OSCILLATORY MAGNETO-ACOUSTIC EFFECT IN METALS

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This note reports the results of a calculation of ultrasonic absorption in metals in the presence of a magnetic field transverse to the direction of propagation. Two cases have been investigated in detail: (a) longitudinal wave, and (b) shear wave with polarization, $\vec{u}$, perpendicular to field, $\vec{H}$ (as well as to the wave-vector, $\vec{q}$). The method of treatment involves the combined application of the Boltzmann transport equation and Maxwell's equations to the free-electron model,[1] and is thus essentially the same as that employed by Rodriguez.[2] However, the present results differ considerably from those of R; in particular, as shown by the accompanying figures, substantial magneto-acoustic oscillations of the general type predicted by Pippard,[3] and observed experimentally,[4] are obtained.

The results are shown in Figs. 1 and 2. These may be regarded as the attenuations plotted versus (ω/H) for a given sample (fixed τ), for a series of different frequencies (corresponding to different ql). The abscissa is the dimensionless variable $\beta = 2\pi v_0 \omega_c^{-1} = 2\pi D\lambda^{-1}$, where D is the equatorial magnetic diameter. The attenuation coefficients (α) may be obtained by multiplying the numbers shown as ordinates by $(mM^{-1})\tau C_S$.[5]

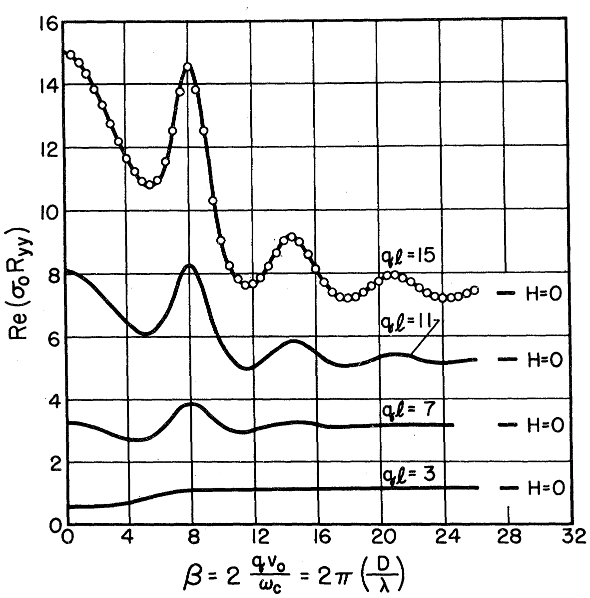

FIG. 1. Magneto-acoustic oscillations for case (a) (longitudinal). All the curves were obtained by computation at the points indicated for $ql = 15$.

The maxima occur at very nearly the same values in both cases. It will be noted that the distances between successive maxima are nearly equal and very close to 2π; the corresponding differences in D are then essentially equal to λ. This confirms the surmise of Pippard[3] that the periodicity is related to the orbit of maximum dimension. The positions of the peaks, however, are not entirely in accord with his predictions. The peak at highest field occurs at $\beta \sim 8.4$, corresponding to the fundamental resonance of an "average" orbit ($\sim 0.75D$). As the field is decreased, accurate periodicity is gradually attained. It seems reasonable to suppose that this periodicity will obtain for many real Fermi-surfaces in the limit of orbit dimension large compared to λ. It should be pointed out, however, that since the oscillations are appreciable only for[6] $\omega_c\tau \gtrsim 1$, rather large values of ql may be required.

The theory requires the calculation, from the Boltzmann equation, of the three "magneto-acoustic" conductivities σ_{xx}, σ_{xy}, and σ_{yy}, which relate the local electron currents to the electromagnetic "deformation fields" accompanying the sound wave. The specific expressions turn out to be

$$\sigma_{yy} = 3\sigma_0(1 + i\,\omega\tau)(ql)^{-2}M_\gamma(\beta), \tag{1}$$

$$\sigma_{xy} = \frac{-3}{ql}\sigma_0\frac{\partial}{\partial\beta}M_\gamma(\beta), \tag{2}$$

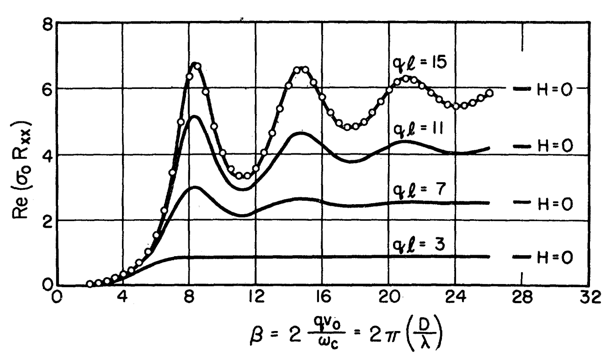

FIG. 2. Magneto-acoustic oscillations for case (b) (triple transverse). All curves were obtained by computation at the points indicated on the curve for $ql = 15$.

$$\sigma_{yy} = \frac{3}{2}\sigma_0(1+i\omega\tau)^{-1}\{1 - [1+(3+4\gamma^2)\beta^{-2}]M_\gamma(\beta) + 3\beta^{-2}L_\gamma(\beta) - 3\beta^{-1}\frac{\partial}{\partial\beta}L_\gamma(\beta)\}. \quad (3)$$

Here

$$\gamma = (\omega_c\tau)^{-1}(1+i\omega\tau), \quad M_\gamma(\beta) = \frac{1}{\beta}\int_0^\beta L_\gamma(\beta')\,d\beta', \quad (4)$$

$$L_\gamma(\beta) = s_{1,2i\gamma}(\beta) = 1 - \frac{\pi\gamma}{\sinh 2\pi\gamma}[\bar{J}_{2i\gamma}(\beta) + \bar{J}_{-2i\gamma}(\beta)]. \quad (5)$$

The functions $s_{\mu\nu}(z)$ and $\bar{J}_\nu(z)$ are referred to by Watson[7] as Lommel's function and Anger's function, respectively. They are defined on pages 345 and 308 of reference 7.

In case (a) the present analytic expression for the attenuation coefficient agrees with that given by R's Eqs. (62) and (64). However, the expression obtained in case (b),

$$\alpha = \frac{m}{M\tau C_s}\,\mathrm{Re}\left\{\frac{\sigma_0\sigma_{yy}}{\sigma_{xx}\sigma_{yy}\sigma_{xy}^2} - 1\right\}, \quad (6)$$

differs from that given by R's Eq. (54). The latter, in the opinion of the present authors, is incorrect; a basic ingredient of its derivation, namely the assumption of zero Hall field in the propagation direction, can be shown to be inconsistent with Maxwell's equations (in particular with the Poisson relation). In fact, under conditions commonly prevalent in metals (high electron densities and conductivities), Poisson's equation is essentially equivalent to the requirement of quasi-neutrality; in the case at hand, this requirement implies zero Hall-current (rather than field). The resulting difference has an especially marked effect on the high-field behavior; in this domain, as shown by Fig. 2, the attenuation is proportional to H^{-2}, rather than to H^2 as concluded by R.

In order to obtain numerical results, the magneto-acoustic conductivities must be computed from Eq. (1) to (5). The replacement of $\bar{J}_\nu(\beta)$ by its expansion[7] in powers of β yields (absolutely convergent) series for the σ_{ij} which coincide with R's Eqs. (29), (30), and (31). On the other hand, the asymptotic ($\beta \gg 1$, $|\gamma| \ll \beta$) expressions for the σ_{ij} [obtained from the asymptotic series for $\bar{J}_\nu(\beta)$ given by Eq. 10.14 (1) of reference 7], differ from R's Eqs. (33), (34), and (35) in that they contain terms proportional to the Bessel functions $J_{\pm 2i\gamma}(\beta)$, as well as to integrals and derivatives of these functions. It has now to be remarked that the oscillatory behavior of the σ_{ij} (and hence of the attenuation) arises entirely from these additional "Bessel" terms; their omission from R's asymptotic formulas may explain the absence of oscillatory behavior in his final results.

[1]A. B. Pippard, Phil. Mag. 46, 1104 (1955).
[2]S. Rodriguez, Phys. Rev. 112, 80 (1958), to be referred to as "R".
[3]A. B. Pippard, Phil. Mag. 2, 1147 (1957).
[4]R. W. Morse and J. D. Gavenda, Phys. Rev. Lett. 2, 250 (1959).
[5]v_0 = Fermi velocity; ω_c, ω = cyclotron and sound (angular) frequencies; τ = electron scattering time; m, M = electron and atom masses; λ, C_s = relevant wavelength and velocity of sound.
[6]This requirement on $\omega_c\tau$ may be noted from Figs. 1 and 2 and is directly deducible from asymptotic expansions of the results below.
[7]G. N. Watson, Bessel Functions (Cambridge University Press, Cambridge, 1952, second edition, Chap. 10, especially Secs. 10.1, 10.11, 10.14, 10.15, and 10.7 to 10.75.

ELECTRON PARAMAGNETIC RESONANCE OF MANGANESE IV IN $SrTiO_3$*

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We have investigated the paramagnetic resonance spectrum of a single crystal of $SrTiO_3$ grown by the Verneuil process with the addition of 0.01 weight percent of MnO_2. The measurements were made at room temperature, liquid air temperature (where the $SrTiO_3$ begins to transform from the high-temperature cubic into a tetragonal phase[1]), and at liquid nitrogen temperature. Several samples cut from the crystal were investigated. The magnetic field was varied parallel to the (110) and (100) planes. A spectrometer working at 3.2 cm was used.

Six main hyperfine line groups arising from the nuclear spin $I = \frac{5}{2}$ of the isotope Mn^{55} were ob-