[9]The importance of including potential scattering for the resistivity has been emphasized by K. Fischer, Phys. Rev. 158, 613 (1967) (see also references contained therein), and J. Kondo (to be published).

[10]It is interesting to note that the predicted temperature dependence of the resistivity, e.g., that a plot of $[1-\rho'(T)/\rho'(T=0)]^{1/2}/T$ vs $\ln T$ yields a straight line, is consistent with the following two experiments: the resistivity in Cu with 0.05% Fe reported by J. P. Frank, F. D. Manchester, and D. L. Martin [Proc. Roy. Soc. (London) A263, 494 (1961)] between 4.0 and 14°K, and the resistivity in Ir with 0.5% Fe found by M. P. Sarachik [Bull. Am. Phys. Soc. 12, 348 (1967) and Phys. Rev. (to be published)] in the temperature range 1.4-38°K. The slope of the straight line is the only free parameter; unfortunately it is determined as a function of of both ϵ_0 and δ_V, as can be seen from (21), so that ϵ_0 and δ_V are not fixed by it separately.

OBSERVATION OF SPIN WAVES IN ERBIUM

A. D. B. Woods, T. M. Holden,* and B. M. Powell
Physics Division, Atomic Energy of Canada Limited, Chalk River, Ontario, Canada
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The energy/wave vector spin-wave dispersion relation, $\hbar\omega(\vec{q})$, for the a and c directions in erbium metal in its conical magnetic phase has been measured at 4.2°K using inelastic neutron scattering. We believe these measurements are the first observations of short-wavelength spin waves in such a structure. The results indicate that the anisotropy energy is much greater than the exchange energies and that the interplanar exchange energies along the c axis are of oscillating sign as required for the stability of the structure of the spiral component. This behavior is very different from that found by Møller and Houmann[1] for ferromagnetic terbium where succeeding interplanar exchange terms had the same sign.

Below 20°K erbium has a conical magnetic structure[2] with a cone angle θ of about 28.5° and a spiral wave vector $\vec{k}_0$ of $(0, 0, 2\pi/4.1c)$, where c is the lattice spacing along the hexagonal axis.

The experiments were carried out using the Chalk River triple-axis spectrometer in its constant-$\vec{Q}$ mode of operation.[3] The crystal was oriented so that the a and c axes were in the scattering plane. Measurements were made along the c axis ($[00\zeta]$ direction) between the reciprocal lattice points (001) and (003) and along four lines parallel to the a axis ($[\zeta\zeta 0]$) passing through the reciprocal lattice positions (002), (001), $(0, 0, 2-k_0c/2\pi)$, and $(0, 0, 1+k_0c/2\pi)$.

The measured spin-wave dispersion curves are shown in Fig. 1. Since there are two atoms in each unit cell the dispersion relation consists of both an optic and an acoustic branch. In the $[00\zeta]$ direction these are continuous at the zone boundary and are shown as a single branch in a double-sized zone. The observed spin-wave energies do not go to zero[4] at $\vec{q}=\vec{k}_0$ but have a finite value for all $\vec{q}$. Furthermore the observed neutron distributions consisted of single groups instead of the three which, in principle, should be present[5,6]; in fact, if the specimen consists of different magnetic domains, six distinct branches should be observable. Both the apparent absence of a zero-frequency mode and the observation of only one neutron group result from the smallness of the cone angle θ.

For $\theta = 28.5°$, neutron groups corresponding to four of these six modes are at least an order of magnitude less intense than those corresponding to the other two, while the splitting in energy between these latter two is too small to be resolved under the present experimental conditions. Even for small θ the neutron groups associated with the $q=0$ modes increase rapidly in intensity as $\omega \to 0$ but they cannot readily be resolved from the very intense

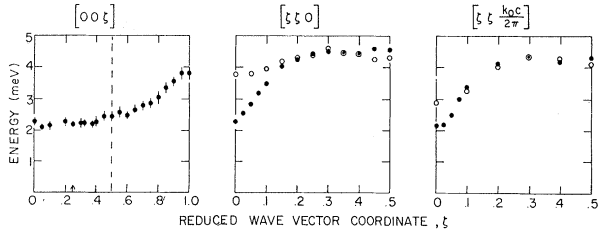

FIG. 1. The spin-wave dispersion curves of erbium measured at 4.2°K. The wave-vector coordinates are measured from the nuclear reciprocal lattice points. Error bars have been omitted for the a axis measurements but, on the average, are ~4%. Note that for the c direction the flatness of the initial half of the branch results in a negative sign for the large second Fourier component. The small arrow indicates the magnetic satellite position on the c axis.

elastic Bragg scattering at $q=0$ and hence could not be observed.

Cooper et al.[4] have derived expressions for the spin-wave energies based on the Hamiltonian given by their Eq. (2). For small values of the cone angle θ, $B_q \ll A_q$,[7] and it can be shown that their Eq. (35) reduces approximately to

$$\frac{\hbar\omega_{q'}}{2S} = J(0) - J(q') + K(0)\cos^2\theta - 2K_4 S^2 \cos^2\theta(1-2\sin^2\theta) - 3K_6 S^4 \cos^4\theta(1-3\sin^2\theta), \quad (1)$$

where $q' = k_0 - q$, and hence is measured with respect to the nuclear reciprocal lattice points, and the equilibrium condition [their Eq. (9)] has been used to eliminate $J(k_0)$. In addition it was assumed that $K(k_0-q') = K(0)$ and that the effect on the spin-wave energies of the wave-vector variation of the small term $\frac{1}{2}\sin^2\theta[J(k_0-q')-J(q')]$ was such that this term could be replaced by its value at $q'=0$. In this approximation $\hbar\omega(q') = \hbar\omega(-q')$. This simplified expression is very similar to that for a ferromagnet and is expected to be valid to within the accuracy of the experimental measurements which is ~5%. Its virtue is that the dispersion curves can readily be fitted to a simple Fourier series of the form

$$\hbar\omega_{q'}/2S = A_0 + \sum_n A_n[1-\cos(nq'/q'_{\max})]$$

with the A_n giving the interplanar exchange constants and A_0 being closely related to the anisotropy energy.

Table I shows the interplanar exchange constants obtained from Fourier analysis and Table II the derived interatomic exchange and anisotropy constants based on an isotropic exchange model extending to four nearest neighbors and neglecting the K_4 and K_6 contributions. The present experiment by itself cannot separate K_4 and K_6 from the quadratic contribution.

The exchange constants show the oscillating behavior required to make the conical structure stable although the cone angle is such that the neutron scattering is more characteristic of a ferromagnet. The derived anisotropy constant is much larger than the exchange constants and is about a factor of 2 larger than that derived from susceptibility measurements.[8] It is difficult to believe that either measurement is wrong by this amount and the discrepancy suggests that the interpretation of the measurements requires the inclusion of the higher order terms in the anisotropy energy.

Table I. Interplanar exchange constants (in meV) derived from the measured spin-wave dispersion curves using the Fourier series defined by Eq. (2) and assuming that Eq. (1) is valid.

	[00ζ]	[ζζ0] Acoustic	[ζζ0] Optic	[ζζ$k_0c/2\pi$] Acoustic	[ζζ$k_0c/2\pi$] Optic
A_0	0.148	0.161	0.251	0.143	0.193
A_1	0.043	0.053	0.018	0.047	0.041
A_2	−0.018	0.030	0.014	0.035	0.026
A_3	0.009	0.018	•••	0.018	•••
A_4	−0.007	0.008	•••	0.008	•••

Table II. Derived constants from the Fourier-series fits based on values of A_0 and the first two Fourier coefficients. These values reproduce the measured dispersion curves to an accuracy of about 10%. All units are meV.

$K(0)$ [a]	J_1 [b]	J_2 [b]	J_3	J_4
0.20	0.005	0.012	0.002	−0.0085

[a] Includes contribution from possible anisotropic exchange (see Ref. 5) and assumes $K_4 = K_6 = 0$. Actually, for this case, $K(0) - 1.1K_4S^2 - 0.73K_6S^4 = 0.20$ meV.

[b] J_1 refers to nearest neighbors in the other sublattice, J_2 to nearest neighbors in the same sublattice.

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[7]This is not true at $q=0$ where $\omega=0$ and $B_q=A_q$. Since this particular mode is not readily observable this limit does not affect the argument significantly. We are indebted to Dr. B. R. Cooper and Dr. R. J. Elliott for bringing our attention to this, and related, points.

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NONPERTURBATIVE CALCULATION OF NUCLEAR MATRIX ELEMENTS FROM TWO-NUCLEON PHASE SHIFTS*

D. S. Koltun
Department of Physics and Astronomy, The University of Rochester, Rochester, New York
(Received 25 August 1967)

A method is presented for calculating two-body shell-model interaction matrix elements from scattering phase shifts, which applies to shifts of any magnitude and to tensor coupling. It is assumed that the average (shell-model) potential, expressed in two-nucleon relative coordinates, is slowly varying within the range of the free two-nucleon interaction. Examples are given for the p shell and compared with the results of other methods, some of which include Pauli corrections.

The idea that interactions between nucleons in a nuclear system might be expressed directly in terms of free two-nucleon scattering amplitudes (or phase shifts), thus avoiding the need for explicit information on the form of the interaction, was used in the original work of Brueckner, Levinson, and Mahmoud[1] on the nuclear-matter problem, and was discussed by others.[2] The approach was more or less abandoned when it was found that the effects of the nuclear medium, on account of the Pauli principle and the binding of the nucleons, might introduce large corrections which could be calculated only by using explicitly the two-nucleon interaction.[3]

For calculating spectra of finite nuclei, it is possible that some of these difficulties are less serious. For "valence" nucleons, the Pauli principle has less effect, and some of the binding corrections are in the effective single-particle potential. Thus it might be possible to obtain interaction energies for nuclear spectra, to a good first approximation, from free-scattering phase shifts, although this approach might not be equally good for total binding energies.

In the harmonic-oscillator shell model, the interaction energy is calculated in terms of a set of matrix elements $\langle n'l'sj|t|nlsj\rangle$ of a reaction matrix or effective interaction between harmonic-oscillator states for relative motion of two nucleons. Kallio[4] has discussed an approximation which gives diagonal $(n', l'=n, l)$ reaction matrix elements in terms of the free two-nucleon phase shifts $\delta_l(E)$, evaluated for certain fixed values of the (free) relative energy, E. More recently, Elliott, Mavromatis, and Sanderson[5] have reported a somewhat different method for calculating the reaction matrix elements, which require energy averages of $\tan\delta_l(E)$. Both approaches are based on perturbation notions and are restricted to small phase shifts: Kallio's by higher order corrections in his long-range interaction v_l, and that of Elliott, Mavromatis, and Sanderson by explicit dependence on the assumption that the entire interaction is weak enough for perturbation methods.

In this paper we present another method for calculating the interaction energy for relative motion of a nucleon pair in a nucleus directly from two-nucleon phase shifts. The method is not restricted to small phase shifts and can therefore be applied to all partial waves, including tensor-coupled waves. The two-nucleon interaction is not treated as a perturbation; it is assumed to be strong and short ranged, as are the current phenomenological potentials, e.g., the Hamada-Johnston potential,[6] but its