$\gtrsim 5$. It would not be possible to attribute deviations at these higher current levels to sample inhomogeneities in view of the even greater deviations shown in the more homogeneous sample of Fig. 1. Also we find that our rolled and unannealed foils with much higher critical currents and inhomogeneity levels show no greater deviations from the Cladis relation, provided the curves are taken to sufficiently high ratios of I/I_c.

Finally we point out that any ideal flux-flow characteristic showing an exactly linear current-voltage characteristic about I_c will show the same kind of deviations from the logarithmic relationship which Cladis attributes to inhomogeneities.

We conclude that the Cladis relation has limited validity. Specifically, it does not hold over an extended range of the linear portion of the flux-flow characteristic, regardless of sample inhomogeneity, and therefore this portion cannot justifiably be called an optical illusion. The present ideas about viscous motion of fluxoids remain unchallenged.

Mr. G. E. Kuhl obtained the data on the lead-bismuth alloys and Mr. C. J. Axt plotted the data of French, Lowell, and Mendelssohn.

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SUPERHEATING IN CYLINDERS OF PURE SUPERCONDUCTING TIN

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Magnetization measurements were carried out on cylindrical samples of pure tin with diameters of 7, 10, and 122 μm and very clean and smooth surfaces in the temperature range from 3.62 to 3.72°K. The magnetization was found to be reversible and linear with the applied field $H_0 < H_{sh}$. Considerable superheating up to fields $H_{sh} = 2.25 H_{cb}$ has been observed.

The theory of Ginsburg and Landau predicts superheated, i.e., metastable superconducting, states in magnetic fields above the thermodynamic critical field H_{cb}. Ginsburg[1] has calculated the maximum superheating field H_{sh}/H_{cb} as a function of the parameter κ for the superconducting half-space. A recent calculation for the same case by Matricon and Saint-James[2] gives the same curve as obtained by Ginsburg.

For a long time experiments showed superheating fields much lower than those given by theory using appropriate κ values estimated from other experiments.[3,4] Measurements on small spheres of indium[5] and tin[6] gave considerable superheating fields. In general, however, the application of the theoretical results for the half-space to experiments with small spheres seems to be uncertain unless $r\kappa/\delta_0 \gg 1$ (r = radius of the spheres, δ_0 = penetration depth).[1] An additional difficulty arises from the unknown actual field at the single spheres, which may differ from the applied field H_0 because of the susceptibility of the sample.

In our experiment we measured the magnetization of three cylindrical samples of pure tin (99.9999 %) with diameters of 7, 19, and 122 μm and equal lengths of 25 mm. The samples were extruded from glass capillaries at a temperature a few degrees below the melting point in helium atmosphere. The surfaces of the samples obtained by this procedure were found to be optically smooth. There were no defects which could be resolved by light microscope. Moreover, an x-ray investigation revealed the samples to be single crystals. Care was taken to avoid any harm to the surfaces, as well as oxidation and plastic deformation of the samples during the unavoidable manipulation.

The magnetization measurements were carried out by means of a superconducting circuit consisting of two coils made of 0.05-mm Nb wire. The sample is surrounded by one of the coils, whereas the second coil is wound around the sensitive cell of a He magnetometer.[7] If the two coils are connected to a closed superconducting circuit,[8] every change of the magnetization of the sample gives a change of the magnetic field at the sensitive cell of the magnetometer. The sensitivity of the whole device for magnetization measurements is about 2×10^{-6} G cm^2. Starting from zero field, the magnetization at constant temperature of all samples was found to be reversible and strictly proportional to the applied field within the limits of experimental error for $H_0 < H_{\rm sh}$. The superheating field is indicated by a sudden irreversible jump of magnetization due to the transition to the normal state. The values for $H_{\rm sh}$ found for the three samples are plotted against the temperature in Fig. 1 (upper curve). The lower points were obtained in decreasing fields from a jump of magnetization due to the transition to the superconducting state. This lower curve agrees well with the $H_{cb}(T)$ curve for tin given by Mapother.[9] Supercooling was not observed. This is easy to understand because in our special arrangement the field coil was shorter than the samples in order to keep the field at the ends of the samples sufficiently low, thereby preventing the destruction of the superheated state at the ends.

The curves for $H_{\rm sh}$ and H_{cb} are nearly the same for all three samples and show a linear dependence on temperature. From their slopes the ratio $H_{\rm sh}/H_{cb}$ can be deduced. Individual values for each sample are given in the following table:

Sample No.	Diameter (μm)	$H_{\rm sh}/H_{cb}$
I	7	2.07
II	19	2.13
III	122	2.25

The fact that, in spite of the very different diameters, all the samples give equal values for $H_{\rm sh}/H_{cb}$ leads to the assumption that $H_{\rm sh}/H_{cb}$ is close to the theoretical limit. It is improbable that in three different samples with different diameters the metastable limit is reduced to the same degree by surface defects or other sources for nonideal behavior.

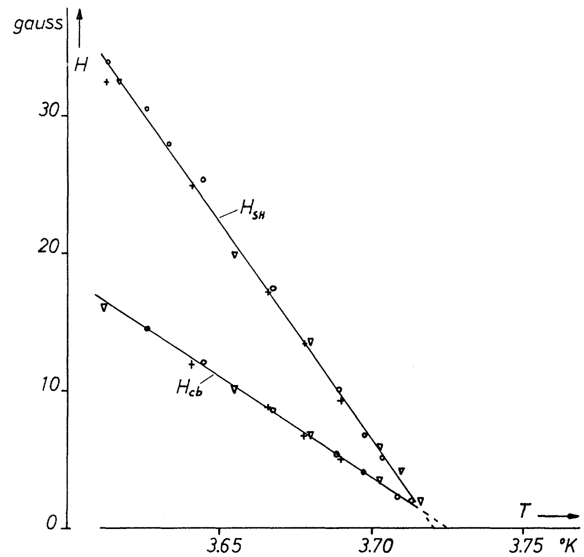

FIG. 1. Superheating field (upper curve) and critical field (lower curve) for tin. Plus signs, Sample No. I, 7 μm diam; inverted open trianges, Sample No. II, 19 μm diam; open circles, Sample No. III, 122 μm diam.

A calculation carried out for a cylinder with radius $r_a = 15\delta_0$ and $\kappa = 0.5$ [10] gives $H_{\rm sh}/H_{cb}$ in good agreement with the results for the half-space.[1,2] Since the radii of our samples are still larger (smallest radius $r_a = 60\delta_0$), comparison of the experimental results with those for the half-space is justified. Using the highest experimental value $H_{\rm sh}/H_{cb} = 2.25$ and the theoretical curves,[1,2] we find $\kappa = 0.165$ which is an upper limit for the parameter κ of tin near T_c.

An estimate of the penetration depth δ_0 at $T/T_c = t = 0.98$ with $\kappa = 0.165$, $H_{cb} = 10.95$ G, and the relation

$$\kappa = \pi 8^{1/2} \delta_0^{\,2} H_{cb}/\varphi_0 \quad (\varphi_0 = 2.06 \times 10^{-7} \text{ G cm}^2),$$

gives $\delta_0 = 1.87 \times 10^{-5}$ cm. This value reduces to $\delta_{00} = 5.2 \times 10^{-6}$ cm, the penetration depth for $t = 0$.

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EXCITATION SPECTRUM OF A LINEAR CHAIN OF PARAMAGNETIC ATOMS WITH SPIN-PHONON INTERACTION

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The low-lying energy levels of a paramagnetic chain in the presence of spin-phonon interaction have been investigated. It is shown that there is no gap in the one-particle excitation spectrum.

The aim of this Letter is to give a first account of the eigenvalue spectrum of a physical model which in recent years has been widely used in connection with research on the spin-phonon interaction.

The model, introduced first by Jacobsen and Stevens (JS),[1] consists of a linear harmonic chain (along the x axis) of N equal paramagnetic atoms, each having a spin $S=\frac{1}{2}$, in the presence of a magnetic field $\vec{H}$ directed along the z axis. Each spin is assumed to interact with the phonon field only, via terms which are linear in the strain.

With the usual notation,[2] the JS Hamiltonian in terms of the normal modes of the chain can be written as

$$\mathcal{H}=\mathcal{H}_0+V_1+V_2,$$

where

$$\mathcal{H}_0=\sum_k \hbar\omega_k \alpha_k^\dagger \alpha_k+\hbar\omega_0\sum_r S_{zr};$$

$$V_1=\sum_{r,k}\epsilon_k(\alpha_k S_{r+}e^{ikr}+\alpha_k^\dagger S_{r-}e^{-ikr});$$

$$V_2=\sum_{r,k}\epsilon_k(\alpha_k S_{r-}e^{ikr}+\alpha_k^\dagger S_{r+}e^{-ikr}).$$

V_1 contains terms which in Ref. 2 have been called "dangerous," and which require a careful treatment, while V_2 has matrix elements only between subspaces of the Hilbert space of the system which are characterized by different numbers of excitations.

Accordingly, we have sought exact solutions of $\mathcal{H}_0+V_1$ and treated V_2 as a perturbation. So far, this has been done for the ground state of the system, and for the subspace containing one excitation only.

The structure of the ground state is obviously not changed by V_1, while the normalized eigenstates of $\mathcal{H}_0+V_1$ in the one-excitation subspace can be written as

$$|k\pm\rangle=(f_{k\pm}\alpha_k^\dagger+g_{k\pm}S_{k+})|0\rangle,$$

each corresponding to the energy

$$\lambda_{k\pm}=\tfrac{1}{2}(\hbar\omega_k+\hbar\omega_0)\pm\{N\epsilon_k^2+[\tfrac{1}{2}(\hbar\omega_k-\hbar\omega_0)]^2\}^{1/2}.$$

In the above expressions

$$S_{k+}=N^{-\frac{1}{2}}\sum_r e^{ikr}S_{r+};$$

$$f_{k\pm}=\frac{\hbar\omega_0-\lambda_{k\pm}+N^{1/2}\epsilon_k}{\{(\hbar\omega_0-\lambda_{k\pm}+N^{1/2}\epsilon_k)^2+(\hbar\omega_k-\lambda_{k\pm}+N^{1/2}\epsilon_k)^2\}^{1/2}};$$

$$g_{k\pm}=\frac{\hbar\omega_k-\lambda_{k\pm}+N^{1/2}\epsilon_k}{\{(\hbar\omega_0-\lambda_{k\pm}+N^{1/2}\epsilon_k)^2+(\hbar\omega_k-\lambda_{k\pm}+N^{1/2}\epsilon_k)^2\}^{1/2}}.$$

One consequence of this is that, as long as ϵ_k vanishes at the boundaries of the Brillouin zone, no gap can exist in the one-excitation spectrum of the system in the absence of any spin-spin relaxation mechanism. This remains valid, at least up to terms of the second order in the coupling constant, when the interaction