(ii) If one is willing to take the "exchange degeneracy hypothesis" [R. C. Arnold, Phys. Rev. Letters 14, 657 (1965)], then it is reasonable to expect a zero of $C_R(t)$ around $t = -0.11$ GeV^2 just like the zero of $C_\rho(t)$, which is required to explain the crossover phenomenon of $\pi^\pm p$ differential cross sections [R. J. N. Phillips and W. Rarita, Phys. Rev. 139, B1336 (1965)]. (iii) Use together of an independent sum rule for $A^{(+)\prime}(\nu,t)/\nu$ [the type of Eq. (19) in III], with Eq. (4), will give us an independent determination of $\alpha_R(t)$ and $C_R(t)$. Discussions will be given elsewhere.

[24]Within the errors of Zovko's values (Ref. 18) we can find a zero in the interval -0.36 GeV$^2 > t > -0.63$ GeV^2.

NEW TEST OF T INVARIANCE IN p-p SCATTERING*

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A test of T invariance in p-p triple scattering has been performed at 430 MeV. The difference in two polarizations which are identical if T is valid was found to be 0.0006 $\pm$ 0.0028. The T-nonconserving amplitude was found to be less than $\frac{1}{2}$% of the T-conserving amplitude. The T-nonconserving phase was found to be less than 0.06 of its maximum value.

This Letter describes a test of time-reversal invariance in strong interactions using p-p triple scattering at 430 MeV. The test is to compare the final polarization in Figs. 1(a) and 1(b) or, as seen in the laboratory, in Figs. 2(a) and 2(b). The two proton-spin and -momentum configurations shown in Fig. 1 in the p-p center of mass are related by time reversal. The Lorentz transformation to the lab frame as shown in Fig. 2 leaves the incident spin directions unchanged, rotates the final momentum forward ($\bar{\theta} \rightarrow \theta$), and changes the angle between final spin and momentum [$\chi_f \rightarrow \chi_f + \theta$ for Fig. 1(a) and $\chi_i \rightarrow \chi_i - \theta$ for Fig. 1(b)].[1]

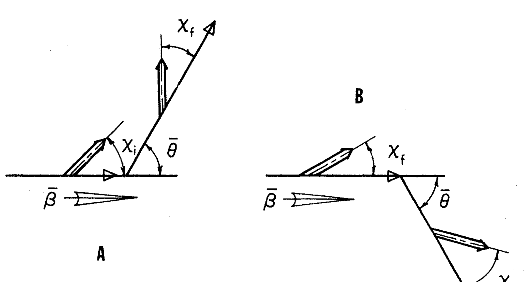

FIG. 1. Two configurations for initial- and final-state proton spin components (double arrows) and momenta for p-p scattering through an angle θ in the center of mass. The angles χ_i and χ_f are arbitrary. The velocity of the center of mass is in the direction of $\bar{\beta}$. Reversing spins and momenta in A gives B after a space rotation.

Thus for arbitrary angles χ_i and χ_f the final-state polarization in Fig. 2(a) along the direction $\chi_f + \theta$ must be equal to the final-state polarization in Fig. 2(b) along the direction $\chi_i - \theta$ if time-reversal invariance is valid. The argument can be phrased in terms of the Wolf-

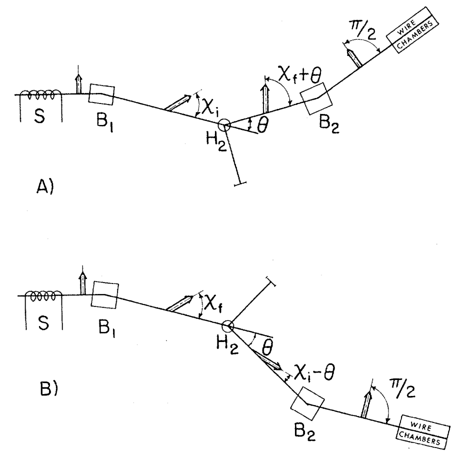

FIG. 2. Transforming the spins and momenta of Figs. 1(a) and 1(b) into the laboratory gives the spins and momenta shown in Figs. 2(a) and 2(b) respectively. The solenoid magnet S and the first bending magnet B_1 prepared the initial spin direction $\chi_i = \chi_f = 45°$. The scattering angle $\theta = 30°$ was chosen. In geometry A the magnet B_2 precessed the spin from 75° to 90° so that it could be analyzed by the wire chambers; in B a precession from 15° to 90° was required. An "up-down" asymmetry resulted from this polarization component.

enstein triple scattering parameters. A and A' are the transverse and longitudinal final polarization, respectively, for an initial beam of 100% longitudinal polarization while R and R' are the same quantities for an initial beam of 100% transverse polarization. For these definitions, "transverse" is restricted to the scattering plane. The relativistic formula relating these four parameters if T invariance is valid, derived by Sprung,[2] is

$$\tan\theta = (A+R')/(A'-R), \tag{1}$$

where θ is the laboratory scattering angle. If the initial polarization is unity in each of the two geometries of Fig. 2, the final polarizations are

$$P_A = (R\sin\chi_i + A\cos\chi_i)\sin(\chi_f+\theta) + (R'\sin\chi_i + A'\cos\chi_i)\cos(\chi_f+\theta),$$

$$P_B = (-R\sin\chi_f + A\cos\chi_f)\sin(\theta-\chi_i) + (-R\sin\chi_f + A'\cos\chi_f)\cos(\theta-\chi_i). \tag{2}$$

The difference between P_A and P_B is given by

$$P_A-P_B = [(A+R')\cos\theta-(A'-R)\sin\theta]\sin(\chi_i+\chi_f). \tag{3}$$

It vanishes if Eq. (1) is satisfied. Thus the comparison of the two configurations shown in Fig. 2 is equivalent to a test of Eq. (1).

The experimental configurations are also shown in Fig. 2. The 430-MeV proton beam had a flux of 10^8 protons/sec and a polarization[3] $P_1 = 0.535 \pm 0.025$ initially oriented normal to the plane of Fig. 2. The solenoid and first bending magnet prepared the spins such that $\chi_i = \chi_f = 45°$. This produced the maximum possible effect ($P_A - P_B$) according to Eq. (3), and allowed the same incident beam and hydrogen target configuration in the two separate measurements. The p-p scattering took place in the 0.9 g/cm² of liquid hydrogen. The proton scattering angle θ was chosen to be 30°. The final spin components of interest, at 75° to the momentum in Fig. 2(a) and at 15° to the momentum in Fig. 2(b), were precessed into directions normal to the momenta by the second bending magnet. These transverse spin components were then analyzed by a polarimeter consisting of a wire spark-chamber system and an on-line computer. The logic required a p-p scatter from hydrogen with both scattered and recoil protons detected, followed by a second scatter of the proton from carbon through an angle between 5° and 20°. The typical trigger rate was 14 events/sec. The wire spark chambers were used to measure the polar and azimuthal angles of the carbon scattering. A right-handed coordinate system $(\hat{x},\hat{y},\hat{z})$ was chosen to describe these angles, with the $\hat{z}$ axis parallel to the incident proton direction, and the $\hat{y}$ axis up, normal to the plane of Fig. 2. Six single-gap wire chambers were placed with their planes normal to the $\hat{z}$ axis. Three were in front of the 13-g/cm² carbon target, and three were behind it. From the x and y coordinates of each spark the incident and scattered proton directions were calculated by the computer. Before accepting an event the computer made several checks to avoid instrumental biases. The polar and azimuthal angles of each accepted event were stored in a matrix which divided the range $\theta = 5°$-$20°$ into ten equal bins, and divided the range $\Phi = 0$-2π into 20 equal bins. This distribution matrix integrated over θ was fitted by the form $N(\Phi) = (1+\epsilon\cos\Phi + \delta\sin\Phi)$ every 2000 accepted events. A more detailed description of this polarimeter system can be found elsewhere.[4] Table I shows the results for one week of running in the A geometry, yielding 495 000 events, and one week in the B geometry, yielding 412 000 events. In each geometry the solenoid was reversed every 50 000 events to change the sign of δ. The difference between $|\delta_A|$ for solenoid plus and $|\delta_A|$ for solenoid minus was ~0.005, a mea-

Table I. Asymmetries of the form $N(\Phi) = 1+\epsilon\cos\Phi + \delta\sin\Phi$ observed in the two geometries of Fig. 2. The solenoidal magnetic field was along the proton direction of motion when the setting was "minus." The signs of ϵ and δ are chosen such that spin up, which scatters to the left from carbon, gives a positive ϵ. The expected value of χ^2 was 18: 20 data points in Φ and two parameters. The combined numbers were obtained by replacing Φ by $\pi-\Phi$ for solenoid "plus."

Geometry	Solenoid	ϵ	δ	N	χ^2
A	plus	+0.1584	−0.1615	234 025	12.6
A	minus	+0.1634	+0.1616	261 520	14.1
B	plus	−0.1453	−0.1658	212 033	12.7
B	minus	−0.1387	+0.1561	200 046	15.9
A	combined	+0.1609	0.1616	495 545	11.3
B	combined	−0.1420	0.1610	412 079	14.4

sure of the up-down bias. The right-left asymmetry parameter ϵ was nonzero because the p-p scatter introduced a polarization component normal to its scattering plane. The difference in $|\epsilon_A|$ for left p-p scattering and $|\epsilon_B|$ for right p-p scattering was ~0.02, a measure of the right-left bias. The combined up-down asymmetries δ_A and δ_B were proportional to P_A and P_B in Eq. (3), with the same proportionality constant: $P_A - P_B = (\delta_A - \delta_B)/C$, where C is the product of the incident beam polarization and the analyzing power of the final carbon scatterer. This test therefore had the advantage that $\delta_A = \delta_B$ if $P_A = P_B$. No absolute numbers were needed for the comparison. From Table I we have $\delta_A - \delta_B = 0.1616 - 0.1610 = 0.0006 \pm 0.0028$. The error is one statistical standard deviation. The constant C was found to be $C = 0.311 \pm 0.006$, giving $P_A - P_B = 0.0019 \pm 0.009$.

The theory of T-invariance violation in p-p scattering has been discussed by Phillips,[5] by Woodruff,[6] and by Thorndike.[7] The difference $P_A - P_B$ can be expressed in terms of the complex coefficients g and T of the p-p spin scattering matrix as

$$I_0(P_A - P_B) = 8\,\mathrm{Re}\,g^*T. \qquad (4)$$

With respect to this result Woodruff's[6] paper is in error by a factor of 2. I_0 is the differential cross section, g is the coefficient of a parity and time-reversal conserving term, and T is the coefficient of the explicitly time-reversal noninvariant but parity-conserving term. Without recourse to a phase-shift analysis, the relative phase of T and g cannot be calculated, but $|T|\cos\alpha$ can be found, where α is this phase. Thus at 430 MeV and $\theta = 30°$, $8|g| \approx 3.4\ \mathrm{mb}^{1/2}$ and $I_0 = 3.6$ mb, giving

$$|T|\cos\alpha = 0.0020 \pm 0.010\ \mathrm{mb}^{1/2}.$$

For one standard deviation $|T| \lesssim \frac{1}{2}\%$ of $\sqrt{I_0}$, assuming α is not too large. With a phase-shift analysis g can be calculated explicitly, and T can be calculated in terms of time-reversal noninvariant phases λ_i. The resulting limits of the λ_i's depend sensitively on the magnitudes of the ${}^3P_2$-${}^3F_2$ and ${}^3F_4$-${}^3H_4$ phase shifts and mixing parameters. Using a set of phase shifts[8] which fits the data well at 430 MeV, we obtain $\sin\lambda_2 \lesssim 0.1$ if $\lambda_4 = 0$, and a comparable limit on λ_4 if $\lambda_2 = 0$. Since the maximum value of $\lambda_i = \frac{1}{2}\pi$, $\lambda_2 \lesssim 0.06$ of its maximum. Uncertainty in the phase shifts leads to perhaps a 50% uncertainty in these limits. The limit on λ_2 is six times larger than the limit on $|T|$ because of the small magnitudes of the phase shifts. Thorndike[7] has analyzed the polarization-asymmetry studies in p-p scattering in the energy range 150-200 MeV by Hwang et al.,[9] Abashian and Hafner,[10] and Hillman, Johansson, and Tibell,[11] and has concluded that $\lambda_2 \lesssim 0.07$. The present experiment has extended the T check to a higher energy and moreover the new measurement technique is not subject to the possible systematic errors which beset any absolute determination of a polarization.

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[1]To prove this assertion we follow V. Bargmann, L. Michel, and V. L. Telegdi [Phys. Rev. Letters 2, 435 (1959)] and regard the spin S as a spacelike four-vector which reduces to the nonrelativistic form in the rest frame of the particle. Let P, E, θ, and χ be, respectively, the momentum, energy, scattering angle, and angle between spin and momentum of a proton in the laboratory frame. Barred quantities refer to the center-of-mass frame. To find the relation between χ and $\bar{\chi}$ it is only necessary to evaluate the scalar product $MS \cdot \mathcal{P}$ of S, the spin four-vector, and $\mathcal{P}$, the total system four-momentum. If coordinate systems are oriented so that the 1-2 plane coincides with the scattering plane and the 1 axis is along the momentum of the particle, then the time and space components of $\mathcal{P}$ and MS in the center-of-mass system are $\mathcal{P} = (2\bar{E}, 0, 0, 0)$; $MS = (\bar{P}\cos\bar{\chi}, E\cos\bar{\chi}, M\sin\bar{\chi}, 0)$; and in the laboratory, $\mathcal{P} = (\Gamma 2\bar{E}, N2\bar{E}\cos\theta, -N2\bar{E}\sin\theta, 0)$, $MS = (P\cos\chi, E\cos\chi, M\sin\chi, 0)$. Γ and N are the energy and momentum of the total system scaled in terms of its mass. Therefore, $MS \cdot \mathcal{P} = 2\bar{E}\bar{P}\cos\bar{\chi}$ in the center-of-mass frame and $MS \cdot \mathcal{P} = 2\bar{E}([\Gamma P - NE\cos\theta]\cos\chi + MN\sin\theta\sin\chi) = 2\bar{E}\bar{P}\cos(\chi - \theta)$ in the laboratory. This latter expression follows from its predecessor on using the relation $N/\Gamma = P/[(E+M)\cos\theta]$, which holds for elastic scattering of equal-mass particles. Since $S \cdot \mathcal{P}$ is an invariant, the connection of χ and $\bar{\chi}$ is the remarkably simple one $\bar{\chi} = \chi - \theta$. Spin components perpendicular to the scattering plane are unaffected by the Lorentz transformation and do not alter the argument. While the results will not be so simple, the same method can be used for spin transformations in other situations.

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PARTIAL-WAVE ANALYSIS OF Σ-π FINAL STATES NEAR 1 BeV *

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The results of a partial-wave analysis of Σ-π final states near 1 BeV are presented.

In this paper we describe our analysis of the reactions $K^-p\rightarrow\Sigma^\pm\pi^\mp$ in the K^-p c.m. energy range from 1735 to 1845 MeV, and $K^-d\rightarrow\Sigma^-\pi^0p$ at 815-, 915-, 1015-, and 1115-MeV/c incident K^- momenta. In addition to the well-known Y_0*(1815) and Y_1*(1765) resonances, an important, perhaps resonant, isospin-0 D_5 amplitude has been observed in this energy region by a CERN-Heidelberg-Saclay (CHS) Collaboration in $\Sigma^\pm\pi^\mp$ and $\bar{K}^0N$ final states.[1] The results of our experiment favor the interpretation of this amplitude as a resonance with mass 1837 ± 11 MeV, and a branching fraction into $\Sigma\pi$ of 0.34 ± 0.03.

These data are from an experiment to study systematically $\bar{K}N$ interactions near 1 BeV. About 750 000 pictures were taken in the Lawrence Radiation Laboratory 25-in. hydrogen bubble chamber exposed to a separated K^- beam from the Bevatron; deuterium was the target liquid during one-third of the run. After cuts were made on the χ^2, fiducial volume, and sigma length, there remained 4190 $K^-p\rightarrow\Sigma^+\pi^-$, 2670 $K^-p\rightarrow\Sigma^-\pi^+$, and 467 $K^-d\rightarrow\Sigma^-\pi^0p$ events. An average Σ^- carried a weight of 1.2 and a typical Σ^+, a weight of 1.45 to correct for the length cut and other scanning biases. The K^- path length at each momentum was determined by a direct beam count and corrected for non-K-meson contamination as calculated from a τ and δ-ray count.

There are two well-established resonances between 1700 and 1850 MeV – the Y_0*(1815) and Y_1*(1765), respectively, having spin and parity $\frac{5}{2}^+$ and $\frac{5}{2}^-$.[2-4] In addition, there are several isobars above and below this range which might affect our data. We assumed that background amplitudes of the form

$$B_{IL}=(a_{IL}+b_{IL}P_K)\exp[i(c_{IL}+d_{IL}P_K)] \qquad (1)$$

are an adequate approximation to nonresonant backgrounds plus possible tails of resonances outside our energy region. Here I is the isospin, L the orbital angular momentum, and a_{IL}, b_{IL}, c_{IL}, and d_{IL} are constants which were varied in the fitting process. Resonant amplitudes were taken to be of the Breit-Wigner form,

$$R_L=X_eX_r/(\epsilon-i),$$

with the dependence of the various partial widths given by

$$\Gamma_i\propto[q_i^2/(q_i^2+Z^2)]^L(q_i/\omega_i),$$

with $Z=350$ MeV, X_e and X_r the elasticity and branching fraction into $\Sigma\pi$, respectively, and q_i the c.m. momenta of the resonance's decay products. For a given set of background and resonant parameters, the differential cross sections and polarizations were calculated through the expressions

$$I(\theta)=|f(\theta)|^2+|g(\theta)|^2,$$

$$\vec{P}(\theta)I(\theta)=2\,\mathrm{Re}[f^*(\theta)g(\theta)]\hat{N},$$

$$g(\theta)=k^{-1}\textstyle\sum_l[(l+1)T_l^{+}+lT_l^{-}]P_l(\cos\theta),$$

$$f(\theta)=k^{-1}\textstyle\sum_l[(T_l^{+}-T_l^{-})]P_l^1(\cos\theta),$$

and compared with the experimental distributions to yield a value of χ^2. The program VARMIT, run on a CDC 6600, then minimized χ^2 with respect to the parameters allowed to vary. In fitting, we arranged the data so that no bin in the angular distributions had less than ten real events and so that the polarization for $\cos\theta$ interval was determined from no less than 40 real events. The total number of data points was 267. Note that our procedure differed from