tons detected, after pion and target-out subtraction, but without corrections for counter efficiencies, dead-time effects, and absorption in the spectrometer. These corrections have been made for the fourth and fifth columns of Table I, however. The fifth column shows the measured cross sections averaged over photon energies between 4.0 and 5.7 BeV. While 4.0 BeV is the threshold photon energy in the case of a hydrogen target, it is much lower for heavier target nuclei. However, an excitation curve obtained by Bertram et al.[1] on nitrogen suggests that there is little coherent antiproton production. For the purpose of comparison Table I also contains some of the results of Ref. 1. The cross sections of the present experiments are somewhat larger but a direct comparison cannot be made because the data were taken at different momenta. A comparison of the experiments with a peripheral-model calculation[3] indicates that not only the magnitude of the cross section but also the momentum dependence is described inadequately by the model. The calculated cross section is two orders of magnitude larger than the experimental one. The predicted momentum dependence gives an increase of the cross section by a factor ~2 when changing the antiproton momentum from 1 to 2 BeV/c, while a decrease by a factor ~2 is observed. The data indicate an approximate $A^{2/3}$ dependence of the cross section.

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SHRINKAGE EFFECTS AND ONE-PION EXCHANGE*

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The experimental data on quasi–two-body final states in π^+p interactions have recently been studied over a fairly large π^+ momentum range.[1] In particular the energy behavior of the cross sections and of the four-momentum-transfer distribution have been investigated.

For the process

$$\pi^+p \to N^{*++}\rho^0 \tag{1}$$

it was found that (a) the width of the forward peak in the differential cross section $d\sigma/dt$ (where t is the square of the four-momentum transfer between the incoming π^+ and the outgoing ρ^0) shrinks with increasing momentum p of the incident π^+; (b) the cross section for the process (1), $\sigma_{N^{*++}\rho^0}$, seems to behave like $\sim p^{-0.5}$. According to the authors, their results disagree with the assumption that the process (1) proceeds via a one-meson exchange.

It is the purpose of this note to show that the experimental data on $d\sigma_{N^{*++}\rho^0}/dt$ and on the energy behavior of $\sigma_{N^{*++}\rho^0}$ are well described by a one-pion-exchange (OPE) model if one takes the finite widths of N^{*++} and ρ^0 into account. The study is part of a rather general investigation of OPE contributions to three- and four-body final states.[2]

The OPE model.—Figure 1 shows the OPE diagram which is supposed to dominate the process (1). The contribution of this diagram will be calculated in a form-factor approach, which is equivalent to a calculation based on the absorption model as long as one integrates over the N^{*++} and ρ^0 decay angles. The differential cross section for the diagram in Fig. 1 is given by[3]

$$\frac{d\sigma}{d|t|\,dm\,dM}=\frac{1}{4\pi^3p^{*2}s}\frac{G^2(t)}{(\mu^2-t)^2}\times m^2q_t\sigma_{\pi^+\pi^-}(m,t)M^2Q_t\sigma_{\pi^+p}(M,t), \quad (2)$$

where s = square of the c.m. rest energy; p^* = c.m. momentum in the initial state; t = square of the momentum transfer between incoming π^+ and outgoing $\pi^+\pi^-$ system (see Fig. 1); $m = \pi_a^+\pi^-$ effective mass; $M = p\pi_b^+$ effective mass; q_t, Q_t = momentum of the exchanged pion in the $\pi_a^+\pi^-$ and $p\pi_b^+$ rest frame, respectively; $\sigma_{\pi^+\pi^-}(m,t), \sigma_{\pi^+p}(M,t)$ = cross sections for the reactions $\pi^+\pi^-\to\pi^+\pi^-$ and $\pi^+p\to\pi^+p$, respectively, where one of the pions has a mass squared of t; and $G(t)$ is a so-called form factor. Dürr and Pilkuhn[4] have suggested relating the off-shell cross sections, $\sigma(m,t)$, to the corresponding on-shell cross sections, $\sigma(m)$, in the following way: Consider the case where the $\pi_a^+\pi^-$ and $p\pi_b^+$ systems come from pure ρ^0 and N_{33}^{*++} decay, respectively. Then

$$q_t\sigma_{\pi^+\pi^-}(m,t)=\left(\frac{q_t}{q}\right)^2\frac{1+R_\rho{}^2q^2}{1+R_\rho{}^2q_t{}^2}q\sigma_{\pi^+\pi^-}(m) \quad (3)$$

and

$$Q_t\sigma_{\pi^+p}(M,t)=\frac{(M+m_p)^2-t}{(M+m_p)^2-\mu^2}\left(\frac{Q_t}{Q}\right)^2\frac{1+R_{N^*}{}^2Q^2}{1+R_{N^*}{}^2Q_t{}^2}Q\sigma_{\pi^+p}(M) \quad (4)$$

with q and Q being the momentum of π_a^+ and the proton in the $\pi_a^+\pi^-$ and the $p\pi_b^+$ rest frame, respectively. m_p is the proton mass.

The values of the elastic scattering cross sections $\sigma_{\pi^+\pi^-}(m)$ and $\sigma_{\pi^+p}(M)$ are taken from experiment.[5,6] Figure 2 shows $\sigma_{\pi^+\pi^-}(m)$ and $\sigma_{\pi^+p}(M)$ for the ρ^0 and N^* region, respectively, as used in the calculation. R_ρ and R_{N^*} are parameters which in nonrelativistic scattering are defined as the radii of interaction.

It is obvious from the formulas (2)-(4) that the off-shell corrections for given values of m, M depend only on t, and not on the incoming momentum.

From a fit of the OPE model to experimental data on the processes $\bar{p}p\to\bar{N}_{33}^{*++}N_{33}^{*++}$, $pp\to N_{33}^{*++}n$, $\pi^-p\to n\rho^0$, and $\pi^+p\to N^{*++}\rho^0$, the

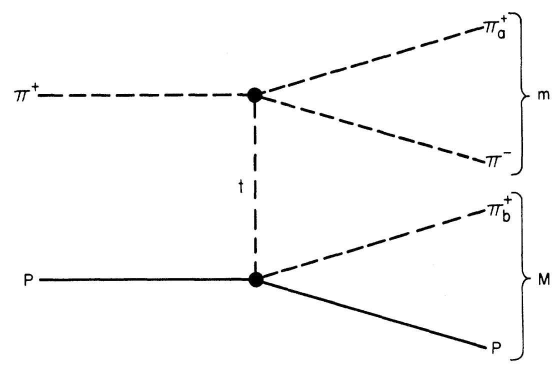

FIG. 1. OPE diagram for reaction $\pi^+p\to p\pi^+\pi^+\pi^-$.

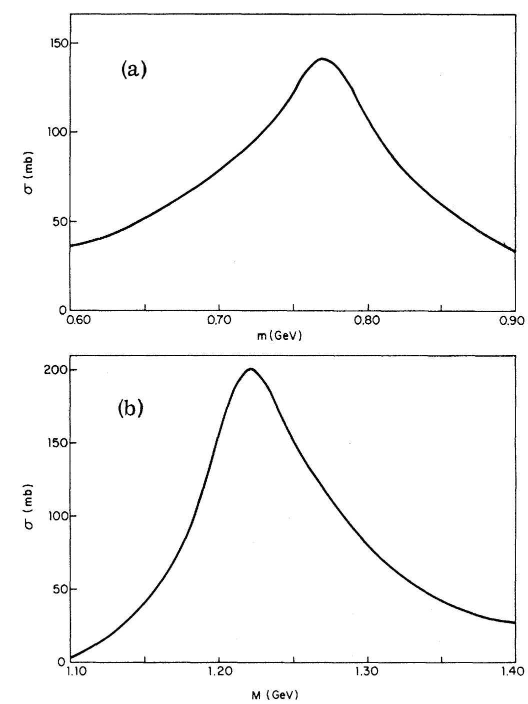

FIG. 2. (a) $\pi^+\pi^-$ elastic scattering cross section in the ρ region. (b) π^+p elastic scattering cross section in the N_{33}^* region.

following results on $G(t)$, R_ρ, and R_{N^*} have been obtained[2]:

With the parametrization $G(t)=(c-\mu^2)/(c+t)$, $c=(2.29\pm0.27)$ GeV^2, $R_\rho=(8.28\pm2.0)$ GeV^{-1} $=(1.64\pm0.40)$ F, $R_{N^*}=(3.97\pm0.11)$ GeV$^{-1}=(0.79\pm0.02)$ F.

A detailed description of the procedure adopted will follow.[2]

Comparison with experiment.—The experiment was done for the differential cross section for all events in the experimentally chosen N^*, ρ^0 mass region.[7] The OPE cross section has been integrated over exactly the same N^*, ρ^0 mass region. In Figs. 3(a)-3(d), the experimental data for the process[1] are given for incoming π^+ momenta of 2.35, 3.5, 4, and 6.95 GeV/c.[8] The result of the OPE calculation is shown by the curves in Fig. 3. The curves are in close agreement with the experimental points. Both the t and the energy dependence of the experimental data are reproduced by the OPE model. No shrinkage effects other than what are predicted by the OPE model are observed. Hence, we conclude that the experimental data on the process (1) are in agreement with the assumption that one-pion exchange is the dominant production mechanism.

The apparent shrinkage in the OPE model comes about by the following:

(1) The large N^* and ρ^0 mass intervals over which the data have been taken: The minimum value of $|t|$ changes considerably over the N^*, ρ^0 mass region and with incoming energy (see Table I).

(2) The differential cross section $d\sigma/d|t|$ does not follow an exponential law $\sim e^{Bt}$; the slope is steeper at small values of $|t|$ and less steep at larger $|t|$ values. With increasing energy, the maximum of $d\sigma/d|t|$ moves towards smaller values of $|t|$; hence the fall of $d\sigma/d|t|$ becomes steeper.

Table I. Minimum four-momentum–transfer squared, $|t|_{min}$, for $\pi^+p\rightarrow N^{*++}\rho^0$ for different ρ and N^* mass values.

Lab. momentum of incoming π^+ (GeV/c)	m_{ρ^0} (GeV)	$M_{N^{*++}}$ (GeV)	$\|t\|_{min}$ (Gev^2)
2.35	0.675	1.185	0.079
	0.825	1.285	0.225
4.0	0.66	1.12	0.027
	0.86	1.32	0.122
6.95	0.64	1.12	0.013
	0.88	1.42	0.081

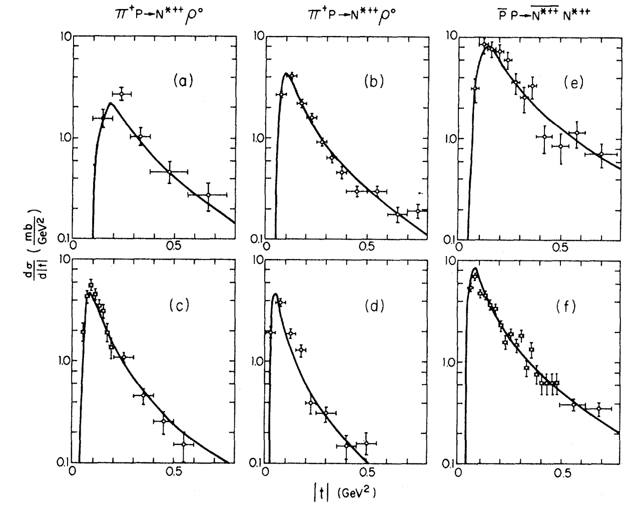

FIG. 3. Differential cross sections $d\sigma/d|t|$ for events in the N_{33}^{*++}, ρ^0 mass region. (a) At 2.35 GeV/c (0.675 GeV$<m<$0.825 GeV, 1.185 GeV$<M<$1.285 GeV). (b) At 3-4 GeV/c (0.68 GeV$<m<$0.86 GeV, 1.12 GeV $<M<$1.32 GeV). (c) At 4 GeV/c (0.66 GeV$<m<$0.86 GeV, 1.12 GeV$<M<$1.32 GeV). (d) At 6.95 GeV/c (0.64 GeV$<m<$0.88 GeV, 1.12 GeV$<M<$1.42 GeV). Differential cross section $d\sigma/d|t|$ for events in the N^{*++}, N^{*++} mass region. (e) At 3.6 GeV/c (1.13 GeV$<M_{N^*,N^*}$ $<$1.33 GeV). (f) At 5.7 GeV/c (1.15 GeV$<M_{N^*,N^*}$ $<$1.35 GeV).

We have compared the OPE model also with the process

$$\bar{p}p\rightarrow\bar{N}^{*++}N^{*++} \qquad (5)$$

which is similar to $N^{*++}\rho^0$ production in that it is likely to be dominated by OPE and it involves the production of broad resonances. The differential cross sections $d\sigma/d|t|$ (where t is now the square of the four-momentum transfer between incoming proton and outgoing isobar) measured at 3.6 and 5.7 GeV/c are shown in Figs. 3(e) and 3(f).[9] The width of the forward peak shrinks on going from 3.6 to 5.7 GeV/c. The OPE calculation as shown by the curves in Fig. 3(e) and 3(f) are in remarkable agreement with the data. Therefore, the observed shrinkage for $\bar{N}^{*++}N^{*++}$ production can be understood in the same way as outlined above for the process (1).

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$$q_t\sigma_{\pi^+\pi^-}(m,t)=\left[c_0+c_1\left(\frac{q_t}{q}\right)^2\frac{1+q^2R_\rho^{\ 2}}{1+q_t^{\ 2}R_\rho^{\ 2}}\right]q\sigma_{\pi^+\pi^-}(m) \quad (3a)$$

with $c_1=0.8+\frac{1}{2}(m-0.6)$, m in units of GeV, and $c_0=1-C_1$.

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GHOST-ELIMINATING ZEROS IN THE REGGE-POLE MODEL

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The superconvergent sum rules for helicity-flip and -nonflip amplitudes controlled by only the R trajectory are used to clarify the ghost-eliminating mechanisms.

The diffraction shrinkage at high energy for the reaction $\pi^-+p\to\pi^0+n$ has been successfully explained by the Regge-pole model based on a single ρ-meson exchange.[1] In addition, the dip phenomenon observed in the above reaction around $t\approx-0.6$ GeV^2 has also been clearly explained in the Regge-pole model with a vanishing helicity-flip amplitude at $\alpha=0$.[2,3] Shrinkage is also seen in another reaction $\pi^-+p\to\eta+n$ which is supposed to be controlled by the R (or A_2) trajectory with even signature.[4] In this case, however, there remain the following unsettled questions[5]:

(i) Experimentally, it is not yet known whether the R trajectory passes through spin zero.[4] If it does, we have a ghost problem. Then to eliminate this ghost[6] the residue function of the helicity-nonflip amplitude will have to vanish.

(ii) It is an open question whether the exchange of the R trajectory can produce helicity flip at $\alpha=0$.[3] If the trajectory simply "chooses nonsense" as suggested by Gell-Mann,[6] the helicity-flip amplitude comes out finite but nonvanishing at $\alpha=0$. On the other hand, if the trajectory "chooses sense," an additional zero occurs at $\alpha=0$ in the residue of the helicity-flip amplitude as implied by the ghost-eliminating mechanism of Chew.[7] Then the helicity-flip term will actually vanish at $\alpha=0$. Therefore, it will be of great interest for the models of ghost elimination whether the helicity-flip amplitude due to the R exchange indeed vanishes at $\alpha=0$. Phenomenologically, the discrimination between these models by Arbab, Bali, and Dash[8] has not yet been convincing since two trajectories contribute, and the theory involves a number of parameters.

The purpose of this Letter is an attempt to answer the above questions (i) and (ii) in connection with the ghost-eliminating mechanisms, applying the previous techniques[9,10] of superconvergence sum rules to the helicity-nonflip and -flip amplitudes controlled by only the R