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¹⁶We wish to mention one interesting observation which might be relevant to our consideration here. The phase-shift analysis of $d + \alpha$ scattering data by McIntyre and Haeberli (Ref. 10) shows that the phase shift varies smoothly with energy except when the excitation energy of the compound system is around 4.5 MeV. It has been mentioned by Wildermuth and McClure (footnote on p. 104 of Ref. 8) that such a behavior might indicate the presence of a resonance level with

a cluster structure other than the $d + \alpha$ structure at this excitation energy.

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¹⁸The first excited state of Li^6 is unbound, which means that the alpha and the deuteron clusters are expected to be spatially quite far apart. On the other hand, it is found from our calculation that the states with $\text{H}^3 + \text{He}^3$ cluster structure are all rather tightly bound. This indicates that there is a poor spatial overlap between the wave function describing a $\text{H}^3 + \text{He}^3$ system and that describing an $\alpha + d$ system. Thus, one expects that there would be only a small admixture of the $\text{H}^3 + \text{He}^3$ configuration in the first excited state of Li^6 . A similar argument should also hold for the second excited state of Li^6 , since here, the alpha and the d^* clusters are only very lightly bound.

ABSOLUTE SPECTROSCOPIC FACTORS FOR (d, p) REACTIONS ON HEAVY DEFORMED NUCLEI*

R. H. Siemssen† and J. R. Erskine

Argonne National Laboratory, Argonne, Illinois

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A reanalysis of a previously reported experiment on the reaction $\text{W}^{182}(d, p)\text{W}^{183}$ still shows a serious discrepancy in the absolute spectroscopic factors if we use distortion parameters that fit the elastic-scattering data. Similar deviations exist for (d, p) reactions on other rare-earth and actinide nuclei.

In a recent investigation of the reaction $\text{W}^{182}(d, p)\text{W}^{183}$ at 7.5- and 12-MeV bombarding energy¹ (hereafter referred to as I), the measured spectroscopic factors were found to be about twice the best theoretical values. (A single-particle rotational model which included the effects of Coriolis band mixing and of pairing correlations was used to calculate the theoretical spectroscopic factors.) The differences in the spectroscopic factors were observed if the experimental spectroscopic factors were extracted with distorted-wave Born approximation (DWBA) calculations with measured distortion parameters. If, on the other hand, "average" optical-model parameters were used, the agreement between the measured and the predicted spectroscopic factors was found to be good. The "average" potentials, however, do not fit the elastic scattering on tungsten, and there is therefore no *a priori* justification to use these potentials for the DWBA analysis. As was also pointed out in I, similar discrepancies in the absolute spectroscopic factors seem to exist for (d, p) reactions on the actinide nuclei, in particular, for the reaction

$\text{U}^{238}(d, p)\text{U}^{239}$ at 12-MeV bombarding energy. The same discrepancy was recently observed by Sheline *et al.*² and is also present implicitly in the study of Iano and Austern.³ The discrepancies of a factor of 2 to 3 in the spectroscopic factors are in contrast to the situation in light- and medium-weight nuclei (as well as for Coulomb stripping on lead), where the agreement with theory is usually to within 20%.⁴

In the optical-model analysis¹ of the deuteron scattering on W^{182} at 12 MeV, a serious ambiguity in the deuteron optical potentials was discovered. With this ambiguity, the imaginary potential W could be varied continuously from 5 to 30 MeV without significantly affecting the quality of the fit so long as the parameters a , a' , and V were correspondingly re-adjusted. It was also found that the scattering data could not be satisfactorily fitted with a set of average geometrical parameters ("set B" from the work of Perey and Perey⁵). These ambiguities in the deuteron potential and the failure to obtain a fit with the average geometrical parameters of Perey and Perey lead one to suspect that the "true" optical-model param-

eters had not been found in the scattering analysis. Thus, one might hope to remove the discrepancies in the absolute spectroscopic factors if the "true" potentials were used for the DWBA analysis. However, as was pointed out in I, the absolute value of the spectroscopic factors did not seem to depend on which of the deuteron optical potentials (within the ambiguities) were used for the DWBA analysis. In order to check whether the observed discrepancies result from the ambiguities in the deuteron potential and to obtain a better understanding of the effects involved, it is therefore desirable to find a less ambiguous deuteron potential—perhaps by extrapolating optical-model parameters from deuteron scattering at higher energies.

Perey and Perey⁶ obtained a good fit to the angular distribution of deuterons elastically scattered from natural tungsten at 15 MeV with the geometrical parameters of set C ($r_0 = 1.3$ F, $a = 0.79$ F; $r_0' = 1.37$ F, $a' = 0.67$ F). Following Perey and Perey, we have used this set of geometrical parameters to reanalyze our deuteron scattering data on W^{182} . A good fit was obtained from a search on V and W alone, in contrast to our previous parameter search with the geometrical parameters of set B (Fig. 1). The strength of V (77 MeV) agrees within 12% and that of W (20.4 MeV) to within 3% with the values Perey and Perey⁶ obtained from their analysis of the 15-MeV data.

Using the "C"-type deuteron potential, we then recalculated the (d, p) angular distributions with the zero-range DWBA code JULIE⁷ and have extracted the absolute spectroscopic factors from the data of I. In order to obtain reasonable agreement between the shapes of the measured and calculated angular distributions, a lower cutoff at the nuclear surface ($R = 7$ F) was required. Absolute spectroscopic factors extracted from these calculations differ by less than 10% from those previously extracted (with measured distortion parameters) in I. A similar result was also obtained with the parameters of Perey and Perey⁶ for the scattering of 15-MeV deuterons from natural tungsten. These results indicate that the observed discrepancies between the measured and the calculated absolute spectroscopic factors found in I are real and cannot be attributed to the ambiguities in the deuteron optical potential.

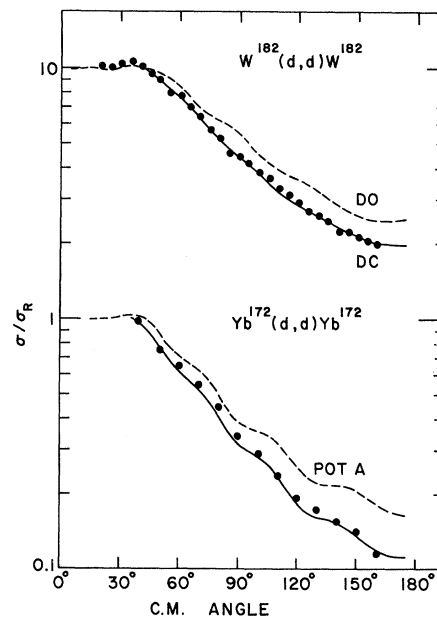


FIG. 1. Angular distributions of deuterons elastically scattered from W^{182} and Yb^{172} at 12 MeV, from Refs. 1 and 18. Potential $D0$ is Perey and Perey's "average" potential described in Ref. 1. Potential DC is the potential with geometrical parameter set C found in the present analysis. Potential $POT A$ is the deuteron potential used by Burke *et al.* (Ref. 16). The solid curve through the Yb^{172} data points is the best fit found in the present analysis ($V = 113$ MeV, $r_0 = 1.15$ F, $a = 0.878$ F, $W = 11.3$ MeV, $r_0' = 1.38$ F, $a' = 0.912$ F, and $r_c = 1.3$ F).

The final analysis in I relied on proton potentials obtained from data on the elastic scattering of protons at 12 MeV, where the angular distribution still has very little structure. Therefore, it also seemed of interest to compare the proton optical potentials from the data of I with those from an analysis of the 17-MeV data of Schrank and Pollock⁸ on the elastic scattering of protons on tungsten. In our analysis of their 17-MeV data (Fig. 2), we used the same geometrical parameters as in I ($r_0 = r_0' = 1.25$ F, $a = 0.65$ F, $a' = 0.76$ F) and searched on V and W alone. The resulting potentials have almost the same values for V and W as in I— $V = 51.9$ MeV and $W = 11.2$ MeV, as compared with $V = 52.4$ MeV (obtained by extrapolating to 17 MeV) and $W = 10.2$ MeV. This shows that the potentials from the 12-MeV proton scattering data used in I are indeed meaningful.

Our next step was to perform DWBA calculations that include finite range and nonlocal-

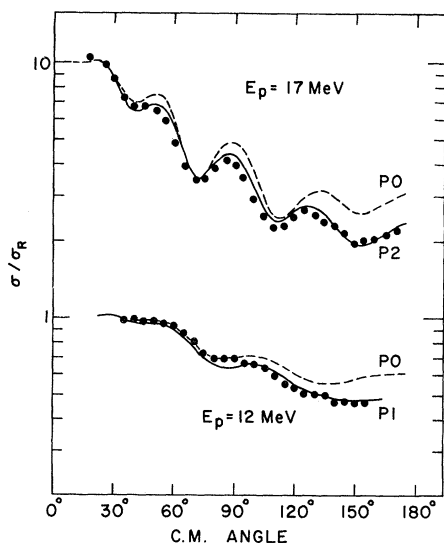


FIG. 2. Angular distributions of protons elastically scattered from tungsten at 12 and 17 MeV, from Refs. 1 and 8. Potential P_0 is Perey's average proton potential described in Ref. 1. Potential P_1 is the potential that best describes the 12-MeV scattering data (Ref. 1). Potential P_2 is the best fit found in the present analysis of the 17-MeV data.

ity corrections (which have not been included in I). The code⁹ used takes these corrections into account in the local-energy approximation.¹⁰ Nonlocality parameters $\beta = 0.85$ F for nucleons and 0.54 F for the deuteron, and a finite-range parameter $r_0 = 1.25$ F were used in the calculations. The inclusion of finite-range and nonlocality corrections increases the tail of the bound-state wave function by approximately 10 to 15%. At $E_d = 12$ MeV the calculated cross sections are increased by approximately 30%.

Spectroscopic factors derived from the DWBA calculation with and without nonlocality and finite-range corrections are listed in Table I. For the calculation of the theoretical cross sections, a deuteron normalization factor of 1.65 (which recent investigations¹¹ indicate to be more appropriate) was taken instead of the factor 1.5 used in I. Also listed in Table I are the theoretical spectroscopic factors from I. As may be seen from Table I, the discrepancies in the absolute spectroscopic factors are reduced if the finite-range and nonlocality corrections are included in the DWBA calculations. This is especially true for the spectroscopic factors from the 7.5-MeV data, which then agree perfectly with theory.

From the foregoing, it seems that improved absolute spectroscopic factors can be obtained if finite range and nonlocality effects are included in the analysis of stripping processes on heavy nuclei. However, this is in contrast to the results of Dost and Hering¹² and Smith, Dost, and Hering¹³ from Coulomb stripping on lead. Here the observed agreement between theory and experiment would be destroyed if these corrections were included in the analysis. Thus, in treating both reactions consistently, there is a discrepancy in the absolute spectroscopic factors of either the reaction $W^{182}(d,p)W^{183}$ or the reaction $Pb^{208}(d,p)Pb^{209}$, which is presently not understood.

A further explanation for the observed discrepancy was sought in an inconsistent use of bound-state wave functions in the calculations. The theoretical spectroscopic factors had been calculated with the assumption of a Nilsson-

Table I. Theoretical and measured spectroscopic factors for the reaction $W^{182}(d,p)W^{183}$.

E_x (keV)	K	I	Spectroscopic factors ^a				$S_{\text{Theor.}}/S_{\text{LEA}} (12 \text{ MeV})$
			Theoretical	Local-energy approximation ^b		Zero range	
				7.5 MeV	12 MeV	12 MeV	
46	$\frac{1}{2}$	$\frac{3}{2}$	0.117	0.12	0.17	0.22	1.42
99	$\frac{1}{2}$	$\frac{5}{2}$	0.168	0.16	0.26	0.34	1.53
209	$\frac{3}{2}$	$\frac{3}{2}$	0.093		0.10	0.13	1.12
292	$\frac{3}{2}$	$\frac{5}{2}$	0.100		0.15	0.20	1.50
412	$\frac{3}{2}$	$\frac{7}{2}$	0.040		0.07	0.09	1.70
453	$\frac{7}{2}$	$\frac{7}{2}$	0.209		0.17	0.23	0.83

^aSpectroscopic factors were extracted from the data of Ref. 1 with "C"-type deuteron potentials and the P_1 proton potentials of Ref. 1. A deuteron normalization factor of 1.65 was used throughout. A cut-off radius of 7 F was used in the zero-range calculations.

^bAbbreviated by LEA.

type deformed wave function. In the Nilsson model¹⁴ the deformed wave function is expanded over a set of spherical harmonic-oscillator wave functions, whereas the DWBA calculation computes the bound-state wave function from a spherical Woods-Saxon well. This inconsistency in the calculation might well be responsible for our observed discrepancy. This possibility was investigated by calculating deformed wave functions with the computer code of Rost,¹⁵ which solves the Schrödinger equation in a deformed Woods-Saxon potential. The radial wave functions obtained were then used in the DWBA code JULIE. The Woods-Saxon well parameters were varied to find the best over-all fit simultaneously to (1) the observed relative differential cross sections of various states within rotational bands associated with the single-particle states (i.e., the signatures of the single-particle states), (2) the known energy ordering of single-particle states in the rare-earth region, and (3) the known binding energy of the states. The best-fit parameters are $a = 0.65$ F, $r_0 = 1.30$ F, and $\lambda = 37.5$. (This set also works well for actinide nuclei.) There are some differences between the signatures calculated with Nilsson and deformed Woods-Saxon wave functions; but when the above parameter set or any set close to it is used, the calculated absolute differential cross sections are not consistently larger as would be necessary to resolve the discrepancy.

To the best of our knowledge, no study as complete as the present investigation has been made to extract absolute spectroscopic factors from (d, p) reactions on rare-earth nuclei. Several authors (Burke *et al.*¹⁶ and also Vergnes and Sheline¹⁷) report generally good agreement between predicted and measured absolute spectroscopic factors. However, neither of these two studies used measured optical-model parameters nor took account of finite-range and nonlocality effects. In fact, as seen in Fig. 1, the deuteron parameters (POT A) used by Burke *et al.* do not fit the elastic-scattering data. The deviations between the data and the calculated angular distributions are the same as were found for tungsten when the average potential $D0$ is used to calculate the $W^{182}(d, d)W^{182}$ angular distribution (Fig. 1). We have analyzed the 12-MeV Yb(d, d) data of Burke *et al.*¹⁸ and have used the resulting best-fit parameters in a DWBA analysis of the Yb(d, p)Yb data. The spectroscopic factors extracted from the

(d, p) data of Burke *et al.* then show the same discrepancy as was found in the reaction $W^{182}(d, p)W^{183}$, especially if a more realistic proton potential (similar to $P1$) with $a' = 0.76$ F is used in the analysis.

The fact that the measured spectroscopic factors are consistently larger than the calculated spectroscopic factors suggests that the discrepancy arises in the DWBA analysis of the reaction mechanism. Any final-state interactions not included in the model calculations would tend to reduce (never to increase) the spectroscopic factors. It should be pointed out that from the investigations^{1,16} of stripping transitions to the ground state of $[510]_{\frac{1}{2}}$ rotational band, it is extremely unlikely that two-step processes (inelastic-scattering effects)^{3,19,20} would account for our observed discrepancy.

The present results may indicate¹ that average potentials rather than best-fit potentials should be used for the (d, p) analysis. The average potentials can be looked at as the (spherical) potentials that would give the best fits to elastic scattering in the absence of strong coupling to the inelastic channels, whereas the measured potentials include the effects of strong coupling on the elastic scattering. These results are in contrast to the DWBA analysis of inelastic scattering, which is best reproduced with measured distortion parameters.²¹

We are indebted to Dr. Dickens and Dr. Perey for making their program for the calculation of the finite-range and nonlocality corrections available to us, and to Dr. Rost for his program for calculations with the deformed Woods-Saxon potential. We also wish to thank Dr. Zeidman for allowing us to use his unpublished data on the elastic scattering of deuterons on Yb¹⁷².

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†Present address: Yale University, New Haven, Connecticut.

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SEARCH FOR $2p-1h$ CONFIGURATIONS IN MIRROR NUCLEI ^{41}Ca AND ^{41}Sc

P. Beuzit, Y. Cassagnou, N. Cindro,* E. Cotton, J. Delaunay, J. P. Fouan, and L. Papineau
Centre d'Etudes Nucléaires de Saclay, Saclay, France

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Recent studies have shown the validity of the shell-model description for nuclear states at very high energies of excitation. This fact has contributed to justify the extension of the model to nuclear-reaction mechanisms by describing the first stages of a reaction in terms of simple shell-model configurations.¹

In a recent Letter Bolsterli et al.² have suggested a method to select $2p-1h$ states in a specific residual nucleus by relating them to their isospin analogs observed as compound states. Applied to mirror nuclei ^{41}Ca and ^{41}Sc , the method consisted in (i) selecting $2p-1h$ candidates in the ^{41}Ca nucleus [by, e.g., reactions $^{39}\text{K}(^3\text{He}, p)$ or $^{40}\text{Ca}(d, p)$] and (ii) exciting the analogs of these states in the ^{41}Sc nucleus by proton inelastic scattering on ^{40}Ca . If a state in ^{41}Ca has a $2p-1h$ character, its analog in ^{41}Sc should have a relatively large proton width for decaying to a $1p-1h$ state in ^{40}Ca . The 3^- state at 3.74-MeV excitation in ^{40}Ca is a $1p-1h$ state and can be used to test these predictions.

In a search for $2p-1h$ states in ^{41}Ca , Bolsterli et al.² eliminated from the $^{40}\text{Ca}(d, p)^{41}\text{Ca}$ spectrum³ all the levels that exhibited a single-particle stripping pattern. The averaged spectrum of the remaining levels showed a structure which they interpreted as due to $2p-1h$ states in ^{41}Ca . On the other hand, Belote et al.⁴ and Seth et al.⁵ interpreted peaks observed in the

reaction $^{39}\text{K}(^3\text{He}, p)^{41}\text{Ca}$ as due to the excitation of states of $2p-1h$ character.

In order to identify in ^{41}Sc the analogs of these $2p-1h$ candidates in ^{41}Ca , we measured (p, p) and (p, p') excitation functions on ^{40}Ca . The measurements were performed at 90° , 125° , 141° , and 160° , in 5-keV steps, with the Saclay tandem accelerator. The over-all resolution was about 5 keV. Inelastic scattering was measured to the first 0^+ , 3^- , and 2^+ states in ^{40}Ca .

The results of the measurement at 160° are shown in Fig. 1 and Table I. As explained be-

Table I. Energies of proton peaks [$E_x(^{41}\text{Ca})$] from the reaction⁵ $^{39}\text{K}(^3\text{He}, p)$ compared with resonance energies $E_x(^{41}\text{Sc})$ of the compound system ^{41}Sc obtained by $p + ^{40}\text{Ca}$. The third column lists the corresponding proton bombarding energies (laboratory system).

$E_x(^{41}\text{Ca})$	$E_x(^{41}\text{Sc})$	E_p
	7.530	6.600
7.58	7.632	6.715
7.68	7.697	6.775
7.76		
7.86	7.860	6.935
7.96	7.960	7.045
8.05	8.047	7.130
8.16	8.154	7.245
	8.192	7.285