polarization for several values of λ for this case is given in Fig. 1 (a). Values of λ as large as unity do not give polarizations very different from that of elastic scattering. This case covers most of the low-lying states whose polarizations have been measured by the Uppsala group[6] at ~156 Mev, viz., the 4.4- and 9.6-Mev levels in C^{12} and the 6.1-Mev group in O^{16}. Experimentally the polarizations from these levels resemble each other and are much the same as for elastic polarization, indicating small values of λ, which is not inconsistent with shell model calculations.[7]

For normal parity change but with $\Delta T = 1$, the resulting angular distributions are shown in Fig. 1 (b) for several values of λ. The very great difference between $\bar{M}_{00}$ and $\bar{M}_{01}$ is reflected in these curves; a marked dependence on λ is seen even for fairly small values of λ and no relation to elastic scattering is found. This case corresponds to the polarization from the group of levels in the 19-21 Mev region in C^{12} and O^{16}, which from photonuclear data consists mainly of levels with $J = 1^-$ and $T = 1$. The polarization in this case is observed to be small.[6]

For "abnormal" parity change, $\Delta\pi = (-1)^{J+1}$, the non-spin-flip matrix element vanishes and the polarization becomes

$$P = 2(1-\rho)\mathrm{Re}(\bar{B}\bar{C}^*)/\{(1-\rho)(|\bar{F}|^2 + |\bar{B}|^2 + |\bar{C}|^2) + 2\rho|\bar{E}|^2\}, \tag{9}$$

and so depends on a single nuclear parameter $\rho = L_3/L_2$ whose values are restricted to lie between 0 and 1. The angular distribution in this case is shown for several values of ρ, for $\Delta T = 0$ in Fig. 1 (c) and for $\Delta T = 1$ in Fig. 1 (d). In both cases the polarization in the forward direction is small. Most of the levels in the 12.5-Mev O^{16} and ~15-Mev C^{12} region, whose polarization has been measured by the Uppsala group[6] and shown to be small, correspond to "abnormal" parity change.

Elastic polarization and polarization from the 2.43-Mev level of Be^9 have been measured by Hafner[8] for 220-Mev protons. This case is of much greater complexity than those discussed above, since the ground state[9] has $J = \frac{3}{2}^-$, $T = \frac{1}{2}$, and the excited state has $J = \frac{5}{2}^-$, $T = \frac{1}{2}$. Explicit calculations can, however, be made on the basis of shell model states. For example, in the case of L-S coupling, for elastic scattering, $\bar{M} = (13/18)M_1 + (5/18)M_0$ and $\tilde{M} = \frac{1}{2}(M_1 + M_0)$, with $\lambda \cong 0.025$ and $\mu \cong 0.003$; for inelastic scattering from the 2.43-Mev level $\bar{M} = (11/15)M_1 + (4/15)M_0$ while $\lambda \cong 0.005$, $\mu \cong 0.002$. As for elastic scattering from C^{12}, $\bar{M} = \frac{3}{4}M_1 + \frac{1}{4}M_0$ and $\lambda = \mu = 0$, the polarization both in the inelastic and elastic scattering from Be^9 should strongly resemble the elastic scattering from C^{12}, as is in fact observed.

High-energy nucleon scattering with very good energy resolution represents another tool for the possible determination of certain nuclear matrix elements. To determine nuclear matrix elements from inelastic nucleon angular distribution data, especially in the forward direction, requires a somewhat more certain determination of the two-nucleon scattering amplitude. Two-nucleon experiments now in progress[10] should provide much of this information.

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*Work performed under the auspices of the U. S. Atomic Energy Commission.

†On leave from and now at Chalk River Laboratory, Chalk River, Ontario.

‡On leave from and now at Los Alamos Scientific Laboratory, Los Alamos, New Mexico.

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Λ INTERACTIONS IN HYDROGEN*

Frank S. Crawford, Jr., Marcello Cresti, Myron L. Good, Frank T. Solmitz, M. Lynn Stevenson, and Harold K. Ticho
Lawrence Radiation Laboratory, University of California, Berkeley, California
(Received January 23, 1959)

While studying the associated-production processes, we have encountered six cases in which the Λ subsequently interacts with a proton in the liquid hydrogen bubble chamber.[1]

The classes of Λ interactions we have seen are

$$\Lambda + p \rightarrow \Lambda + p \text{ (4 events)}, \tag{1}$$

$$\Lambda + p \rightarrow \Sigma^+ + n \text{ (2 events)}. \tag{2}$$

We have not yet seen any examples of the reaction

$$\Lambda + p \rightarrow \Sigma^0 + p. \tag{3}$$

[Charge independence predicts that Reaction (3) should be half as frequent as Reaction (2).]

The momentum spectrum of the Λ produced in our associated production experiment is shown in Fig. 1. The letters "Λ" and "Σ^+" in this figure indicate the Λ momentum that produced the above reactions. The threshold for the endothermic reaction (2) is 635 Mev/c. Table I contains the pertinent information concerning the reaction dynamics for each event.

Figure 2 shows one of the Λ elastic scattering events. The original Λ was produced in the direction indicated by arrow No. 2. (The associated K^0 did not decay in the chamber.) At Point A, the Λ scatters in the direction indicated by arrow No. 3. At point B, the Λ decays into a pion (Track 4) and a proton (Track 5). Track 6 is the recoil proton.

Figure 3 shows one of the $\Lambda + p \rightarrow \Sigma^+ + n$ interactions. Tracks 3 and 4 are the negative and positive decay pions of the K^0 (arrow No. 2) that was produced in the primary reaction $\pi^- + p \rightarrow K^0 + \Lambda$.

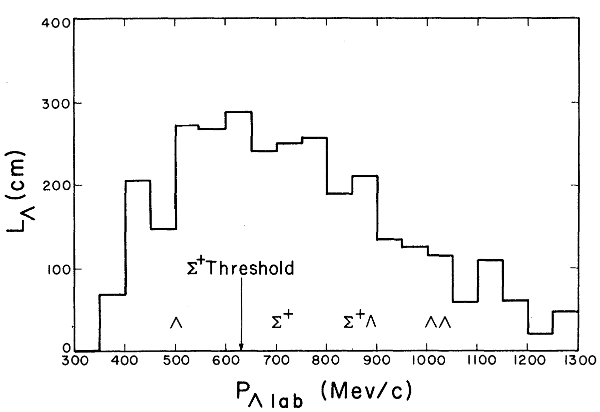

FIG. 1. This figure shows the Λ path length, L_Λ, as a function of the momentum of the Λ's prior to the interactions. The momenta of the Λ elastic scatterings are indicated by the "Λ" symbols, and the two $\Lambda + p \rightarrow \Sigma^+ + n$ reactions by the "Σ^+" symbols.

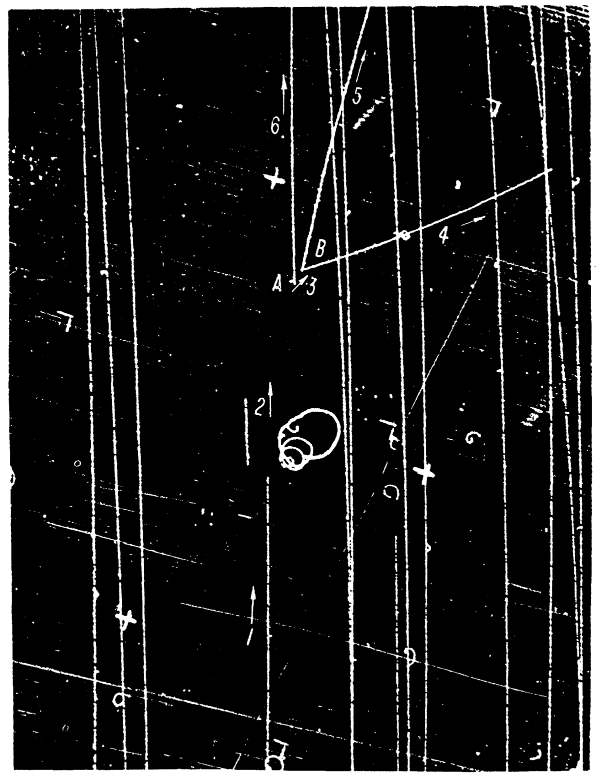

FIG. 2. The primary reaction is π^- (Track 1)$+p \rightarrow \Lambda$ (arrow No. 2)$+K^0$ (not seen). The Λ travels 3.7 cm and scatters elastically at point A in the direction indicated by arrow No. 3. Track 6 is the recoil proton. At point B the scattered Λ decays into a π^- (Track 4) and a proton (Track 5).

The Λ (arrow No. 5) interacts at point A to produce a Σ^+ (Track 6) and a neutron. The Σ^+ then decays at point B into a π^+ (Track 7) and a neutron.

The 655 Λ's that we have observed to decay in the chamber have traversed a total path length in liquid hydrogen of 2200 cm.

In addition to the observed Λ decays there were 195 cases in which only the K^0 decays. From the observed K^0 we can predict the direction in which the unseen Λ traverses the chamber. We then search along this direction for a recoil proton or a "recoil" Σ^+. We estimate that Λ's of this category traversed 700 cm of liquid hydrogen. From the combined path length in liquid hydrogen of 2900 cm, we obtain an elastic scattering cross section[2]

$$\sigma_{\Lambda\Lambda} = 40 \pm 20 \text{ mb}.$$

If the cross section is assumed to have the energy dependence of $\sigma = \epsilon\pi\lambda\!\!\!^{-2}(\text{c.m.})$, with ϵ con-

FIG. 3. The primary reaction is π^- (Track 1)$+p \rightarrow \Lambda$ (arrow No. 5)$+K^0$ (arrow No. 2). The K^0 decays into a π^- (Track 3) and a π^+ (Track 4). The Λ subsequently interacts at point A to produce a Σ^+ (Track 6) plus a neutron. The Σ^+ decays at point B into a π^+ (Track 7) and a neutron.

stant, then by averaging over the Λ momentum spectrum we obtain

$$\epsilon_{\Lambda\Lambda} = \sigma_{\Lambda\Lambda}/\pi\langle \lambda\!\!\!^{-2}(\text{c.m.})\rangle = 2.4 \pm 1.2.$$

Here $\lambda\!\!\!^{-}$(c.m.) is the de Broglie wavelength of the Λ in the c.m. system.

Of the total path length of 2900 cm, 2000 cm was traversed by Λ's of momentum higher than the threshold for the $\Lambda+p \rightarrow \Sigma^+ + n$ reaction. From the two observed reactions of type (2) we obtain

$$\sigma_{\Lambda\Sigma^+} = 30 \pm 20 \text{ mb}.$$

By averaging over Λ momenta above threshold, we find the corresponding value

$$\epsilon_{\Lambda\Sigma^+} = 3 \pm 2.$$

Furthermore, by detailed balancing we can predict the cross section for the inverse reaction,[3] $\Sigma^+ + n \rightarrow \Lambda + p$, i.e.,

$$\sigma_{\Sigma^+ n} = (p/p')^2 \sigma_{\Lambda\Sigma^+} = 120 \pm 80 \text{ mb}.$$

The two events had momenta of 120 and 220 Mev/c in the $\Sigma^+ + n$ c.m. system. If charge independence holds, we expect the $\Sigma^0 + p \rightarrow \Lambda + p$ cross section to be 60 ± 40 mb. (We mention this because of the evidence for such an interaction in the $K^- + d$ experiment of Horwitz et al.[4])

We are grateful to Professor Luis Alvarez for his stimulation and guidance during this experiment. We are indebted to Don Gow and the bubble chamber crew, and to Hugh Bradner and the scanners for their help. We thank George Kalbfleisch and Roger Douglass for their assistance with the data analysis.

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[*]This work was done under the auspices of the U. S. Atomic Energy Commission.

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[2]A Λ that scatters less than 10 degrees produces a recoil proton that would escape detection. Our elastic scattering cross section therefore corresponds to scatterings greater than 10 degrees.

[3]We assume that both the Λ and the Σ have the same

Table I. Reaction dynamics.

Event frame number	Initial momentum (lab) (Mev/c)	Hyperon angle (c.m.) (degrees)	Momentum in center of mass of "scattering" Incoming	Outgoing
A. $\Lambda + p \rightarrow \Lambda + p$				
318425	1000	65	420	420
321560	1000	120	430	430
389195	500	80	220	220
439748	880	170	380	380
B. $\Lambda + p \rightarrow \Sigma^+ + n$				
232018	700	40	310	120
309771	840	90	360	220

spin.
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RADIATIVE DECAY OF THE MUON*

Toichiro Kinoshita
Laboratory of Nuclear Studies,
Cornell University, Ithaca, New York,

and

Alberto Sirlin
Physics Department,
Columbia University, New York, New York
(Received January 26, 1959)

The inner bremsstrahlung accompanying the muon decay ($\mu \to e + \nu + \bar{\nu} + \gamma$) has been studied previously, and its contribution to the radiative correction of the decay spectrum, the lifetime, and the asymmetry has been analyzed in detail.[1-3] Recently, some other aspects of this problem have been discussed by several authors.[4-6] The purposes of this note are to derive the spectrum of the photon emitted in the decay of muons in the framework of the two-component theory of the neutrino and to point out the possibility that this effect may be observed with present techniques.

The probability that a photon and an electron are emitted in muon decay with any given energies and momenta has been given by Eq. (17b) of reference 2. To find the photon spectrum in which we are now interested, we have only to integrate it with respect to the energy and angle of the electron, taking account of the conservation law of energy and momentum. The result is given by

$$\frac{dN}{N_0} = \frac{\alpha}{3\pi}(1-y)dy\left\{\left[2\ln\frac{m_\mu}{m_e} - \frac{17}{6} + \ln(1-y)\right]\right.$$
$$\left.\times\left[\frac{3}{y} - 2(1-y)^2\right] - \frac{1}{12}(1-y)(22-13y)\right\}, \quad (1)$$

where N_0 is the total probability of muon decay without radiative corrections and $y = 2\omega/m_\mu$ is the photon energy in units of its maximum value. In deriving Eq. (1) the mass of the electron has been neglected in comparison with its energy whenever this approximation has not led to spurious divergences. This is certainly justified when the condition $(1-y)^2 \gg (2m_e/m_\mu)^2$ is fulfilled.

Integrating Eq. (1) further over the range of photon energy between $y = y_0 = 2\omega_0/m_\mu$ and $y = 1$, one finds the rate for muon decay accompanied by photons of energy greater than ω_0:

$$R(y_0) = \frac{\alpha}{3\pi}\left\{\left(\ln\frac{m_\mu}{m_e} - \frac{17}{12}\right)\left[6\ln\frac{1}{y_0} - 6(1-y_0) - (1-y_0)^4\right]\right.$$
$$+ 3[L(1) - L(y_0)] - \tfrac{1}{2}[6 + (1-y_0)^3](1-y_0)\ln(1-y_0)$$
$$\left. + \frac{1}{48}(1-y_0)(125 + 45y_0 - 33y_0^2 + 7y_0^3)\right\}, \quad (2)$$

where

$$L(x) = \int_0^x \frac{dt}{t}\ln|1-t|. \quad (3)$$

For $y_0 \ll 1$, Eq. (2) coincides with the expression obtained by integrating Eq. (25b) of reference 2 over all electron energies.

Equation (2) shows that about 4.9% of muon decay is accompanied by photons of energy greater than $2m_e$, the threshold energy for the production of electron-positron pairs. Even for $\omega_0 \sim 20m_e$, the rate is still as large as 1.2%. Thus it seems likely that the inner bremsstrahlung accompanying the muon decay may be detected and investigated experimentally with techniques now available.

Equations (1) and (2) constitute a definite and clear-cut prediction of the two-component neutrino theory and of the usual laws of quantum electrodynamics as applied to the electron and muon fields. In particular, they are independent of the relative magnitudes of the V and A interactions in the order $(e\mu)(\nu\nu)$ or, what amounts to the same thing, of the S and V interactions in the order $(e\nu)(\nu\mu)$. Thus, if the predictions of Eqs. (1) and (2) were to be contradicted by experiments, the consequences would be far reaching indeed.

The question naturally arises as to whether measurements of the radiative decay rate can give us new information about the nature of the weak interaction in the more general framework of the four-component neutrino theory. This is now being studied.

One of us (A.S.) would like to thank Professor T. D. Lee for his comments and Dr. M. Bardon, Dr. D. Berley, and Dr. J. Lee for useful conversations.

*Supported in part by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

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