error is statistical. This does not imply that there are no other errors; it is only a reflection of the fact that we are ignorant of any substantial systematic errors.

If instead we correct using known annihilation[2] and pion cross sections[3] and some auxiliary measurements performed on secondaries from π^+ and π^- interactions in Freon-filled bubble chambers,[4] we find corrections of $(+0.25 \pm 0.05)\times 10^{-3}$ for positron annihilation, and $+(0.25 \pm 0.15)\times 10^{-3}$ for differential pion interactions, for a total correction

$$\delta_{\text{correction}} = (+0.5 \pm 0.16)\times 10^{-3}$$

and

$$\delta = (+2.11 \pm 0.3)\times 10^{-3} \quad \text{(computed correction)}.$$

We combine these results to report what we believe to be the best value at this time:

$$\delta = (+2.24 \pm 0.36)\times 10^{-3} \quad \text{(average correction)}.$$

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[1]This is a consequence of a phenomenological analysis of experimental results on other decay modes. References to the phenomenological analysis and to the experiments are given in the following Letter [Phys. Rev. Letters 19, 997 (1967)].

[2]P. A. M. Dirac, Proc. Cambridge Phil. Soc. 26, 361 (1930).

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[4]We wish to thank Professor D. H. Perkins and Professor H. H. Bingham for making these heavy-liquid bubble-chamber photographs of π^+ and π^- interactions available to us.

NONORTHOGONALITY OF THE LONG- AND SHORT-LIVED NEUTRAL KAON STATES AND PHENOMENOLOGICAL ANALYSIS OF EXPERIMENTS ON CP NONCONSERVATION IN K^0 DECAY*

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It is noted that the nonzero charge asymmetry in the $(K_L)_{e3}$ decay demonstrates the nonorthogonality of the K_L^0 and K_S^0 states. The result of the foregoing Letter on the charge asymmetry, combined with other measurements relevant to CP nonconservation in K decay, are found to be consistent with a phenomenological analysis, which yields a pion-pion scattering phase shift consistent with other indirect observations only if the mass difference $m_L - m_S$ is negative.

The result on the charge asymmetry reported in the preceding Letter can be related to other properties of neutral kaon decay.[1] Here we wish to call attention to two essentially separate connections, one to the nonorthogonality of the long- and short-lived K^0 states, the other to the CP-nonconserving amplitudes in the two-pion decay.

We assume CPT symmetry in the absence of any evidence to the contrary. It was shown by Lee, Oehme, and Yang[2] that the long- and short-lived states can then be written in terms of the eigenstates of hypercharge, K and $\bar{K}$:

$$|L\rangle = p\,|K\rangle + q\,|\bar{K}\rangle, \tag{1a}$$

$$|S\rangle = p\,|K\rangle - q\,|\bar{K}\rangle, \tag{1b}$$

$$|p|^2 + |q|^2 = 1,$$

where p and q are as yet undetermined coefficients, which are equal in the case of CP conservation. If CP symmetry is violated, $|L\rangle$ and $|S\rangle$ are in general not orthogonal. Let α be the overlap:

$$\alpha = |\langle L|S\rangle| = |p|^2 - |q|^2.$$

Consider now the four leptonic decay amplitudes

$$\mathrm{Amp}(K^0 \to \pi^- e^+ \nu) = f, \tag{2a}$$

$$\mathrm{Amp}(\bar{K}^0 \to \pi^+ e^- \bar{\nu}) = f^*, \tag{2b}$$

$$\mathrm{Amp}(K^0 \to \pi^+ e^- \bar{\nu}) = g^*, \tag{2c}$$

$$\mathrm{Amp}(\bar{K}^0 \to \pi^- e^+ \nu) = g. \tag{2d}$$

It is a consequence of CPT symmetry that two independent amplitudes suffice for the description of the two reactions. The f amplitude [Eqs. (2a) and (2b)] corresponds to $\Delta S = \Delta Q$, the g amplitude [Eqs. (2c) and (2d)] to $\Delta S = -\Delta Q$. Let $x = g/f$ be the ratio of $\Delta S = -\Delta Q$ to the $\Delta S = +\Delta Q$ amplitudes, and δ be the charge asymmetry in K_L decay as defined in the foregoing Letter. Then

$$\delta = \frac{(\Gamma_{e^+} - \Gamma_{e^-})}{(\Gamma_{e^+} + \Gamma_{e^-})} = \alpha \frac{(1 - |x|^2)}{|1 + x|^2}. \tag{3}$$

A nonzero δ implies that the long-lived and short-lived K states are not orthogonal. The most direct result of the foregoing experiment, is, therefore, that K_L and K_S are not orthogonal.

In order to proceed it is necessary to know x. From experiment (see, for instance, the summary[3] by Bell and Steinberger) only an upper limit of ~0.2 can be put on $|x|$. In the following we assume that $x = 0$. This is attractive from a theoretical point of view, and in any case, no gross error is likely. We then have the second result:

$$\alpha = |p|^2 - |q|^2 = \delta = (2.24 \pm 0.36) \times 10^{-3}.$$

It has been shown by Wu and Yang[4] that this result can be related to the two-pion decay amplitudes. The analysis has been applied by Gaillard et al.[5] to the data existing at that time.

Let

$$\eta_{+-} = \frac{\langle \pi^+, \pi^- | T | L\rangle}{\langle \pi^+, \pi^- | T | S\rangle},$$

$$\eta_{00} = \frac{\langle \pi^0\pi^0 | T | L\rangle}{\langle \pi^0\pi^0 | T | S\rangle},$$

$$\epsilon = \frac{p-q}{p+q},$$

and

$$\epsilon' = \frac{i}{\sqrt{2}} e^{-i(\delta_2 - \delta_0)} \mathrm{Im} \frac{\langle 2|T|K\rangle}{\langle 0|T|K\rangle},$$

where δ_0 and δ_2 are the pion-pion scattering phases in the isospin 0 and 2 states, respectively. Using the phase convention $\mathrm{Im}\langle 0|T|K\rangle = 0$, and assuming CPT symmetry, the following phenomenological relations can be obtained (see Ref. 3 or 4):

$$\eta_{+-} = \epsilon + \epsilon', \tag{4a}$$

$$\eta_{00} = \epsilon - 2\epsilon'. \tag{4b}$$

If, in addition, the CP-nonconserving amplitudes in decay modes other than the two-pion mode can be neglected (see, for instance, Ref. 3 or 4), the unitary requirement yields

$$\mathrm{Im}\epsilon / \mathrm{Re}\epsilon = 2\Delta m / \Gamma_S, \tag{4c}$$

where $\Delta m = m_L - m_S$.

From the definitions of ϵ and α, and from the fact that they are small, it follows that

$$2\,\mathrm{Re}\epsilon = \alpha.$$

For the following analysis it is further assumed that $x \simeq 0$, that is, that the $\Delta S = -\Delta Q$ amplitude is negligible, so that $\delta = \alpha$. This gives the fourth of the relations (4):

$$\delta = 2\,\mathrm{Re}\epsilon. \tag{4d}$$

We summarize: The relations (4a)-(4d) are derived with the assumption of CPT invariance, smallness of CP nonconservation in other than the 2π decay mode, and $\Delta S = \Delta Q$. They have as consequences that the five measurable parameters $|\eta_{+-}|$, phase of η_{+-}, $|\eta_{00}|$, phase of η_{00}, and δ are expressible in terms of three parameters $|\epsilon|$, $|\epsilon'|$, and phase of ϵ'.

Relevant existing measurements are given in Table I, and shown in Fig. 1.

The relations (4a) to (4d) imply that it is possible in Fig. 1 to draw a straight line, the line $-3\epsilon'$, from η_{+-} through ϵ to η_{00}, such that the length from η_{+-} to ϵ is one-half that from ϵ to η_{00}. This line is also shown in Fig. 1 and it can be seen that a good fit is possible. On the basis of this fit,

$$|\epsilon'| = (1.69 \pm 0.2) \times 10^{-3},$$

$$\varphi_{\epsilon'} = (147 \pm 4)^\circ = (2.56 \pm 0.07)\ \mathrm{rad},$$

$$\varphi_{\eta_{00}} = (-9 \pm 5)^\circ = (-0.16 \pm 0.09)\ \mathrm{rad},$$

Table I. Experimental data relevant to the phenomenological relations (4a)-(4d).

Quantity	Value	Reference
$\lvert\eta_{+-}\rvert$	$(1.96\pm0.09)\times10^{-3}$	b
$\lvert\eta_{00}\rvert$	$(4.3^{+1.1}_{-0.8})\times10^{-3}$	c
	$(4.17\pm0.30)\times10^{-3}$	d
φ_{+-}	1.47 ± 0.3[a]	e
$\Delta m/\Gamma_S$	0.44 ± 0.028	f
	0.48 ± 0.026	g
δ	$(2.24\pm0.36)\times10^{-3}$	h

[a]The phase of φ_{+-} is based on positive Δm. For negative Δm the sign of φ_{+-} reverses and the signs of all other angles in Fig. 1 reverse as well.

[b]J. Cronin, compilation presented in Proceedings of the Rochester Conference on High Energy Physics, August, 1967 (unpublished).

[c]Ref. 5.

[d]J. Cronin, P. Kunz, W. Risk, and P. Wheeler, Phys. Rev. Letters 18, 25 (1967), and private communication by J. Cronin. We wish to thank Professor Cronin for making the more recent results available to us prior to publication.

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[h]Preceding Letter [Phys. Rev. Letters 19, 993 (1967)].

and

$$\delta_2-\delta_0=(57\pm4)^\circ=(1.0\pm0.07)\text{ rad}.$$

The errors are approximate only.

The above solution of Eqs. (4a)-(4d), which is also that shown in Fig. 1, is based on $m_L > m_S$, since there is mounting evidence[6-8] for this sign of Δm. The resultant phase for pion-pion scattering, $\delta_2-\delta_0$, has then however the opposite sign from that which follows from the analysis of pion production in pion-nucleon collisions, although the magnitude, $(57\pm4)^\circ$, is quite close to the magnitude of 53° reported by Walker et al.[9] in a recent analysis of pion-production experiments.

Another solution to the CP-nonconservation data is possible, however, based on negative Δm. The solution is obtained by changing the sign of the imaginary axis in Fig. 1. All magnitudes remain the same, and all phases change sign. The $\delta_2-\delta_0$ phase shift obtained on the basis of $m_L < m_S$ agrees with the results of the analysis of the pion-production experiments.

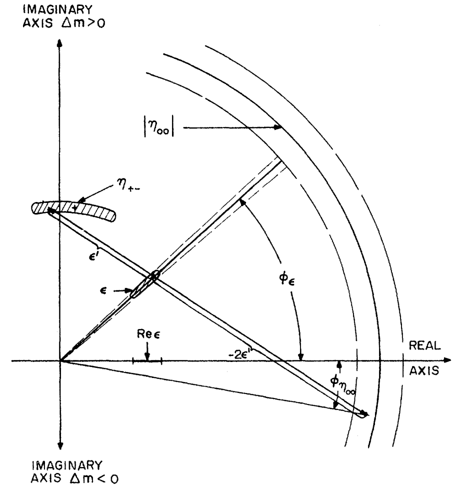

FIG. 1. Experimental results for η_{+-}, η_{00}, ϵ, and ϵ' in the complex plane.

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