Berkeley, 1955).

<sup>7</sup>F. Bloch and N. E. Bradbury, Phys. Rev. <u>48</u>, 689 (1935). Other types of atoms or molecules may act as the third body. Mixture experiments in gases, such as He-O<sub>2</sub> and N<sub>2</sub>-O<sub>2</sub>, give K values for He and for N<sub>2</sub> as third bodies which are about 250 and 50 times smaller, respectively, than those for O<sub>2</sub>.

<sup>8</sup>These curves are an average of the data of H. L. Brose, Phil. Mag. <u>50</u>, 536 (1925); R. A. Nielsen and N. E. Bradbury, Phys. Rev. <u>51</u>, 69 (1937); R. W. Crompton (private communication); and P. Herreng, reference 3.

<sup>9</sup>These curves are an average of the data of R. H. Healey and J. W. Reed, <u>The Behavior of Slow Elec-</u> <u>trons in Gases</u> (Amalgamated Wireless, Sydney, 1951), p. 94 ff; H. L. Brose, Phil. Mag. <u>50</u>, 536 (1925); R. W. Crompton (private communication). We have taken the average energy to be equal to 1.5 times the experimentally determined values of  $D/\mu$ .

<sup>10</sup>Craggs, Thorburn, and Tozer, Proc. Roy. Soc. (London) <u>A240</u>, 473 (1957).

<sup>11</sup>T. E. Bortner and G. S. Hurst, Health Phys. <u>1</u>, 39 (1958); G. S. Hurst and T. E. Bortner, Proceedings of the International Congress of Radiation Research, August, 1958 (to be published), and Phys. Rev. (to be published).

<sup>12</sup>Each point shown is derived from the slope of a plot of resultant K as a function of the fractional oxygen concentration at very low oxygen concentrations, e.g., 1-5%.

<sup>13</sup>J. D. Craggs, <u>Third International Conference in</u> <u>Ionization Phenomenon in Gases</u>, Venice, Italy, June, <u>1957</u> (The Italian Physical Society, October, 1957), p. 207.

 $^{14}$ W. A. Rogers and M. A. Biondi (private communication).

## **PROPOSAL FOR A METHOD OF POLARIZING 1-Mev DEUTERONS**

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The purpose of this note is to point out that the elastic scattering of deuterons from He<sup>4</sup> can yield a rather large deuteron polarization with a high cross section. The differential cross section in d-He<sup>4</sup> elastic scattering has been measured to 3% or better in the region of 1-Mev incident deuteron energy.<sup>1</sup> The scattering in this region is characterized by a resonance at 1.070 Mev which corresponds to the formation of the first excited state of Li<sup>6</sup>, to which is given the assignment  $J=3^{+,2}$  If one uses the phase-shift analysis as calculated from these data assuming a single resonance level with J=3, L=2,<sup>2</sup> then one can construct the tensor invariants of the polarization for the scattered deuteron beam as a function of energy.<sup>3</sup> This is done by writing the scattered wave for incident spin projection  $m_s$  in the form

$$\psi(m_{S}) = \sum_{m_{S'}} a_{m_{S'}} m_{S'} \chi_{m_{S'}}^{S}, \qquad (1)$$

where  $\chi_{m_S}$ ,<sup>S</sup> is an eigenvector of  $S_z$ , and  $a_{m_S}$ ,<sup> $m_S$ </sup> is a coefficient involving the phase shifts and spherical harmonics. Here we choose the z axis in the incident direction, and the y axis normal to the scattering plane. Then the density matrix for the scattered beam is

$$\langle m_{S}' | \rho | m_{S}'' \rangle = (2s+1)^{-1} \sum_{m_{S}} a_{m_{S}'} m_{S} a_{m_{S}''} m_{S}^{*} a_{m_{S}''}$$
(2)

normalized such that  $\operatorname{Tr}\{\rho\} = d\sigma/d\Omega = I_0$ . Following reference 3, we express the density matrix in terms of the unit matrix plus five independent tensor invariants:

$$\rho = \sum_{JM} A_{JM} T_{JM}, \qquad (3)$$

where  $T_{JM}$  transforms under rotation of the quantization axis like the spherical harmonic  $Y_M^{J}(\theta,\phi)$ . The expectation values  $\langle T_{JM} \rangle$  can be calculated from the general rule

$$\langle T_{JM} \rangle = I_0^{-1} \operatorname{Tr} \{ \rho T_{JM} \}, \qquad (4)$$

once the density matrix is known in terms of the phase shifts.

Figure 1 shows the cross section,  $I_0$ , and Figs. 2 and 3 give the tensor invariants for the scattered wave, with the z axis as chosen above, as a function of the incident deuteron energy in the region of the resonance. Coulomb forces are included in the calculation. The definitions of the invariants as shown on the figures are the same as in reference 3. The curves are reliable only in the region near the resonance energy, because the approximation  $\tan \beta_2^3 = \Gamma / [2(E_0 - E)]$  was used for the resonant phase shift  $\beta_2^3 \cdot \langle T_{10} \rangle$  of course, vanishes because of parity conservation, and  $\langle S_{\chi} \rangle$  vanishes because of the choice of the y axis. Thus  $i \langle T_{11} \rangle = (\sqrt{3}/2) \langle S_{\nu} \rangle$ .



FIG. 1. Differential cross section in the center-ofmass system for d-He<sup>4</sup> elastic scattering as a function of incident deuteron energy in the lab system, calculated from the phase-shift analysis of reference 2.

In the center -of-mass system this term has angular dependence proportional to  $\sin\theta\cos\theta$ , which is a maximum at  $\theta = 45^{\circ}$ . Due to interference effects,  $\langle T_{11} \rangle$  and  $I_0$  do not peak at the same energy. At 45°,  $\langle T_{11} \rangle$  is 10 kev lower, and the difference is not eliminated by varying  $\theta$  in the region of 45°. Because of their behavior near resonance and their small magnitude, both  $\langle T_{22} \rangle$ and  $\langle T_{21} \rangle$  may be neglected.

In order to consider the effect of this polarization on a second reaction, the axis of quantization should be rotated so that the tensor components are described in terms of the new incident direction as the z direction. Since the orientation of the deuteron spin vector in its own center-of-mass system is not affected by adding an arbitrary velocity to the deuteron, the desired angle of rotation places the quantization axis parallel to the lab momentum vector after the first scattering. For deuterons incident on  $\text{He}^4$ ,  $\theta_{\text{c.m.}} = 45^\circ$  corresponds to  $\theta_{\text{lab}} = 30^\circ$ . The transformation into the center-of-mass system of the second reaction does not change the relative orientation of the z and quantization axes. Thus Eq. (2.8) of reference 3 can be used



FIG. 2. Tensor invariants of the deuteron polarization in d-He<sup>4</sup> elastic scattering as defined in reference 3, calculated from the phase-shift analysis of reference 2. Since  $N = S_{\chi} + i S_{\gamma}$ , in no reaction can  $i \langle T_{11} \rangle$  exceed  $\sqrt{3}/2$ .



FIG. 3. Tensor invariants of the deuteron polarization in d-He<sup>4</sup> elastic scattering as defined in reference 3.

directly, after the rotation is performed on  $i \langle T_{11} \rangle$  and  $\langle T_{20} \rangle$ .<sup>4</sup> The result is as follows:

$$I(\theta,\phi) = I_0(\theta) + \frac{5}{8} \langle T_{20} \rangle A(\theta)$$
  
+  $\left[\frac{3}{8}(2)^{1/2} \langle T_{20} \rangle B(\theta) + i \langle T_{11} \rangle C(\theta)\right] \sin\theta \cos\phi$   
+  $\frac{1}{16}(6)^{1/2} \langle T_{20} \rangle D(\theta) \sin^2\theta \cos 2\phi,$  (5)

where  $I_0$  is the unpolarized cross section; A, B, C, and D are polynomials in  $\cos\theta$ , and are dependent upon the reaction matrix elements. Because of the large magnitude of  $i \langle T_{11} \rangle$  after the initial scattering and the low energy, one would expect the  $\cos\phi$  term in the asymmetry to dominate; the "right-up" type of asymmetry characteristic of spin one would probably be much smaller. Professor W. M. MacDonald has sug-

gested to the author that such deuterons may be a useful tool in investigating (d,p) and (d,n) reactions. Dr. A. S. Langsdorf has pointed out that polarized deuterons of higher energy may be obtained by accelerating He<sup>4</sup> nuclei on a deuterium target, rather than the inverse.

<sup>1</sup>Galonsky, Douglas, Haeberli, McEllistrem, and Richards, Phys. Rev. <u>98</u>, 586 (1955).

<sup>2</sup>A. Galonsky and M. T. McEllistrem, Phys. Rev. <u>98</u>, 590 (1955).

<sup>3</sup>W. Lakin, Phys. Rev. <u>98</u>, 139 (1955).

<sup>4</sup>For the rotation matrices see, for example, M. E. Rose, Elementary Theory of Angular Momentum

(J. Wiley and Sons, New York, 1957), Chap. IV.

## MAGNETIC PERTURBATION ON CIRCULAR POLARIZATION OF EXTERNAL BREMSSTRAHLUNG

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It is well known that the external bremsstrahlung (EB) produced by  $\beta$ -rays is circularly polarized. The degree of circular polarization, according to theoretical predictions,<sup>1-3</sup> is a function of the quantum energy and attains its maximum value when the quantum has the same energy as the radiating electron. The EB polarization has been measured by some investigators<sup>4-7</sup> whose results appear to agree reasonably as far as concerns the energy dependence. A lack of agreement seems to exist regarding the dependence of the polarization on the atomic number of the absorber. A strong Z dependence has been reported by Cohen et al.<sup>6</sup> but it was not confirmed by Galster and Schopper.<sup>5</sup> Recently we<sup>8</sup> have found that the atomic number of the target affects the EB polarization to an extent which seems to substantiate the calculations of Neamtan<sup>9</sup> on electron depolarization in matter.

With a view to obtaining further information on the influence of matter on EB polarization, we have investigated the circular polarization-energy relation for EB quanta, by using a magnetized iron target as  $\beta$  absorber. The circular polarization of  $\gamma$ -rays was analyzed through Compton scattering with polarized electrons available in magnetized iron. The experimental apparatus and procedure were the same as previously described.<sup>7-8</sup> The source of  $\beta$ -rays (Y<sup>90</sup>) was placed just behind the target (about 8 mm thick) which was magnetized perpendicularly to the polarimeter axis ( $B = 16\,000$  gauss).

The measurements consisted of several 6minute counting runs, one for each opposite polarimeter field direction, made alternately with magnetized and unmagnetized targets. Every run showed that the asymmetry in the counting rate for opposite polarimeter field direction was lower when the target was magnetized. Taking into account the correction factor of any instrumental effect, and the efficiency of the polarimeter, we obtain the polarization-energy relation for EB quanta produced in the unmagnetized target. The relation for EB quanta produced in the magnetized target is obtained from the preceding relation by multiplying by the ratio of the measured asymmetries. This is right, in view of the fact that the interesting spectral distributions for the two magnetization states of the target have identical shapes. The two relations are shown in Fig. 1. For each curve a total of  $6 \times 10^7$ pulses were counted. The quoted errors are only statistical.

It is noteworthy that at low energies (E < 1.4