MESON DECAY RATES IN O(4, 2) AND THE INTRODUCTION OF SU(3) *

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The relativistic O(4, 2) dynamics is applied in its simplified version to meson decays with spin parity $2^+\to 1^-+0^-$, $2^+\to 0^-+0^-$, $1^-\to 0^-+0^-$ in good agreement with experiment.

Strong decay rates of higher spin mesons have been analyzed in the SU(3) framework with an assumed spin-dependent phase space (or transmission barrier) factor, without[1,2] or with[3] an intrinsic symmetry breaking. The decay of spin-2^+ mesons has also been calculated recently in the quark-rearrangement model.[4] Within the framework of noncompact dynamical groups the question of meson decays has been considered in the $SL(2,C)$ [5] and in $SL(6,C)$ [6] formalisms. In the former case the theory leads to some unobserved selection rules and the group is not large enough to account for all observed decays, unless more unusual towers of representations are introduced. In the second case the straightforward application of the theory forbids all three-meson vertices.[6]

From the considerations of form factors and spectra, the infinite-dimensional representations of the group $O(4,2)\sim SU(2,2)$ have been recently introduced[7] to label the baryon levels with the same intrinsic quantum numbers. The essential ingredient of this latter theory is the occurrence of a mixing effect [that is also essential in the transitions between hydrogenic levels, also described[8] within the framework of O(4, 2)] that cannot occur in the $SL(2,C)$ theory. This effect is crucial in the behavior of scalar and electromagnetic form factors.

In this paper we generalize the technique that was used in the calculation of baryon decays in O(3, 1) dynamics[9] to O(4, 2) and apply it to strong meson decays. The procedure is as follows: In the rest frame the mesons are assigned to two (parity-doubling) simplest representations of O(4, 2) with definite charge-conjugation and G-parity properties. The group SU(3) of the internal quantum numbers is taken, in the rest frame, simply as a direct product[10] which, however, in an arbitrary frame introduces a definite symmetry breaking. The states are consequently labeled by

$$|\alpha;a\rangle\equiv|n,J^{PG},J_3;I,I_3,Y\rangle, \tag{1}$$

where n is the principal quantum number in O(4, 2), the other labels being the usual ones. On this Hilbert space one defines the generators of the pure Lorentz transformations $\vec{M}$ (boosters), and for electromagnetic interactions, the unique vector Γ_μ and other tensors $\Gamma_{\mu\nu}$, etc. The states of momentum p are defined by $|\alpha;a;p\rangle=e^{-i\vec{\xi}\cdot\vec{M}}|\alpha;a\rangle$ with $\tanh\xi=p/E$. In addition, the interaction can be pictured as a mixing effect

$$|\bar{\alpha};\bar{a};p\rangle=\exp(i\theta_{\alpha a}T)|\alpha;a;p\rangle, \tag{2}$$

where T is a scalar operator with respect to the rotation subgroup of O(4, 2), and hence does not mix the J and $J_3=m$ values; it mixes the n values (as in the H atom). We also take, as far as simple decay processes are concerned, the minimal symmetry breaking defined by the fact that T does not mix the internal quantum numbers a; i.e., T is an operator in O(4, 2) alone.

Without loss of generality, we can choose the boosters to be $M_i=L_{i5}$. Then there is still some freedom in the choice of T. In most of the meson decays, however, T will not be an important factor because of the small momentum transfer involved, and we shall take the mixing angle in (2) to be zero, except in the decays $2^+\to 1^-+0^-$ where for kinematical reasons the first-order mixing is taken with $T=L_{45}$, the lowest order being forbidden.

The transition probability amplitude for the decay is then given by

$$A=\langle nJm;a;p|e^{i\theta T}|n'J'm';a';q\rangle\otimes|n''J''m'';a'';r\rangle, \tag{3}$$

where the two-particle states have been defined as the direct product. If we take out the boosting operations in the rest frame of the initial particle [note that the boosters are independent of the SU(3) quantum numbers], and insert intermediate states, we obtain

$$A=\langle nJm;a|e^{i\theta T}|\bar{n}_1\bar{n}_2\bar{m}a'\rangle\otimes|\bar{\bar{n}}_1\bar{\bar{n}}_2\bar{\bar{m}}a''\rangle\langle\bar{n}_1\bar{n}_2\bar{m}|e^{-i\xi'M_3}|n'J'm'\rangle\langle\bar{\bar{n}}_1\bar{\bar{n}}_2\bar{\bar{m}}|e^{-i\xi''M_3}|n''J''m''\rangle. \tag{4}$$

Here the intermediate states have been used in the O(3)⊗O(3) diagonalization of the O(4) subgroup. The matrix elements occurring in the last equation are given by[11]

$$\langle n_1n_2m|e^{-i\xi M_3}|n'J'm'\rangle=\delta_{m'm}(-1)^{m+m'}(2J'+1)^{\frac{1}{2}}\begin{pmatrix}\frac{1}{2}(n'-1) & \frac{1}{2}(n'-1) & J'\\ \frac{1}{2}(m-n_1+n_2) & \frac{1}{2}(m+n_1-n_2) & -m'\end{pmatrix}$$
$$\times V^{\frac{1}{2}(m+1)}_{n_1+\frac{1}{2}(m+1),\,n_1'+\frac{1}{2}(m+1)}(-\xi)\,V^{\frac{1}{2}(m+1)}_{n_2+\frac{1}{2}(m+1),\,n_2'+\frac{1}{2}(m+1)}(-\xi), \tag{5}$$

where the V functions are matrix elements of suitable finite O(2, 1) transformations and occur in all transition-probability calculations:

$$V^{\frac{1}{2}(m+1)}_{n_1+\frac{1}{2}(m+1),\,n_1'+\frac{1}{2}(m+1)}(-\xi)=\theta_{n_1n_1'}(\cosh\tfrac{1}{2}\xi)^{-(n_1+n_1'+m+1)}(i\sinh\tfrac{1}{2}\xi)^{n_1-n_1'}$$
$$\times F(-n_1',-n_1'-m,1+n_1-n_1';-\sinh^2\tfrac{1}{2}\xi),$$

$$\theta_{n_1n_1'}=\frac{1}{(n_1-n_1')!}\left[\frac{n_1!(n_1+m)!}{n_1'!(n_1'+m)!}\right]^{1/2},\quad n_1>n_1'$$

(for $n_1<n_1'$, interchange n_1 and n_1'). Finally, in the O(3)⊗O(3) diagonalization the O(4, 2) states are given by

$$|n_1n_2m\rangle=[n_1!(n_2+|m|)!n_2!(n_1+|m|)!]^{-\frac{1}{2}}\times a_1^{\dagger n_2+m}a_2^{\dagger n_1}b_1^{\dagger n_1+m}b_2^{\dagger n_2}|0\rangle,\quad m>0,$$
$$\times a_1^{\dagger n_2}a_2^{\dagger n_1+m}b_1^{\dagger n_1}b_2^{\dagger n_2+m}|0\rangle,\quad m<0. \tag{6}$$

To evaluate the first matrix element in Eq. (4), one has, strictly speaking, to define precisely two-particle states, for example, by the reduction of the direct product. In the present calculation we simply represent each state by the corresponding creation and annihilation operators, take their direct products for both states, and treat these creation operators for both particles as indistinguishable. This is the simplified version referred to in the Abstract. We illustrate one case: $2^+\to0^-+0^-$.

From (6), the initial 2^+ state is given by

$$|2^+\rangle=(1/2\sqrt{6})(a_1^{\dagger 2}b_2^{\dagger 2}+4a_1^\dagger a_2^\dagger b_1^\dagger b_2^\dagger+a_2^{\dagger 2}b_2^{\dagger 2})|0\rangle$$

and from (4), three intermediate states contribute

$$A=\frac{1}{2\sqrt{6}}\langle a_1^{\dagger 2}b_2^{\dagger 2}+4a_1^\dagger a_2^\dagger b_1^\dagger b_2^\dagger+a_2^{\dagger 2}b_2^{\dagger 2}|\Big\{|0\rangle\otimes|2^+\rangle\frac{1}{\cosh^2\frac{1}{2}\xi}\frac{-3}{\sqrt{6}}\frac{\sinh^2\frac{1}{2}\xi''}{\cosh^4\frac{1}{2}\xi''}$$
$$+|2^+\rangle\otimes|0\rangle\frac{-3}{\sqrt{6}}\frac{\sinh^2\frac{1}{2}\xi''}{\cosh^4\frac{1}{2}\xi''}\cdot\frac{1}{\cosh^2\frac{1}{2}\xi}+\left|\frac{1}{\sqrt{2}}(a_1^\dagger b_2^\dagger+a_2^\dagger b_1^\dagger)\right\rangle\otimes\frac{1}{\sqrt{2}}|(a_2^\dagger b_1^\dagger+a_1^\dagger b_2^\dagger)\rangle\frac{\sqrt{2}i\sinh\frac{1}{2}\xi'}{\cosh^3\frac{1}{2}\xi'}\frac{\sqrt{2}i\sinh\frac{1}{2}\xi''}{\cosh^3\frac{1}{2}\xi''}\Big\}.$$

Summing these terms, we have

$$A(2^+\to0^-+0^-)=-\frac{3}{\sqrt{6}}\left[\frac{\sinh^2\frac{1}{2}\xi'}{\cosh^4\frac{1}{2}\xi'\cosh\frac{1}{2}\xi'}+\frac{\sinh^2\frac{1}{2}\xi''}{\cosh^2\frac{1}{2}\xi'\cosh^4\frac{1}{2}\xi''}\right]-\frac{8}{\sqrt{6}}\frac{\sinh\frac{1}{2}\xi'\sinh\frac{1}{2}\xi''}{\cosh^3\frac{1}{2}\xi'\cosh^3\frac{1}{2}\xi''}. \tag{7}$$

Table I. Comparison between calculations and experiment.

Decay mode	SU(3) with phase space[a]	Quark model and phase space[b]	O(4,2)	Experiment[c]
$2^+ \to 0^- + 0^-$				
$f \to 2\pi$	100[d]	100[d]	100[d]	110 ± 14
$f \to K\bar{K}$	2.4	0	2.45	2.3 ± 0.6
$f \to \eta\eta$	0.2	$4.2 \sin^4\alpha$	0.828	small
$f' \to \pi\pi$	1.7	0	2.95	$< 12 \pm 9$
$f' \to K\bar{K}$	31	53	15.9	$> 51 \pm 10$
$f' \to \eta\eta$	9	$67 \cos^2\alpha/\sin^2\alpha$	3.95	not seen
$A_2 \to K\bar{K}$	6[d]	0	2.93	3.06 ± 1
$A_2 \to \eta\pi$	11	$94 \sin^2\alpha$	16.1	2.3 ± 1.5
$K_V \to K\pi$	42	42	54.9	48 ± 5
$K_V \to K\eta$	1.4	$38.4 \sin^2\alpha$	1.1	1.9 ± 2.8
$1^- \to 0^- + 0^-$				
$\rho \to 2\pi$	160[d]	160[d]	160[d]	160
$\phi \to K^+K^-$	3.3	3.1	1.55	1.9 ± 0.5
$\phi \to K^0\bar{K}^0$	2.2	2.0	1.32	1.6 ± 0.4
$K^* \to K\pi$	42	59.5	38.1	49.8 ± 1.7
$2^+ \to 1^- + 0^-$				
$A_2 \to \rho\pi$	75[d]	75[d]	75[d]	75 ± 7
$K_V \to K^*\pi$	28.5	37.8	26.5	33 ± 3
$K_V \to K\rho$	8.2	11.1	1.73	8.2 ± 4
$K_V \to K\omega$	3	2.9	0.52	0.92 ± 1.5
$f' \to K^*\bar{K} + \bar{K}^*K$	18	31.8	12.4	$< 34 \pm 10$

[a]See Ref. 2.
[b]See Ref. 4.
[c]From A. Rosenfeld et al., Rev. Mod. Phys. 39, 1 (1967).
[d]Values underlined are inputs.

Finally, we have to multiply the square of the invariant amplitude (7) by the invariant phase space p/M_{initial} and multiply it by the square of the coupling constant. The spin dependence of the amplitude or transmission-barrier factor are all given by the theory. For infinite-component theories with a tower of particles of different masses, the coupling constant G cannot be just a constant but must be a matrix.[12] We see this simply by the fact that for the electromagnetic coupling, the charge is the diagonal matrix element of Γ_0 which is proportional to n, so that the "coupling constant" must be e/n in order for all excited states to have the same charge e. Similarly, we expect that the scalar coupling must be done with a matrix coupling constant. This remaining ambiguity will be determined by the actual mass spectrum and gauge principles. At this stage we have taken the following mass factors in the coupling constant from dimensional arguments that give the best agreement with the experimental numbers:

$$\Gamma = [G^2/(2J+1)](\mathrm{CG})^2(M_i^{\,2}/m_1 m_2)P_f |A|^2.$$

Here G is now dimensionless, A the invariant amplitude and (CG) the SU(3) Clebsch-Gordan coefficients. The results, together with the previous calculations and experimental numbers, are shown in Table I. Only one input is used in each class of decays. In the future, if the mixing angle θ is known, all the three classes can be compared with a single input. In fact, the purpose of using an irreducible representation of a noncompact group is eventually to relate all higher spin resonance decays with each other.

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[10]For a discussion of $O(3,1)\otimes SU(3)$ see A. O. Barut, High Energy Physics and Elementary Particles (International Atomic Energy Agency, Vienna, Austria, 1965), p. 690, and Ref. 5.

[11]The same matrix elements occur in H-atom calculations: see A. O. Barut and H. Kleinert, Phys. Rev. 160, 1149 (1967).

[12]The charge matrix has been briefly introduced in a previous work: Barut and Kleinert, Ref. 8.