with proton interpretation is compared with that expected in all views. Tracks with interpretations other than proton are not used, since at our incident energy the K, π ambiguity is in most cases difficult to resolve.

[17] A comparison with visual identification indicates a maximum possible misidentification of 3% for Reactions (1) and (2).

[18] This enhancement was called the Q effect by the authors of Ref. 2. The quoted cross section, for the enhancement with N^{*++} removed, is estimated from Reactions (1) and (2) separately and corrected for invisible K^0 decays and FSD efficiencies.

[19] J. Bartsch et al. [Aachen-Berlin-CERN-Imperial College (London)-Vienna Collaboration], Phys. Letters 22, 357 (1966).

[20] The $K^*(890)$ mass band is defined from 860 to 940 MeV, whereas the ρ-meson band is defined from 650 to 850 MeV. The N^{*++} band is defined from 1160 to 1320 MeV.

[21] To avoid confusion in nomenclature we refer to the peak observed here at 1360 MeV as the $K^*(1320)$.

[22] It was observed in the study of $K^+p \rightarrow K^{+,0}\pi^{-,0}\pi^+p$ at 4.6 BeV/c (Ref. 8) that the $K^*(1320)$ resonance was mainly associated with the equatorial region of $\cos\alpha$. That observation is quite contrary to our data at 9 BeV/c, which indicate that both the $K^*(1250)$ and $K^*(1320)$ have $\cos^2\alpha$ shape distributions in the $K\pi$ scattering angle. One possible explanation for this difference is that the production mechanisms of the $K^*(1250)$ and $K^*(1320)$ may be a sensitive function of incident energy, giving rise to different decay angular distributions. This can also give rise to the energy dependence of the phase between the two resonance amplitudes, as described in Ref. 11. Without invoking the interference between these two resonances, it is difficult to account for all the different observations by various experimental groups at different incident momenta and the fact that the $K^*(1250)$ production rises extremely slowly with incident momentum far above its kinematical production threshold.

INTERFERENCE PHENOMENA ASSOCIATED WITH BOSON RESONANCE PRODUCTION*

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It is suggested that the peculiar variation in structure observed in the $K\pi\pi$ mass enhancement in the region 1.1 to 1.4 BeV as a function of incident momentum could be primarily due to interference between two resonances with the same J^P values.

As mentioned in the previous Letter,[1] the structure of the very large enhancement observed in the $K\pi\pi$ system in the mass region 1.1 to 1.5 BeV has shown distinct and statistically significant differences for various incident momenta. The $K\pi\pi$ enhancement can be separated into two parts: the $K^*(1420)$, and the region roughly between 1.1 and 1.4 BeV recently called the "Q enhancement."[2] We wish to suggest here that the observed variation in structure in the Q enhancement is primarily due to interference effects between two adjacent resonances with equal spin and parity. In addition, a coherent background term is probably also present and must be taken into account.[3] As has been suggested,[1] $J^P = 1^+$ is the most likely value for both K^* resonances as well as for the background.[3,4]

In general a mass distribution corresponds to an average over all decay angular distribution. Thus K^*'s with different J^P values, such as a 1^+ K^* and the 2^+ $K^*(1420)$, will not give an interference effect in the $K\pi\pi$ mass distributions. However, two K^*'s with equal J^P values will add coherently.

In what follows we consider a very simple model corresponding to the coherent addition of two resonances together with a third added incoherently.[5] Here we express each resonance by a Breit-Wigner amplitude and allow an arbitrary phase between two of them.

Let $B_k = \frac{1}{2}\Gamma_k/(E_k - E - i\frac{1}{2}\Gamma_k)$, with $k = 1$, 2, and 3, correspond to the Breit-Wigner amplitude for each of these resonances; then the resulting mass distribution can be expressed as

$$d\sigma/dM \propto (|a_1B_1 + B_2e^{i\varphi}|^2 + |a_3B_3|^2)P,$$

where E_k and Γ_k are the resonant masses and widths, respectively, φ is a relative phase angle, and a_1 and a_3 relative amplitudes, all of which must be determined from experiment, and P is a phase-space factor. As an illustration, this expression was evaluated for $E_1 = 1250$ MeV, $\Gamma_1 = 50$ MeV; $E_2 = 1320$ MeV, $\Gamma_2 = 80$ MeV; $E_3 = 1420$ MeV, $\Gamma_3 = 90$ MeV; $a_1 = 1$; $a_3 = 2^{-1/2}$; and values of φ from 0 to $9\pi/5$ in ten equal steps. Figure 1 shows the resulting

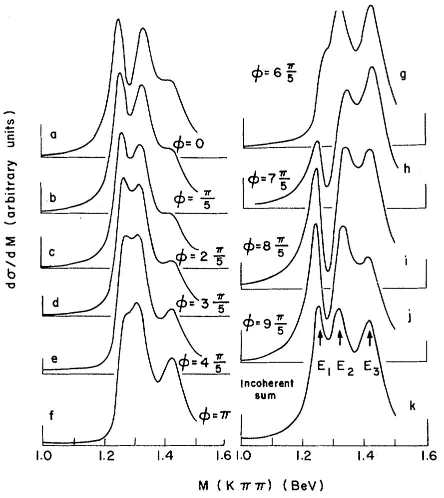

FIG. 1. Computation of the interference patterns in the $K\pi\pi$ mass distribution for two K^* resonances at 1250 and 1320 MeV added coherently and a third at 1420 MeV added incoherently. The computation was done for a series of values of the phase angle φ between the two coherent amplitudes as described in the text, and is shown in parts *a* to *j*. In part *k* the incoherent sum of the three resonances is shown.

mass distributions. As may be noted, aside from the $K^*(1420)$ peak, the shape of the mass distribution in the Q enhancement can appear as a single peak at E_2 for $\varphi \approx 6\pi/5$, a broad flat-topped peak centered at $\approx \frac{1}{2}(E_1 + E_2)$ for $\varphi = \pi$, indications of a single peak at E_1 for $\varphi \approx 3\pi/5$, and two separate peaks at $\approx E_1$ and $\approx E_2$ for $\varphi \approx \pi/5$ or lower, as well as for $\varphi \approx 8\pi/5$. Thus, in a variation of φ, the mass distribution can go through an entire gamut of shapes, some of which are very similar to the experimental K^+p data in the region 4.6-9 BeV/c.[6]

A more realistic model must include both a coherent and an incoherent background term, where the phase of the former can also vary relative to the two Breit-Wigner amplitudes.[7]

We have not yet attempted actually to "fit" the available experimental data with this model, primarily because observation of the splitting between the "1250-MeV" and "1360-MeV" peaks discussed in the preceding Letter for the 9-BeV/c data is strongly dependent on the experimental resolution, a quantity which is not known to us for much of the data in the literature. Furthermore, for a significant fit, considerably higher statistical accuracy will be required at many of the momenta studies.

It is clear, however, that a fit to the experimental data can be obtained, although such a fit will not be unique without more detailed information than given by the mass distributions alone. A possible approximate solution for the K^+p interaction which ignores background interference effects would be $\varphi \approx 6\pi/5$ rad at 4.6 BeV/c and either $\varphi \approx 2\pi/5$ to $\pi/5$ rad or $\varphi = 8\pi/5$ rad at 9 BeV/c. Thus $\Delta\varphi = \varphi(4.6) - \varphi(9.0) \approx \pi$ rad or $\approx -2\pi/5$ rad. These assignments would then imply that in the first case at some intervening momenta the Q enhancement should appear as a single wide peak centered at $\approx$1285 MeV, while this is not so in the second case.

The above cases should be considered as examples only and do not represent an exhaustive search for solutions.

The significance of φ.—Finally, if we accept this model, we must ask about the physical significance of φ and whether it is likely to vary with incident momentum. We will show that in terms of the quark model, a variation of φ with incident momentum is plausible. On the $q\bar{q}$ model for bosons we expect two 1^+ nonets: ${}^3P_1$ and ${}^1P_1$ with $J^{PC} = 1^{++}$ and 1^{+-}, respectively.[8] Although the assignments are by no means settled, the $A_1(1080)$ and $B(1220)$ have been considered as the isovectors of these two nonets.[9] If we consider the corresponding K^*'s, a new and so far unique situation can arise.[10] While the isovectors in the two nonets are eigenstates of G, and their neutral members and the isoscalars are eigenstates of C as well, mixing can occur[11,12] between the two 1^+ K^*'s. If the two K^*'s are indeed mixed, we may then have production amplitudes for the two physical K^*'s which consist, for example, of the sum and difference of A_P and A_M with appropriate coefficients. Here A_P is the amplitude for Pomeranchukon exchange and A_M is the amplitude for exchange of one or more isoscalar mesons[13]: ω, f_0, or P'. If this is the case, one amplitude, A_P, remains essentially constant while the other A_M, may decrease rapidly with increasing incident momentum. The phase angle φ would then reflect this relative change in the production amplitudes of the two K^*'s.

One other feature which follows is that the alignment of the two K^*'s may also change with incident momentum as the relative strength of A_P and A_M change. Thus at low momentum we would expect an alignment characteristic of a considerable component due to meson exchange which, as the momentum increases, should go over to an alignment characteristic of Pomeranchukon exchange.

There is actually some experimental evidence for a change in the alignment of the structure in the Q enhancement. Thus in the K^+p experiment[6] at 4.6 BeV/c it was observed that the $K^*(1320)$ peak corresponds primarily to a K_{in}-K_{out} scattering angle α in the $K^*(890)$ c.m. system in the equatorial region, i.e., $|\cos\alpha| < 0.8$. The same result can be noted for the peak at 1270 MeV observed in a K^-p experiment at 3.8 BeV/c by Field et al.[14] These results are thus indicative of an isotropic or $\sin^2\alpha$ distribution for the respective peaks observed at these momenta. On the other hand, in the K^+p experiment[1] at 9 BeV/c, both peaks observed in the Q enhancement occur in both the equatorial and the polar regions in $\cos\alpha$. This corresponds to a $\cos^2\alpha$ distribution, which, as pointed out in the preceding Letter, is indicative of 1^+ resonance formation by Pomeranchukon exchange.

Finally, it should be noted that φ may also be a function of Δ_p^2, the invariant four-momentum transfer to the $K\pi\pi$ system. The experimental mass distributions may thus be more complex in that the peaks in Fig. 1 can depend both on incident momentum and Δ_p^2 cuts.

I would like to thank G. F. Chew and H. Harari for helpful discussions on this subject.

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[1] G. Goldhaber, A. Firestone, and B. C. Shen, preceding Letter [Phys. Rev. Letters 19, 972 (1967)]. See also references to earlier work quoted there.

[2] J. Berlinghieri, M. S. Farber, T. Ferbel, B. Forman, A. C. Melissinos, T. Yamanouchi, and H. Yuta, Phys. Rev. Letters 18, 1087 (1967).

[3] M. Ross and Y. Y. Yam, Phys. Rev. Letters 19, 546 (1967).

[4] C.-Y. Chien, P. M. Dauber, D. J. Mellema, P. Schreiner, W. E. Slater, D. H. Stork, and H. K. Ticho, Bull. Am. Phys. Soc. 12, 506 (1967).

[5] Interference phenomena in mass distribution have been invoked previously in the ρ-ω mixing for the 2π decay of the ω, for example, by J. Bernstein and G. Feinberg, Nuovo Cimento 25, 1343 (1962), and more recently in connection with the splitting of the A_2 by G. Chikovani et al., Phys. Letters 25B, 44 (1967).

[6] The 4.6-BeV/c K^+p data show two resonance features in the $K\pi\pi$ mass at 1320 and 1420 MeV on a large background. [B. C. Shen, I. Butterworth, C. Fu, G. Goldhaber, S. Goldhaber, and G. H. Trilling, Phys. Rev. Letters 17, 726 (1966).] The 9-BeV/c data are described in the preceding Letter (Ref. 1).

[7] It is interesting to note that if we consider the A_1 as a resonance, some of the peculiar features observed in the $\pi\rho$ system, namely the appearance of the A_1 at some incident momenta and not at others, could be similarly interpreted as the interference of one 1^+ resonance and a coherent (1^+) background term. This implies that the corresponding φ' would have to vary with incident momentum.

[8] See, for example, R. H. Dalitz, in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, 1966 (University of California Press, Berkeley, California, 1967), p. 215.

[9] Here it must be stressed that in view of the new experimental evidence (Ref. 1) and the present arguments, the earlier suggested assignment (Ref. 5) of A_1 and K^* (1320) to the same nonet is no longer valid.

[10] It should be noted that here we have not used the quark model in any essential way but only to justify the assumption of two adjacent 1^+ nonets with opposite charge conjugation.

[11] This was pointed out by H. Lipkin, in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, 1966 (University of California Press, Berkeley, California, 1967).

[12] The K^*'s are not eigenstates of C. There is however a wider generalization of C referred to as "unitary parity" which has been introduced by Dothan and is called $\mathcal{C}$ in the literature. The precise definition and references are given by B. W. Lee in Proceedings of the Seminar on High Energy Physics and Elementary Particles, Trieste, 1965 (International Atomic Energy Agency, Vienna, Austria, 1965). The K^*'s are considered eigenstates of $\mathcal{C}$ and mixing between them involves SU(3) breaking. See also G. L. Kane, Phys. Rev. 156, 1739 (1967).

[13] Here we must remember one further experimental fact: The K^-p data at various momenta have shown very clearly that the production of the Q peak disappears completely when charge exchange occurs at the nucleon vertex, such as in the reaction $K^-p \to K^{*-}\pi^+n$. This feature suggests that Q production is suppressed for isovector exchange, leaving the possibility of Pomeranchukon and isoscalar exchange. [J. Bartsch et al., Phys. Letters 22, 357 (1966); T. P. Wangler et al., Bull. Am. Phys. Soc. 12, 540 (1967); P. J. Dornan et al., Bull Am. Phys. Soc. 11, 342 (1966).]

[14] J. Field, T. Hendricks, O. Piccioni, and P. Yager, Phys. Letters 24B, 639 (1967).