of inelastic cross sections for the 3^- and 2^+ levels of ^{40}Ca, $\sigma_{3^-}/\sigma_{2^+}$, is shown on the lower part of Fig. 1. The position of maxima of this ratio (arrows up) should correspond to the position of peaks in the $^{39}K(^3He,p)$ proton spectrum[5] of ^{41}Ca (arrows down). Table I compares these energies in the range common with both experiments; a certain agreement is visible, although a detailed comparison is hampered by the lack of statistics, the 50-keV resolution in the $(^3He,p)$ spectra,[5] and also by the shifts in energy existing between corresponding levels in mirror nuclei. Nevertheless, an important point to stress is that the spacings between groups of presumed $2p$-$1h$ resonances in ^{41}Sc are of the same order of magnitude as those between proton peaks in ^{41}Ca. Furthermore, it would be of interest to obtain the characteristics of the very strong resonance of the 3^- level around 8.15-MeV excitation in ^{41}Sc.

In order to compare our results with the predictions[2] for average "expurged" (d,p) spectra, we have extended the measurement to protons between 5.380 and 5.600 MeV, corresponding to excitations in the ^{41}Sc nucleus between 6.326 and 6.540 MeV. The Coulomb barrier reduced drastically the inelastic cross section; it was difficult to explore a wider range of energy. Nevertheless, in this narrow range the agreement between the excitation function for inelastic scattering to the 3^- level (curve a, Fig. 2) and the averaged (d,p) spectrum (curve b) is fair. The comparison with the (mutually consistent) $^{39}K(^3He,p)^{41}Ca$ spectra of Belote et al.[4] and Seth et al.[5] (curves d and c) is less clear. Taking into account the experimental uncertainties and the shift in the energy of levels in mirror nuclei, the large peak centered around 6.470-MeV excitation in ^{41}Ca could correspond to the resonance around 6.420 MeV in ^{41}Sc.

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SUPERCONVERGENT AMPLITUDES IN NUCLEON COMPTON SCATTERING

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(Received 2 June 1967)

A superconvergent sum rule for nucleon Compton scattering is derived. The agreement with experiment is good.

About a year ago, Drell and Hearn[1] wrote down an extremely simple sum rule relating the proton magnetic moment to an integral over the quantity $(\sigma_P-\sigma_A)$, σ_P (σ_A) representing the absorption cross section of a circularly polarized photon by a proton with the photon spin parallel (antiparallel) to the proton spin. Reasonable agreement with experiment was found when the π-N intermediate state, dominated by the N^* and N^{**} resonances, was retained. The sum rule was obtained by combining the Low–Gell-Mann–Goldberger low-energy theorem with an unsubtracted dispersion relation for the relevant amplitude. The latter assumption can be justified by a Regge-pole model. In this Letter, we wish to consider some further sum rules for certain other amplitudes in nucleon Compton scattering, which from a Regge-pole model have a superconvergent behavior. Superconvergence relations have in recent times been considered in other processes[2-5] with reasonable success, and because of their simplicity, they are certainly useful for correlating experimental data. Our present sum rule will be a relation between the nucleon magnetic moment and an integral over the imaginary part of the superconvergent amplitude; the latter we shall evaluate by retaining the π-N intermediate state, the imaginary part then being expressible as bilinear combinations of photomeson production amplitudes.

The kinematics and dispersion relations for nucleon Compton scattering have been studied by Hearn and Leader[6]; we shall follow their notation. If the high-energy behavior of the Compton scattering amplitude at finite t is assumed to be given in terms of Regge poles in the t channel, we can construct the following three amplitudes which behave like $s^{\alpha(t)-2}$ as $s\to\infty$ with $\alpha(t)$ the leading Regge trajectory in the crossed channel and which are free of s-channel kinematic singularities[7]:

$$R_1(s,t)=\frac{\langle\vec{k};1,-1|R|k;\frac{1}{2}\,\frac{1}{2}\rangle}{(\sin\frac{1}{2}\psi)^2(\cos\frac{1}{2}\psi)^2},\tag{1a}$$

$$R_2(s,t)=\frac{\langle\vec{k};1,-1|R|k;\frac{1}{2}\,-\frac{1}{2}\rangle}{(\cos\frac{1}{2}\psi)^3(\sin\frac{1}{2}\psi)},\tag{1b}$$

$$R_3(s,t)=\frac{\langle\vec{k};-1,1|R|k;\frac{1}{2}\,-\frac{1}{2}\rangle}{(\cos\frac{1}{2}\psi)(\sin\frac{1}{2}\psi)^3}.\tag{1c}$$

Under the crossing transformation $s\rightleftarrows u$, R_1 is even, whereas R_2 and $-R_3$ transform into each other. The superconvergence relation for R_1 is therefore trival, whereas a nontrivial superconvergence relation exists for

$$R(s,t)\equiv R_2(s,t)+R_3(s,t),\tag{2}$$

if $\alpha(t)<1$. In terms of the conventional invariant amplitudes,[6] we have

$$R(s,t)=\frac{1}{8\pi^2 t}\frac{t(A_4-A_5)+2(u-s)A_6}{1-(u-s)^2/t(t-4m^2)}.\tag{3}$$

The function $R(s,t)$ will satisfy a superconvergence relation for $\alpha(t)<1$, which is satisfied if we keep t negative. However, we cannot work at an arbitrarily large negative value of t, since that would involve the evaluation of the amplitude at unphysical points far removed from the physical region. We shall use a partial-wave expansion for evaluating the amplitude and hence shall work inside the Lehmann ellipse. We choose t to be $-m_\pi^2$. With this choice, our superconvergence relation reads

$$(\mu_p-1)^2=\frac{1}{\pi}\int_{(m+m_\pi)^2}^{\infty}ds\,\mathrm{Im}R^p(s,t),\tag{4a}$$

$$\mu_n^{\,2}=\frac{1}{\pi}\int_{(m+m_\pi)^2}^{\infty}ds\,\mathrm{Im}R^n(s,t),\tag{4b}$$

where μ_p (μ_n) denotes the total magnetic moment of the proton (neutron) in units of nuclear magnetons.

To evaluate $\mathrm{Im}R(s,t)$, we retain only the π-N intermediate state. The expression for $\mathrm{Im}R(s,t)$ then can be written in terms of bilinear combinations of photomeson production amplitudes; this is simple and straightforward algebra, the details of which we do not present here. We retain s, p, and d waves in the photomeson production amplitude; for the p and d waves, we use the energy-dependent parametrization carried out recently by Walker.[8] For the s wave, we use the model used by Adler and Gilman[9] and Ref. 5, which essentially incorporates the analysis of low-energy photoproduction results carried out by Schmidt.[10] The integration of the p- and d-wave contributions in Eqs. (4a) and (4b) were carried out numerically up to $s=150m_\pi^2$, which more than covers the resonating $N^*(1230)$ and $N^{**}(1520)$ regions. The integration of the nonresonating s-wave part was carried out only in the low-energy region up to $s=100m_\pi^2$, as was done in Refs. 9 and 5.

Our results are summarized in Table I; for the values of μ_p and μ_n we obtain $\mu_p=3.0\mu_N$ and $|\mu_n|=1.75\mu_N$. The neutron value however depends on the ratio of the coupling of the $N^{**}(1520)$ to isovector and isoscalar photons. Following an earlier result of Bietti,[11] we have taken the coupling to be purely isovector, our result essentially giving some a fortiori justification.

We have a few comments on the result. The agreement with experimental values certainly is good. We regard this as indicating the usefulness of simple superconvergence relations in correlating experimental results. Secondly, it has been shown[12] that in the case of "charged" photons by nucleons, a fixed pole in the complex angular-momentum plane at $J=1$ is expected. The presence of such fixed poles in the present case will introduce the residue as an additive term in our Eqs. (4a) and (4b). The success of our calculation seems to indicate that the fixed pole residue, if the pole is present at all, is certainly small.

Table I. The evaluation of the sum rules given by Eqs. (4a) and (4b).

Quantity	Partial-wave contributions s	p	d	Total
$(\mu_p-1)^2$	-1.2	4.3	0.9	4.0
$\mu_n^{\,2}$	-2.1	4.3	0.9[a]	3.1

[a]On the assumption that the $N^{**}(1520)$ is excited by isovector photons only.

I am grateful to my friend Dr. S. Nussinov for discussions.

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MEASUREMENT OF THE RELATIVE PARTIAL DECAY RATES FOR $K^\pm \to \pi^\pm + \pi^+ + \pi^-$†

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(Received 19 May 1967)

Recent experiments[1,2] have indicated that *CP* nonconservation occurs in the weak parity-nonconserving process $K^0 \to 2\pi$ through the $\Delta I \geq \frac{3}{2}$ transitions. It is important to know whether *CP* nonconservations occur in other channels of the weak interaction. The experiment reported in this paper investigates the possibility of a violation of *CP* invariance in the charged $K \to 3\pi$ channel that is parity-conserving and strangeness-changing.

We measure the relative partial rates of 3.5-BeV/c positive and negative kaons decaying in the tau mode, $K^\pm \to \pi^\pm + \pi^+ + \pi^-$,

$$\Gamma(--+)/\Gamma(++-). \tag{1}$$

If the ratio in (1) differs from unity, then there is a *CP*-invariance violation possibly due to the presence of $\Delta I \geq \frac{5}{2}$ transitions. Barrett and Truong[3] propose a model in which the *CP* nonconservation is due to the interference between the totally symmetric $I = 3$ and $I = 1$ three-pion isospin states if strong final-state interactions exist.[4] They assume that the matrix element for tau decay is linear in the energy of the odd pion and that *CPT* invariance is valid.[5]

The value we obtained after minor corrections is

$$\frac{\Gamma(--+)}{\Gamma(++-)} = \frac{\tau^-/K^-}{\tau^+/K^+} = 1.005 \pm 0.009,$$

where $\tau^\mp$ is the number of tau decays observed and $K^\mp$ is the number of kaons from which these tau decays occurred. Thus there is no deviation from unity within experimental error.

The tau decays were observed by photographing the three pions in thin-foil spark chambers. The identifying feature was three particles from a common vertex with no particle making an angle greater than 10° with the direction of the decaying kaon.[6] Table I shows that the maximum possible background due to competing kaon decays that could be confused with the tau was 0.069 of the tau rate. There is no mechanism for *CP* nonconservation in the background from K decays that is observable in our measurement. Therefore this background cannot introduce other than negligible corrections. Other backgrounds due to kaon interactions are given in Table II. Possible bias due to these backgrounds is discussed later.

The experiment was performed using a kaon beam derived from the external proton beam of the Argonne zero-gradient synchrotron. The 3.5-BeV/c kaon beam had $|\Delta p|/p = 0.01$. The kaons were identified by a CO_2 gas differential Cherenkov counter that detected kaons and rejected pions. A Freon-13 threshold Cherenkov counter which detected pions was used in anti-

Table I. Background contributions from competing kaon decays.

Decay mode	Rate/tau rate (%)
$K \to \pi + \pi^0 \to \pi + e^+ + e^- + \gamma$	4.2
$K \to \pi + \pi^0 + \pi^0 \to \pi + \pi^0 + e^+ + e^- + \gamma$	0.5
$K \to \pi^0 + \mu + \nu \to e^+ + e^- + \gamma + \mu + \nu$	1.0
$K \to \pi^0 + e + \nu \to e^+ + e^- + \gamma + e + \nu$	1.2