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HYDRODYNAMICS OF LIQUID HELIUM II*

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This note gives a brief account of a macroscopic hydrodynamic theory of liquid helium II which can describe most of the experimental observations on the thermal and mechanical behavior of liquid helium below the λ-point. The theory evolves from a one-fluid concept of liquid helium in which a long-range order exists in the momentum space. The transfer of long-range order (disorder) is then the superconductivity of heat. By assuming that one is dealing with a classical thermomechanical medium, it can be mathematically proved that there is a momentum transfer associated with the reversible energy transfer even when the momentum of the fluid is zero. This is a typical feature of the usual two-fluid theory; indeed, a two-fluid model can be constructed, in a purely mathematical manner, out of the present one-fluid theory. The basic equations, one governing the rate of change of momentum and the other governing the rate of change of the reversible heat flux vector, can then be reduced to the ordinary equations of Landau. There is, however, no longer any compelling reason to believe that the motion of the superfluid component must be irrotational. (See Lee and Yang.[1])

Accordingly, we propose to abandon the often-used basic assumption that the superfluid component of the flow is irrotational. (See Lifšic and Halatnikov.[2]) We next include certain dissipative terms in our equations of motion dependent on the gradients of physical quantities, in agreement with Lifšic and Halatnikov. Since the equations of motion now contain the second-order space derivatives of the superfluid component, we need a boundary condition for the tangential component of the superfluid velocity. Bearing in mind the requirement of covariance with respect to coordinate transformations, we propose the following general form for the boundary condition:

$$\partial \vec{v}_{s,T}/\partial N = f(\vec{q}^2)\vec{q}, \tag{1}$$

where $\vec{q}$ is the tangential component of the difference between superfluid velocity and wall velocity, and $\partial \vec{v}_{s,T}/\partial N$ is the tangential component of the normal derivative of the superfluid velocity. Before a molecular theory can be developed, the form of $f(\vec{q}^2)$ must be determined by experiments. For small velocities, we assume

$$f(\vec{q}^2) = \alpha + \beta\vec{q}^2 + \ldots \quad . \tag{2}$$

From Andronikashvili's disk pile experiment, we know that α must be negligibly small. Thus, the simplest form for $f(\vec{q}^2)$ is

$$f(\vec{q}^2) = \beta\vec{q}^2. \tag{3}$$

With this simple assumption, we can derive the following results for experiments involving simple geometry.

(A) For rotating bucket experiments, the theory predicts uniform rotation of the superfluid component, but at a speed lower than that of the normal component, because of the boundary condition (1). The ratio of angular momentum L actually acquired by helium II to the ratio

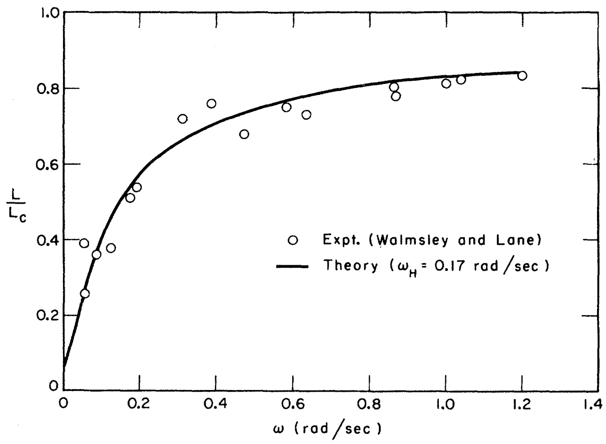

FIG. 1. Angular momentum of liquid helium II in a rotating bucket.

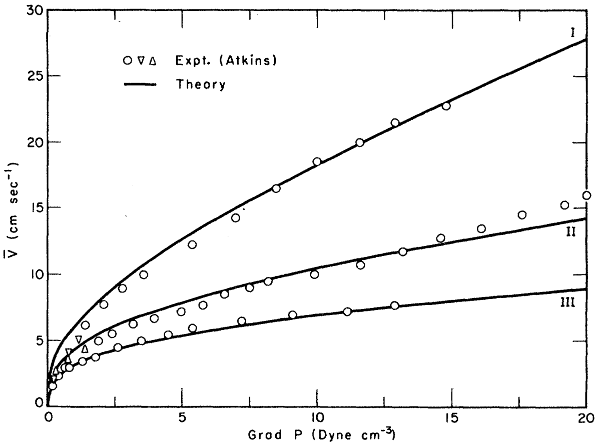

FIG. 2. Velocity of flow of liquid helium II through tubes of various diameters. (I) $d = 4.40\times 10^{-2}$ cm (∇ for short tube). (II) $d = 2.03\times10^{-2}$ cm (Δ for short tube). (III) $d = 0.815\times10^{-2}$ cm.

L_C for ordinary fluids is given by

$$L/L_C = s + x\,(1-s),$$
$$\omega/\omega_H = \tfrac{1}{2}\,[s/(1-s)^3]^{1/2}, \qquad (4)$$

where x is the fraction of normal fluid present, ω is the angular velocity of the bucket, and ω_H is related to the molecular parameter β by

$$\omega_H{}^2 = 4\beta a^{-3}. \qquad (5)$$

In this formula, a is the radius of the bucket.

(B) For isothermal flow through a tube (or channel), the amount of flow depends on the pressure gradient Γ in the following manner:

$$\text{Mean velocity } \bar{v} = Ad^2\Gamma + Bd^{1/3}\,\Gamma^{1/3}, \qquad (6)$$

where A and B are molecular parameters, and d is the diameter of the tube (or width of the channel).

In Fig. 1, one set of experimental results of Walmsley and Lane[3] is compared with Eq. (4). There is only one other set of data at the same temperature which is, however, not sufficiently accurate to test the validity of (5). In Fig. 2, the experimental results of Atkins[4] are compared with (6). One may notice that the theoretical and experimental results agree for tubes of various sizes.

Preliminary analyses also show agreement of the theory with experimental results for fountain effect and for damping experiments at finite amplitudes.[5] These will be discussed in a more complete paper. It also follows from the present concepts that second sound should propagate through rotating liquid helium II in essentially the same manner as in the nonrotating case—a fact observed by Hall and Vinen.[6] The detailed examination of attenuation of second sound in rotating helium is left for further investigation; preliminary considerations also show general agreement with their results.

I should like to express my thanks to Professor C. N. Yang for many discussions during the development of the one-fluid concept of liquid helium II while both of us were at Madison during the summer of 1958. I wish also to thank Professor C. T. Lane for private communications and comments on the data shown in Fig. 1, and to thank Mr. Alfred Clark, Jr., for his help in the numerical analysis of the experimental data shown in Fig. 2. Thanks are also due to my colleagues L. Tisza, Kerson Huang, and Charles E. Chase for their interest and discussions.

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