-moment radii indicate that this resonance should occur at $\nu_r \sim 3.5 m_\pi^2$ (square of the pion momentum in the π-π barycentric system). In Fig. 1 the function $|F_\pi(s)|^2$ is plotted for this value of ν_r and several values of the width Γ. In Fig. 2 the pion form factor is plotted for $s < a$. Since it is less than one over most of this region, its appearance in the denominator of the integral of Eq. (9) will produce an additional enhancement.

In conclusion, our Eq. (9) for the weight functions together with the approximation given in Eq. (10) for the pion-pion scattering amplitude suggests that a π-π resonance of suitable position and width could lead to agreement between dispersion theory and many aspects of nucleon electromagnetic structure. Detailed calculations are in progress.

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ENERGY OF THE GRAVITATIONAL FIELD*

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Einstein's equations for the gravitational field are valid for any system of coordinates and make it difficult for one to distinguish physical effects from effects of the curvature of the coordinate system. In consequence there is no obvious definition for the energy of the gravitational field. The usual definition, in terms of a component t_0^0 of the stress pseudotensor, makes the energy depend very much on the system of coordinates and is thus not satisfactory.

For physical problems one can restrict the gravitational field to be weak, of the order of γ, the gravitational constant. One can then use a system of coordinates for which the $g_{\mu\nu}$ differ from their values in special relativity by quantities of the order γ. Even if one restricts oneself to such coordinate systems (and renounces, for example, the use of polar coordinates), one can still make arbitrary changes of order γ in the coordinate system and such changes produce changes in the energy of the same order as the energy itself, so the difficulty persists.

In discussing the question people have usually lost sight of the primary requirement for the energy, that it shall be a useful integral of the equations of motion. The development of the Hamiltonian form of gravitational theory[1] enables one to make a new attack on the problem, taking this utility requirement into account.

In the Hamiltonian form one deals with the state at a certain time x^0, which state is described by dynamical variables for all values of x^1, x^2, x^3 for this one value of x^0. It is found that the only variables needed to describe the gravitational field are the six g_{rs} $(r, s = 1, 2, 3)$ and their conjugate momenta p^{rs}. The four $g_{\mu 0}$ do not enter into the description of the state at a certain time. They are needed only to provide a connection between the state at one time and the state at a neighboring time.

We are thus led to the condition that the energy at a certain time should involve only the g_{rs}, p^{rs}, and the nongravitational variables and should not depend on the $g_{\mu 0}$. The usual definition in terms of t_0^0 does not satisfy this condition, nor does the definition recently proposed by Møller.[2]

We shall take over the Hamiltonian worked out in reference 1, keeping the same notation except that we shall change the sign of all the $g_{\mu\nu}$ to make g_{00} negative (which entails changing also the sign of the p^{rs}). Thus we have the Hamiltonian

$$H=\int\left\{(-g^{00})^{-\frac{1}{2}}\mathcal{H}_L+g_{r0}e^{rs}\mathcal{H}_s\right\}d^3x, \tag{1}$$

where $\mathcal{H}_L$ and $\mathcal{H}_s$ are functions only of the g_{rs}, p^{rs}, and nongravitational variables, namely

$$\mathcal{H}_L=K^{-1}(p^{rs}p_{rs}-\tfrac{1}{2}p_r{}^r p_s{}^s)+\tfrac{1}{4}K^{-1}(K^2e^{rs})_v e^{uv}\times(2g_{rus}-g_{rsu})+\left\{K^{-1}(K^2e^{uv})_u\right\}_v+\mathcal{H}_{ML}, \tag{2}$$

$$\mathcal{H}_s=p^{uv}g_{uvs}-2(p^{uv}g_{us})_v+\mathcal{H}_{Ms}, \tag{3}$$

where a lower suffix added to a field quantity denotes an ordinary derivative. We have also the constraints

$$\mathcal{H}_L\approx 0, \quad \mathcal{H}_s\approx 0. \tag{4}$$

In an ordinary Hamiltonian theory one takes the Hamiltonian itself to be the total energy, as it is always an integral of the motion. This will not do in the present theory, because, on account of the constraints (4), the Hamiltonian (1) equals zero. So long as one considers the exact equations of motion there does not seem to be any useful integral that one could take to be the total energy.

For the weak-field approximation it is reasonable to divide the Hamiltonian into two parts, a part that gives the main motion corresponding to $g_{\mu 0}=-\delta_{\mu 0}$ and a part that gives the correcting terms due to small deviation of the $g_{\mu 0}$ from the values $-\delta_{\mu 0}$. These two parts are

$$H_{\text{main}}=\int(w+\mathcal{H}_{ML})\,d^3x, \tag{5}$$

where

$$w=p_{rs}p_{rs}-\tfrac{1}{2}p_{rr}p_{ss}+\tfrac{1}{4}(g_{rsu}g_{rsu}-g_{rru}g_{ssu})+\tfrac{1}{2}(g_{rsr}g_{uus}-g_{urs}g_{usr}), \tag{6}$$

and

$$H_{\text{cor}}=\int\left\{\tfrac{1}{2}(1+g_{00})(g_{rsrs}-g_{rrss}-\mathcal{H}_{ML})-g_{r0}(2p_{rss}-\mathcal{H}_{Mr})\right\}d^3x. \tag{7}$$

The constraints in this approximation are

$$g_{rsrs}-g_{rrss}-\mathcal{H}_{ML}\approx 0, \quad 2p_{rss}-\mathcal{H}_{Mr}\approx 0. \tag{8}$$

The constraints now cause H_{cor} to have the value zero, but not H_{main}. The difference has arisen because of the neglect of a surface term at infinity in the derivation of (5), such neglect being justifiable because a term of this nature in the Hamiltonian does not influence the equations of motion.

Let us now consider an example for which, at large distances r from the origin, there is no matter present and g_{rsu} and p_{rs} are of order r^{-2}. Such examples often occur in practice. We now have w of order r^{-4} at large distances, so H_{main} converges. It is a constant of the motion, provided we take values for the $g_{\mu 0}$ which preserve its convergence. H_{main} is now a useful integral of the motion, because its constancy is not a consequence merely of the constraints (8). We may thus reasonably define H_{main} to be the total energy.

For an example in which there is continuous emission of gravitational waves, g_{rsu} and p_{rs} at great distances are of order r^{-1}. H_{main} now does not converge, corresponding to the total energy of the gravitational waves being infinite. The energy of physical importance is now the total energy within a large region R. To be able to obtain such an energy we need an expression for the energy density, at any rate for large values of r.

The expression (5) for the total energy in the case of convergence suggests that we look upon $w+\mathcal{H}_{ML}$ in general as the energy density, so that w is the energy density of the gravitational field. Let us examine whether this is permissible taking first the special case when our coordinate system is such that

$$g_{\mu 0}=-\delta_{\mu 0}. \tag{9}$$

The conservation of energy would require that

$$\partial(w+\mathcal{H}_{ML})/\partial x^0=k_{rr}, \tag{10}$$

where k_r is some 3-vector, which can be interpreted as the energy flux.

Now the first of the constraints (4), if evaluated from (2) to the second order in γ, gives

$$w+\mathcal{H}_{ML}=-\left\{K^{-1}(K^2e^{uv})_u\right\}_v. \tag{11}$$

This leads immediately to (10) with

$$k_v=-\left\{K^{-1}(K^2e^{uv})_u\right\}_0.$$

But this k_v cannot be interpreted as the energy flux because it is of order γ, whereas the energy density w in the absence of matter is of order γ^2.

We have, with the conditions (9),

$$\partial(w+\mathcal{H}_{ML})/\partial x^0 = \int [w+\mathcal{H}_{ML}, w'+\mathcal{H}'_{ML}]d^3x'. \quad (12)$$

We see from (6) that w involves only undifferentiated momentum variables and dynamical coordinates differentiated not more than once, and we may assume that $\mathcal{H}_{ML}$ is similar. It follows that $[w+\mathcal{H}_{ML}, w'+\mathcal{H}'_{ML}]$ cannot involve $\delta(x-x')$ differentiated more than once. Since it is antisymmetrical between the two points x and x', we can infer that it must be of the form

$$[w+\mathcal{H}_{ML}, w'+\mathcal{H}'_{ML}] = (k_r+k_r')\delta_r(x-x'), \quad (13)$$

for some 3-vector k_r. Substituting this into (12), we get just the result (10). The gravitational part of k_r, which comes from $[w, w']$, is linear homogeneous in the p_{rs} and linear homogeneous in the g_{rsu}, so it is of order γ^2, the same as w. So this k_r can be interpreted as the energy flux and the conservation law is verified.

When the conditions (9) do not hold, the above deduction gets spoilt by the extra change in $w+\mathcal{H}_{ML}$ produced by H_{cor}, given by (7). The source of the trouble is that $w+\mathcal{H}_{ML}$ gets altered if one makes a change in the coordinate system of order γ. The alteration consists of two parts, corresponding to the two terms in the integrand in (7). If the coordinates x^r are changed by $x^r \to x^r + b^r$, where the b^r are functions of x^1, x^2, x^3 of order γ, the change in $w+\mathcal{H}_{ML}$ is

$$\delta_1(w+\mathcal{H}_{ML}) = -\int b_r'[w+\mathcal{H}_{ML}, 2p'_{rss'} - \mathcal{H}'_{Mr}]d^3x'. \quad (14)$$

If the hypersurface $x^0 = \text{constant}$ is changed by each point of it being shifted normally through a distance a, where a is a function of x^1, x^2, x^3, of order γ, the change in $w+\mathcal{H}_{ML}$ is

$$\delta_2(w+\mathcal{H}_{ML}) = \int a'[w+\mathcal{H}_{ML}, g'_{rsr's'} - g'_{rrs's'} - \mathcal{H}'_{ML}]d^3x'. \quad (15)$$

The energy density, defined as $w+\mathcal{H}_{ML}$, is subject to these two uncertainties.

It may easily be verified that, when the conditions (9) hold, the component $t_0{}^0$ of the stress pseudotensor is just equal to w, so the conservation law proved above is not a new result. When the conditions (9) do not hold, $t_0{}^0$ differs from w by terms involving derivatives of the $g_{\mu 0}$. If the energy density is defined in terms of $t_0{}^0$, it is subject to the above two uncertainties and a further uncertainty because of its dependence on the $g_{\mu 0}$. So the use of w instead of t_{00} makes some improvement.

When the total energy is convergent it can be expressed as a surface integral at infinity. With the present definition of energy this integral is, according to (11),

$$-\int K^{-1}(K^2 e^{uv})_u dS_v. \quad (16)$$

With the usual definition in terms of the pseudotensor, it is

$$-\int\left\{(-g^{00})^{-1/2}K^{-1}(K^2 e^{uv})_u - g_{r0}p^{rv}\right\}dS_v. \quad (17)$$

The two expressions agree provided $g_{\mu 0} = -\delta_{\mu 0} + O(r^{-1})$ at large distances.

On account of the uncertainties (14), (15), there does not seem to be any general definition for the energy density independent of the coordinate system. However, there is an important special case when these uncertainties vanish, namely, when there is no matter present and the gravitational field, to the first order of accuracy, consists only of waves moving in one direction.

In the absence of matter, (14) and (15) give, with the help of (6),

$$\delta_1 w = \tfrac{1}{2}g_{rru}(b_{uss} - b_{sus}) + g_{rsr}b_{uus} - g_{rsu}b_{urs}, \quad (18)$$

$$\delta_2 w = -2p_{rs}a_{rs}. \quad (19)$$

Let us suppose the gravitational field consists only of waves moving, say, in the direction x^3. Then the derivatives g_{rsu} and p_{rsu} will vanish unless $u=3$. If we now make a change in the coordinate system so as to preserve the condition that the gravitational field consists only of waves moving in the direction x^3, we can introduce only coordinate waves moving in the direction x^3. This requires that the derivatives b_{ru} and a_u shall also vanish unless $u=3$. We now get $\delta_1 w = 0$ and

$$\delta_2 w = -2p_{33}a_{33}.$$

The second of the constraints (8) now gives $p_{r3} = 0$, so $\delta_2 w$ also vanishes.

We can conclude that the energy density of gravitational waves moving in a single direction is well-defined, independent of the coordinate system. It is only the interference energy density of waves moving in different directions that is subject to uncertainties.

Let us go back to the problem of determining the total energy within a large region R surrounding some accelerating masses that are continuously emitting gravitational waves. Let us take a solution of the field equations in terms of retarded potentials without any ingoing waves. Then for large values of r, the main part of the gravitational field, of order r^{-1}, consists of waves moving in only one direction at each point, namely radially outward. The energy density w at large distances r is now well-defined, independent of any transformation of coordinates that preserves the character of the solution of being expressible in terms of retarded potentials, and does not introduce any ingoing coordinate waves.

The total energy within the region R, defined by

$$\int_R (w + \mathcal{H}_{ML})\, d^3x, \qquad (20)$$

is now also well-defined, because any transformation of coordinates that affects only the central part of R will not change (20), on account of (11), while any permissible transformation of coordinates in the outer part does not affect w and so does not affect (20). Thus the uncertainties in the energy density defined by $w + \mathcal{H}_{ML}$ do not affect calculations of the emission of energy by gravitational waves.

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POSSIBLE DETERMINATION OF HYPERON PARITIES AND COUPLING STRENGTHS

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The notions of "global" or "universal" symmetries, wherein the Σ and Λ hyperons are treated as members of the same multiplet structure in certain or all of their strong interactions, have been widely discussed,[1-3] although a decisive experimental determination of the relative Σ-Λ parity is yet to be carried out. These detailed theories require this relative parity to be even, but the possibility that it is odd has also been mentioned.[4,5] Several rather difficult experiments have been suggested to determine the relative Σ-Λ parity.[6-8] Furthermore, analyses of the forward angle K-meson–proton dispersion relations have been performed in order to determine simultaneously the relative Σ-Λ parity and the relative K-Λ parity.[9,10] It was emphasized some time ago,[11] and again recently,[12] that there are likely to be serious ambiguities in this procedure. In particular, one must at present make the somewhat subjective and certainly unjustifiable assumption that, in the case of odd relative Σ-Λ parity, the renormalized pseudoscalar coupling of K mesons is not an order of magnitude greater than the renormalized scalar coupling. In this note we shall discuss another experiment for determining the relative Σ-Λ parity which may be feasible at this time.

Chew[13] has suggested a method of extracting information from differential cross sections by their extrapolation to nonphysical values of momentum transfer. Thus an examination of the 400-Mev neutron-proton angular distribution in the backward directions has yielded a new evaluation of the pion-nucleon coupling constant.[14] We should like to suggest an analogous procedure applied to the hyperon-nucleon data concerning such processes as:

(a) $\Sigma^- + p \rightarrow \Lambda + n$,

(b) $\Sigma^- + p \rightarrow \Sigma^0 + n$,

(c) $\Sigma^\pm + p \rightarrow \Sigma^\pm + p$,

(d) $\Xi^- + p \rightarrow \Lambda + \Lambda$,

(e) $\Xi^- + p \rightarrow \Xi^- + p$.

Since at this time only processes (a) and (b) have been abundantly seen, we confine ourselves to a discussion of what we may be able to learn