separation of the lower two being $3e^2qQ/20$ and of the upper two, $6e^2qQ/20$. This splitting scheme leads to a specific heat $A_0T^{-2}=(14/9)Ra_0^2T^{-2}$ in the high-temperature limit $T>>a_0$, where a_0 has been written for $3e^2qQ/20$. Comparing this result with the experimental value of A_0, we obtain $a_0=0.002$ deg and the maximum field gradient $eq=0.14\times10^{16}$ esu. Since the ingot is polycrystalline and contains impurities, the value of eq may differ from that of a perfect crystal.

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[1] This ingot, furnished to us by Dr. M. P. Garfunkel, has a history similar to specimen 4C of J. K. Hulm and B. B. Goodman [Phys. Rev. 106, 659 (1957)], for which: T_C = 1.81°K with a transition breadth of 0.05 deg. The known impurities are Fe = 0.0011%, Si < 0.01%, and $W\sim0.03\%$. We are grateful for the loan of this sample.

[2] Bardeen, Cooper, and Schrieffer, Phys. Rev. 108, 1175 (1957).

[3] See, for instance, M. H. Cohen and F. Reif, Solid State Physics, edited by F. Seitz and D. Turnbull (Academic Press, New York, 1957), Vol. 5, p. 321.

[4] T. P. Das and E. L. Hahn, Solid State Physics, edited by F. Seitz and D. Turnbull (Academic Press, New York, 1957), Supplement 1.

NUCLEAR SPECIFIC HEAT OF GALLIUM AND ZINC

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Recently the specific heats of gallium[1] and zinc[1-3] have been measured in the superconducting state below 1°K. However the data have all been analyzed without concern of the possible effect of an interaction of the nuclear quadrupole moment with the gradient of the crystalline field. Now that such an effect has been shown to be of considerable importance for the specific heat of rhenium,[4] it becomes necessary to review the previous results.

Knight et al.[5] measured the pure quadrupole resonance spectra for Ga^{69} and Ga^{71} in the metallic state. From their results it becomes possible to calculate the quadrupole coupling constant for both isotopes and also the contribution of the nuclei to the specific heat of naturally occurring gallium (60.2% Ga^{69}, 39.8% Ga^{71}). This contribution is given by $C=4.6/T^2$ ergs/mole deg when $T>>0.001$°K. It is only 5% of the specific heat of the conduction electrons at the lowest temperatures measured by us. Therefore we did not observe a departure of the electronic specific heat in the superconducting state from the expected exponential decrease with temperature:

$$C_{es}=\gamma T_C a\exp(-bT_C/T). \quad (1)$$

Recalculating our data, after subtracting this nuclear term, the constants for Ga in Eq. (1) change: a from 7.0 to 7.5 and b from 1.35 to 1.38. The fit of the experimental data and Eq. (1) is extended to somewhat higher temperatures: $T_C/T>1.6$.

Zinc has a hexagonal crystal, and hence an appreciable field gradient can exist at the nuclei. However the single isotope which is magnetic, Zn^{67} (spin 5/2), has a natural abundance of only 4%. Phillips[2] noted deviations of the specific heat from the exponential temperature dependence, Eq. (1), outside the experimental error. It is reasonable to assume that these deviations are nuclear in origin and that they have a temperature dependence proportional to $1/T^2$. From his graph it follows that the additional term is then given by $C\sim1.5/T^2$ erg/mole deg for naturally occurring zinc. Hence for Zn^{67}, $C\sim37/T^2$ erg/mole deg and the nuclear quadrupole coupling constant for Zn^{67} is approximately 70 Mc/sec. If the additional term is subtracted from the specific heat data of Phillips, his constants of Eq. (1) change: a from 5.8 to 6.2 and b from 1.22 to 1.24 (using his values of γ and T_C). We reported values $a=6.4$, and $b=1.27$, which are only slightly higher. They were obtained at higher temperatures where this additional term can be neglected.

In the normal state, at the lowest temperatures Phillips notes a 5% deviation from the expected value of the heat capacity but allows the possibility of errors in his thermometers. However, this value of the deviation is to be expected if the splitting of nuclear energy levels of the Zn^{67} isotope contributes to the specific heat. It should be possible to substantiate our assumption by a quadrupole resonance experiment.

It is difficult to obtain data from Zavaritskii's[3] curves for zinc, but it appears that his results are not inconsistent with the above conclusions.

Zavaritskii[3] uses the relation $a=4b^2$ for the

constants in Eq. (1). This relation holds very well for all the elements[1] for which these constants are known.

We would like to use this opportunity to correct a statement in our previous article.[1] Dr. D. E. Mapother pointed out that on cooling through the transition temperature in a magnetic field a superconducting shell does not expel flux lines. Therefore our measurements were performed in the earth's magnetic field.

*National Science Foundation Postdoctoral Fellow.

[1] G. Seidel and P. H. Keesom, Phys. Rev. 112, 1083 (1958).

[2] N. E. Phillips, Phys. Rev. Lett. 1, 363 (1958).

[3] N. V. Zavaritskii, J. Exptl. Theoret. Phys. (U.S.S.R.) 34, 1116 (1958)[translation: Soviet Phys. JETP 34, 773 (1958)].

[4] P. H. Keesom and C. A. Bryant, preceding Letter [Phys. Rev. Lett. 2,260(1959)].

[5] Knight, Hewitt, and Pomerantz, Phys. Rev. 104, 271 (1956).

THREE-LEVEL MASERS AS HEAT ENGINES*

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The purpose of this note is to demonstrate that three-level masers[1,2] can be regarded as heat engines. The principal conceptual difference between these and conventional heat engines is that in the 3-level maser one is concerned with the discrete energy levels of a particle's internal energy whereas in a conventional heat engine one is concerned with the continuous spectrum of energies associated with external motion of the working substance. In treating a 3-level maser as a prototype of heat engine, a particular advantage is, in our opinion, the resulting conceptual simplicity. Especially, it is easily shown that the limiting efficiency of a 3-level maser is that of a Carnot engine.

Consider the system shown in Fig. 1. A three-level system is assumed with all transitions allowed and with no appreciable relaxation processes. The usual 3-level maser terminology is introduced by correlating transition $1 \leftrightarrow 3$ with pump frequency ν_p, $1 \leftrightarrow 2$ with signal frequency ν_s, and $2 \leftrightarrow 3$ with idler frequency ν_i. As a further convention, the length of each energy level line is drawn proportional to its population.

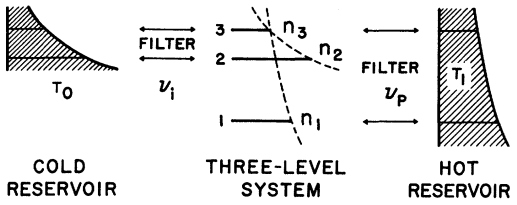

FIG. 1. Three-level system in thermal contact with two heat reservoirs.

The levels 1 and 3 are in thermal contact, through a filter passing frequencies in the vicinity of ν_p and rejecting frequencies in the vicinity of ν_i and ν_s, with a heat reservoir at temperature T_1. The temperature is indicated in the figure by showing schematically the Boltzmann distribution of this heat reservoir. Levels 2 and 3 are in thermal contact with a reservoir at a lower temperature T_0 through a filter which passes frequencies in the vicinity of ν_i but rejects those close to ν_p and ν_s.

Experimentally, the high-temperature reservoir might be realized by a gas noise lamp and the filter by a wave guide cutting off the lower frequencies. For practical purposes, however, the single mode present in a wave guide does not provide good thermal contact. The assumed coupling situation to the low-temperature reservoir, on the other hand, was closely approximated by experimental conditions in some maser experiments.[3] There, the idler transition of the gadolinium three-level system was coupled, through spin-spin interaction at frequency ν_i, to a transition of the same frequency of cerium ions within the same crystal. Thus, through the resultant short spin-lattice relaxation time, good thermal contact to the lattice heat reservoir at T_0 was established.

In the system described, for each quantum $h\nu_p$ supplied by the hot reservoir, the energy $h\nu_i$ is passed to the cold reservoir. The smaller quantum $h\nu_s$ can be extracted at the signal transition if maser action prevails, that is if $n_2/n_1 \geq 1$. Thus the efficiency of this idealized system in maser operation is

$$\eta_M = \nu_s/\nu_p. \tag{1}$$

From the Boltzmann factors involved, we find

$$\frac{n_2}{n_1} = \frac{n_2}{n_3}\frac{n_3}{n_1} = \exp\left(\frac{h\nu_i}{kT_0}\right) \times \exp\left(-\frac{h\nu_p}{kT_1}\right). \tag{2}$$