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FINE STRUCTURE IN THE ZEEMAN EFFECT OF EXCITONS IN GERMANIUM*

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Exciton formation observed in the experimental study[1] of the indirect transitions in Ge produced two strong absorption edges at zero magnetic field which were identified with the split ground level. The original data were obtained by observing the transmission of linearly polarized infrared radiation through a 6-mm sample at 1.5°K using fields up to 38 9000 oersteds and a spectral slit width of 7×10^{-5} ev. With increasing magnetic field, the two absorption edges move to higher energy and a large number of smaller edges appear superimposed upon them as the Zeeman fine structure develops. These smaller edges also move to higher energy with increasing field, and this motion is best illustrated when the edges are transformed to maxima by taking the derivative of the transmission traces as shown in Fig. 1. Figure 2 shows a diagram of energy levels as a function of magnetic field intensity where the bars show the positions of the maxima, including those of Fig. 1, and the lines represent theoretical results.

The theory was based on the spin Hamiltonian derived by considering the symmetry properties of the exciton band edge. In Ge, the band edge for this exciton is at four equivalent [111] points on the reduced zone boundary,[2] which are assumed to contribute independently to the spectrum. States at each of these points are characterized by their symmetry with respect to the point group D_{3d}. The Hamiltonian is a linear combination of D_{3d} invariants constructed from

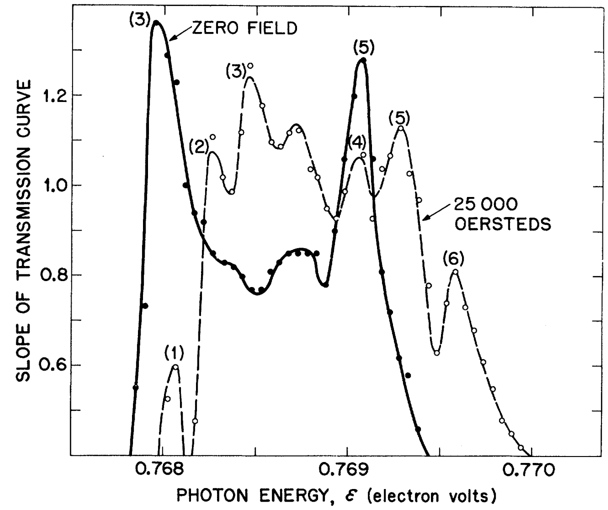

FIG. 1. Zeeman fine structure in the indirect exciton in Ge for $\vec{H} \parallel [100]$, $\vec{E} \parallel \vec{H}$ at 1.5°K. The two prominent maxima at zero field move to higher energy with increasing field and develop satellites which also move to higher energy.

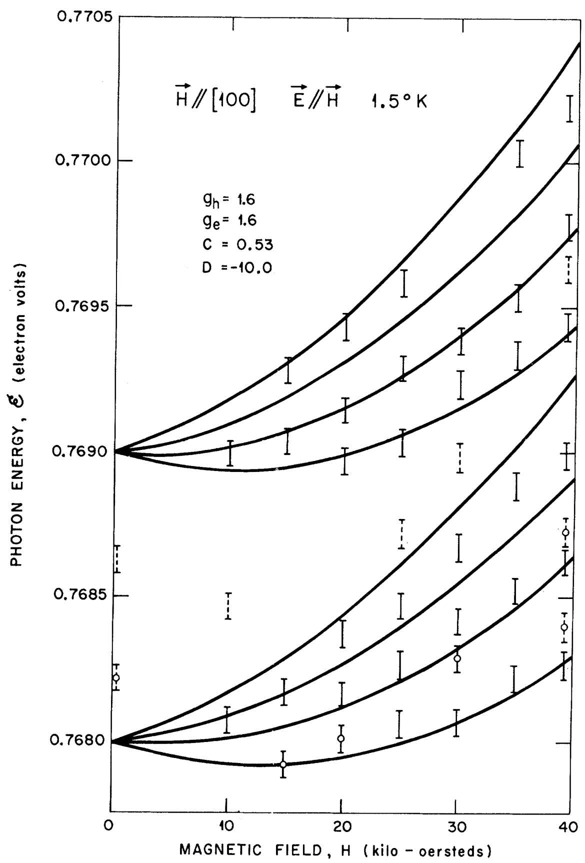

FIG. 2. Diagram of energy levels of the indirect exciton vs magnetic field intensity. The bars represent experimental maxima such as those of Fig. 1; those containing circles represent inflections and the dashed bars are not accounted for by the theoretical lines. The lines represent solutions of Eq. (1).

the angular momentum J of the hole ($\vec{J}=3/2$), the spin $\vec{S}$ of the electron ($S=\frac{1}{2}$), and the magnetic field $\vec{H}$. A simple combination which was found to fit the experimental energy spectrum is

$$\mathcal{H}=D(J_z^2-\vec{J}^2/3)-g_h\beta\vec{J}\cdot\vec{H}+g_e\beta\vec{S}\cdot\vec{H}+CH^2, \quad (1)$$

where β is the Bohr magneton. The first term represents the zero-field splitting, the z axis lying along a trigonal direction, the second and third terms represent Zeeman energies of the hole and electron, and the fourth represents an isotropic diamagnetic contribution which becomes significant above 10 000 oersteds.

The first two terms of Eq. (1) yield a 4×4 matrix. The effect of the magnetic field is to split the two zero-field levels into four. Addition of electron Zeeman splitting results in eight levels for each exciton valley. The experimental results for $\vec{H}\parallel[100]$, for which all valleys give the same spectrum in Ge, are compared with the theoretical spectrum in Fig. 2. The values of the coefficients obtained by fitting the experimental data for this orientation are $2D=(-10.2\pm5\%)\times10^{-4}$ ev, $g_h=1.6\pm10\%$, $g_e=1.6\pm15\%$, and $C=(0.53\pm5\%)\times10^{-12}$ ev/oe². Actually, any value within the range of uncertainty fits reasonably well.

Experimental results were also obtained for the magnetic field in the [111] direction. For $\vec{E}\parallel\vec{H}$, a reasonable fit was achieved by using the above parameter values and considering only the three valleys oriented equivalently with respect to $\vec{H}$. For $\vec{E}\perp\vec{H}$, more structure appeared, which is consistent with the appearance of a contribution from the fourth valley.

A detailed interpretation of the coefficients of the spin Hamiltonian can be considered with the effective-mass approximation. A theoretical estimate[3] of D agrees with the experimental value. The magnitude of D depends both on the electron anisotropy and the difference in mass of the heavy and light holes. The value of g_h obtained is only a quarter of the value 6.4 for the "free" hole estimated by Luttinger.[4,5] Thus, binding effects must be important. A theoretical estimate indicates that the electron g-factor should be less than two and highly anisotropic.[3] The spectra observed are not inconsistent with this. It may be possible to determine this anisotropy by performing a more accurate experiment, but in that case it may also be necessary to include other anisotropic terms in the spin Hamiltonian.

The indirect exciton data can be improved by using higher fields, circularly polarized radiation and an electronic method to differentiate the transmission signal continuously. The use of a sample at least one centimeter thick will improve the definition by appropriately matching the absorption coefficient. Alternatively, the emission technique[6] at liquid He temperature should produce discrete lines from the annihilation of the indirect exciton. The use of a Weierstrass sphere with larger-volume magnetic fields would focus the emergent radiation.

A similar analysis of the direct exciton[7] has been undertaken using a Hamiltonian analogous to Eq. (2) with $D=0$. Transitions to six levels are expected in the region below about 15 000 oersteds where the spin Hamiltonian should be valid. The six allowed transitions have not been properly resolved, although the fine structure has definitely been indicated by the three- to four-fold increase in line width at 38 900 oersteds. The electron spin g-factor[5] ($g=-2.6$) implies a Zeeman splitting of about 5×10^{-4} ev at maximum field. To resolve this structure, either higher fields will be required or the present line width of $\sim10^{-3}$ ev will have to be reduced by the use of thinner, strain-free samples.

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