LIMITS ON $|\Delta T|=\frac{3}{2},\frac{5}{2}$ AMPLITUDES AND CP NONINVARIANCE IN $K_{\pi 3}$ DECAY

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An important matter concerning processes involving CP-noninvariant amplitudes is the question of the isospin properties of these amplitudes. For example, from the CP-noninvariant decays $K_2^0\to 2\pi$, one learns that[1,2] $\mathrm{Im}(A_2/A_0) \lesssim 3\times10^{-3}$. The central question of whether the CP-noninvariant phase of the $\Delta T>\frac{1}{2}$ amplitude A_2 is very small is the question of whether $\mathrm{Re}(A_2/A_0)$ is in fact about an order of magnitude larger than the above number. This would be inferred from the experimental ratio of the amplitude for $K^+\to 2\pi$ to that for $K_1^0\to 2\pi$ if one assumes that $\mathrm{Re}(A_2/A_0)=(\frac{2}{3})^{1/2}|A_2^+|/|A_0|\approx 4.5\times10^{-2}$. This assumption must be tested experimentally in the $K_1^0\to 2\pi$ branching ratio,[1,3,4] and should it be found to fail, one would have evidence that both $|\Delta T|=\frac{3}{2}$ and $\frac{5}{2}$ occur in $K_{\pi 2}$ decays and the CP-noninvariant parts of these amplitudes need not be very small.[5]

In this note we examine the two questions, which are also related, of violations of the nonleptonic $|\Delta T|=\frac{1}{2}$ rule and CP noninvariance in the $K_{\pi 3}$ decays. CP noninvariance has not been established experimentally in these decays. However, we have been able to sharpen significantly the phenomenological analysis of these decays by using the results of two recent measurements: (1) the measurement of the difference in partial rates for τ^+ and τ^- to an accuracy of 0.2%,[6] and (2) the measurement of the K_2^0 total rate with a factor of ~3.5 improvement in the uncertainty.[7] To the extent that one considers significant a three-standard-deviation effect in a "consensus analysis" of the experimental data on K decays, we are able to draw the conclusion that one or both of the following statements hold: (1) CP noninvariance occurs in the $K_{\pi 3}$ amplitude with $|\Delta T|=\frac{1}{2}$. (2) There is a $K_{\pi 3}$ amplitude with $|\Delta T|=\frac{3}{2}$ in magnitude of the order of 4-20% of the $|\Delta T|=\frac{1}{2}$ amplitude, and this amplitude may have a CP-noninvariant part. The above statements are independent of the possible presence of a small $K_{\pi 3}$ amplitude with $|\Delta T|=\frac{5}{2}$, which the analysis severely limits in magnitude.

The assumptions utilized in writing expressions for the partial rates are the following: (a) neglect of $|\Delta T|=\frac{7}{2}$ in $K_{\pi 3}$, and (b) writing of the K_2^0 state as $|K_2^0\rangle=(2)^{-1/2}(|K^0\rangle-|\bar{K}^0\rangle)+O(\epsilon)$, where[2] $|\epsilon|<3\times10^{-3}$. Then we have

$$R_{+00}=|-A_1^{+}+2A_3^{+}|^2\varphi_{+00}, \tag{1a}$$

$$R_{++-}=4|A_1^{+}+\tfrac{1}{2}A_3^{+}|^2\varphi_{++-}, \tag{1b}$$

$$R_{--+}=4|e^{2i\delta_1}(A_1^{+})^*+\tfrac{1}{2}e^{2i\delta_3}(A_3^{+})^*|^2\varphi_{--+}, \tag{1c}$$

$$R_{+-0}=2|A_1^0+\tfrac{3}{2}A_3^0|^2\varphi_{+-0}, \tag{1d}$$

$$R_{000}=3|-A_1^0+A_3^0|^2\varphi_{000}. \tag{1e}$$

In these expressions, the subscripts on the R and φ represent the charge state of the three pions. The φ represent the respective phase-space integrals.[8] The explicit expressions for the K^+ and K_2^0 amplitudes in terms of the reduced matrix elements[9] and the CP-nonconserving and average strong-interaction phases are

$$A_1^{+}=\{|\lambda_{1/2}|e^{i\Delta_{1/2}}-\tfrac{1}{2}|\lambda_{3/2}|e^{i\Delta_{3/2}}\}\times e^{i\delta_1}=|A_1^{+}|e^{i\Delta_1^{+}}e^{i\delta_1}, \tag{2a}$$

$$A_1^0=\{|\lambda_{1/2}|\cos\Delta_{1/2}+|\lambda_{3/2}|\cos\Delta_{3/2}\}e^{i\delta_1}, \tag{2b}$$

$$A_3^{+}=(\tfrac{2}{3})^{1/2}|\eta_{5/2}|e^{i\Delta_{5/2}}e^{i\delta_3}=|A_3^{+}|e^{i\Delta_3^{+}}e^{i\delta_3}, \tag{2c}$$

$$A_3^0=(\tfrac{2}{3})^{1/2}|\eta_{5/2}|e^{i\delta_3}\cos\Delta_{5/2}=|A_3^{+}|e^{i\delta_3}\cos\Delta_3^{+}. \tag{2d}$$

In Eqs. (2a)-(2d) the constant reduced matrix elements with $|\Delta T|=\frac{1}{2}$ and $\frac{3}{2}$ for decay into the totally symmetric $T=1$ state of three pions characterized by the average strong-interaction phase shift δ_1 are denoted by $\lambda_{1/2}$ and $\lambda_{3/2}$ with CP nonconserving phases $\Delta_{1/2}$ and $\Delta_{3/2}$; the constant reduced matrix element with $|\Delta T|=\frac{5}{2}$ into the totally symmetric $T=3$ state characterized by the average strong-interaction phase shift δ_3 is denoted by $\eta_{5/2}$ with CP nonconserving phase $\Delta_{5/2}$.

We seek combinations of experimental data which isolate terms linear in $|A_3^{+}|$. If we neglect terms quadratic in the ratio $|A_3^+|/|A_1^+|$, we have, with $\Gamma = R/\varphi$,

$$\sin(\delta_1-\delta_3)\sin(\Delta_1^+-\Delta_3^+)|A_3^+|/|A_1^+| = -\tfrac{1}{2}[\Gamma_{++-}/\Gamma_{--+}-1], \quad (3)$$

$$\cos(\delta_1-\delta_3)\cos(\Delta_1^+-\Delta_3^+)|A_3^+|/|A_1^+| = \tfrac{1}{5}[\Gamma_{++-}/4\Gamma_{+00}-1]-\tfrac{1}{2}[\Gamma_{++-}/\Gamma_{--+}-1]. \quad (4)$$

In addition, the following relation is independent[10] of terms linear in $|A_3^+|$:

$$|A_1^0|^2/|A_1^+|^2 = (\Gamma_{+-0}+\Gamma_{000})/(\Gamma_{++-}+\Gamma_{+00}). \quad (5)$$

As an alternative to this last equation, we can write

$$|A_1^0|^2/|A_1^+|^2 = \Gamma_{+-0}/2\Gamma_{+00}. \quad (6)$$

This expression has the advantage that the energy dependence of the two processes is the same if only the totally symmetric $T=1$ state is produced at the weak vertex. It does, however, require that we neglect terms linear in $|A_3^+|$. Since the experimental results for the two branching ratios in Eqs. (5) and (6) are almost identical and highly correlated, we choose to work with Eq. (5) only.

The various partial rates were determined as part of a least-squares analysis[11] of the various K-meson decay rates. The least-squares fitting techniques employed were similar to those of Trilling.[12] We used the data quoted by Trilling, updated wherever possible, and all relevant results revealed in a literature search including those of Refs. 6 and 7. The fit was heavily overdetermined and was found to be insensitive to the presence or absence of individual pieces of data. Chi-squared tests showed that the results were acceptable, i.e., $P(\chi^2)=8\,\%$ for K^+ and $P(\chi^2)=75\,\%$ for K_2^0. The procedure yielded a "best-fit" set of partial rates for K decay and a variance matrix. All quantities dealt with in the present paper were derived from these fitted rates. Full use was made of all appropriate terms in the variance matrix in calculating the uncertainties quoted.

The terms on the right-hand side of Eqs. (3)-(6) have been measured directly[6] or can be calculated by means of the least-squares analysis. We have

$$\Gamma_{++-}/\Gamma_{--+}-1 = 0.000\pm 0.002, \quad (7)$$

$$\Gamma_{++-}/4\Gamma_{+00}-1 = 0.002\pm 0.027, \quad (8)$$

$$(\Gamma_{+-0}+\Gamma_{000})/(\Gamma_{++-}+\Gamma_{+00}) = 0.849\pm 0.043, \quad (9)$$

$$\Gamma_{+-0}/2\Gamma_{+00} = 0.806\pm 0.039, \quad (10)$$

as the presently available data.[13]

The results of substituting Eqs. (7) and (8) into Eqs. (3) and (4) are presented in Fig. 1.[14] We have presented these results for $\delta_1-\delta_3$ equal to both 10° and 30°.[15,16] We feel that these choices of average strong-interaction phases bound the range of reasonable values, representing as they do the assumptions (1) $\delta_3 \approx 0$[15] and (2) either a sizable S-wave $T=0$ π-π scattering length[15] ($\delta_1 \approx 30°$) or the much smaller scattering length desired by Weinberg ($\delta_1 \approx 10°$).[16] Combinations of $|A_3^+/A_1|$ and $\Delta_1^+-\Delta_3^+$ which are consistent with the experimental data to within one standard deviation lie between the base line of the graph and the curves drawn. Subject to the above assumptions, we conclude that $|A_3^+/A_1^+|$ is less than 1 % and we neglect it in the remaining analysis.

If CP is conserved in $|\Delta T| = \frac{1}{2}$ transitions, and if the $|\Delta T| = \frac{1}{2}$ rule is exactly true, then the value of the ratio presented in Eq. (9) should be identically equal to 1.0. The result indicates a deviation from this value by approximately three standard deviations. A wide range of interpretations is possible. We present three cases:

Case I: CP is not conserved in $|\Delta T| = \frac{1}{2}$ transitions, $|\Delta T| = \frac{1}{2}$ rule is true.—In this case $\lambda_{3/2} = 0$ and $\cos\Delta_{1/2} = 0.935 \pm 0.022$, i.e., $\Delta_{1/2} = 21° \pm 4°$.

Case II: CP is conserved in $|\Delta T| = \frac{1}{2}$ transitions, $|\Delta T| = \frac{1}{2}$ rule is violated.—Here we have a relationship which limits the allowable combinations of the ratio $|\lambda_{3/2}/\lambda_{1/2}|$ and the phase

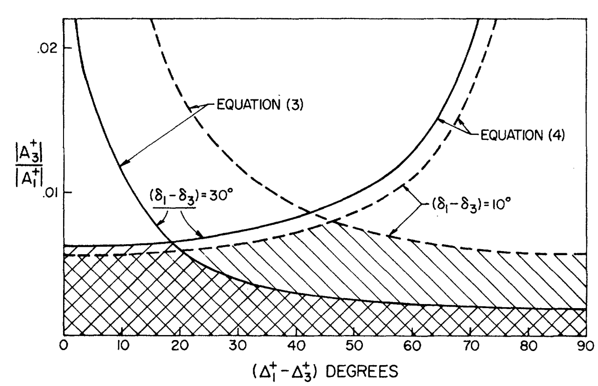

FIG. 1. Allowed combinations of $|A_3^+|/|A_1^+|$ and $\Delta_1^+-\Delta_3^+$ for $\delta_1-\delta_3=10°$, 30°. Combinations which lie within one standard deviation of the experimental data lie in the shaded areas.

$\Delta_{3/2}$. (The assumptions in this case require $\Delta_{1/2}=0$.) Figure 2 represents the experimental limits placed on these variables. The dashed lines represent one-standard-deviation limits.

Case III: CP-nonconserving phases $\Delta_{1/2}$ and $\Delta_{3/2}$ are both of order of the K_2^0 parameter[3] $|\eta_{+-}|\approx 2\times 10^{-3}$.—Then Fig. 2 allows us to conclude that the ratio $|\lambda_{3/2}|/|\lambda_{1/2}|=0.044\pm 0.014$. This is remarkably close to $|A_2^+|/|A_0|\approx 0.055$ observed in $K_{\pi 2}$ decay.

It is important to note that these cases give somewhat different predictions for $r=R(K_1^0\rightarrow 3\pi)/R(K_2^0\rightarrow 3\pi)$.[17] Compared with the rates for K_1^0 expected from transitions to the CP-allowed $T=0$ and 2 states of three pions,[3] Case I gives the relatively large value of $r\sim\tan^2\Delta_{1/2}=0.14\pm 0.06$. Case II gives $r\sim|\lambda_{3/2}/\lambda_{1/2}|^2\sin^2\Delta_{3/2}$ which varies between 0 and 0.05 using Fig. 2. Case III gives $r\approx 10^{-5}$.[3]

From our analysis it appears that if CP noninvariance with $|\Delta T|=\frac{1}{2}$ is indeed very small[1,5] (our Cases II or III), then it is not unlikely that an amplitude with $|\Delta T|=\frac{3}{2}$ comparable with that in $K_{\pi 2}$ decays occurs in $K_{\pi 3}$ decays, and this amplitude may have a CP-noninvariant part. An important test of the latter possibility would be a measurement of the rate for $K_1^0\rightarrow 3\pi$. Another test would be a high-precision comparison of the energy spectra in τ^+ and τ^- decays.[15]

In view of the results quoted above, it is appropriate to mention several models which might describe $K_{\pi 3}$ decay. (1) In general, one expects ordinary second-order electromagnetic corrections to introduce small amplitudes with $|\Delta T|=\frac{3}{2}$ and $\frac{5}{2}$ even if the strangeness-changing nonleptonic decay interaction satisfies the $|\Delta T|=\frac{1}{2}$ rule. (2) If, however, the latter interaction has in addition a piece which transforms like a 27-plet member, one might expect violation of the rule only through a $|\Delta T|=\frac{3}{2}$ amplitude intrinsic to the weak interaction, and this might violate CP invariance.[1,5] (3) The special case of a combination of the ordinary electromagnetic interaction and a C-nonconserving electromagnetic current[18] which transforms as an isoscalar would give rise to second-order electromagnetic effects which violate CP invariance in the $|\Delta T|=\frac{1}{2}$ and $\frac{3}{2}$ amplitudes only, with the larger effect in the latter amplitude.[19]

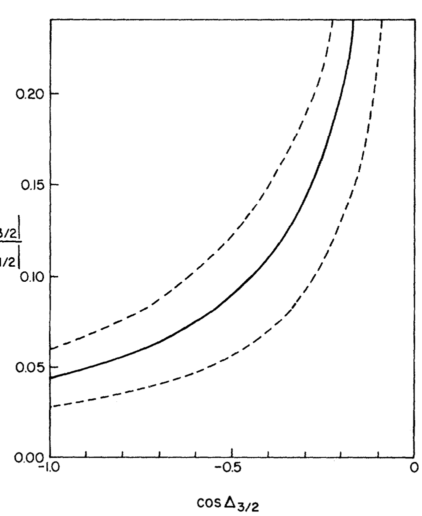

FIG. 2. Allowed combinations of $|\lambda_{3/2}/\lambda_{1/2}|$ and $\Delta_{3/2}$. Dashed curves represent one-standard-deviation limits.

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Note added in proof.—New measurements of branching ratios involving $K_2^0\rightarrow 3\pi^0$ have been made at the CERN heavy-liquid bubble chamber.[20] Their combined results can be expressed as

$$R_{000}/R_{+-0}=1.69\pm 0.12.$$

The individual branching ratios measured directly by each of the three groups have been added as inputs to the least-squares analysis and they strengthen considerably the conclusions drawn in the present paper. We now have for Eq. (9) the value 0.825 ± 0.031, and for Eq. (10), 0.820 ± 0.035. We see that the branching ratio (9) is almost six standard deviations away from agreement with the $\Delta T=\frac{1}{2}$ rule.

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[8]Terms in the matrix element which are linear in the pion energies contribute to the total rate in the case of Eqs. (1a) and (1d) because of unequal charged and neutral pion masses. In addition quadratic energy terms in the squared matrix element contribute to all the rates. Although no quadratic terms have been observed, some experimental limits can be placed on their existence (R. Plano, private communication). These effects and their uncertainties have been included in the phase-space corrections.

[9]We follow the notation and sign conventions of the detailed isospin analysis of $K_{\pi 3}$ decays by G. Barton, C. Kacser, and S. P. Rosen, Phys. Rev. 130, 783 (1963).

[10]R. H. Dalitz, Rev. Mod. Phys. 31, 823 (1959); S. Okubo, R. E. Marshak, and E. C. G. Sudarshan, Phys. Rev. Letters 2, 12 (1959).

[11]T. J. Devlin (unpublished). The decay rates used are

$$R_{++-} = (4.48 \pm 0.03) \times 10^6 \ \mathrm{sec}^{-1},$$
$$R_{00+} = (1.36 \pm 0.04) \times 10^6 \ \mathrm{sec}^{-1},$$
$$R_{+-0} = (2.36 \pm 0.09) \times 10^6 \ \mathrm{sec}^{-1},$$
$$R_{000} = (4.44 \pm 0.45) \times 10^6 \ \mathrm{sec}^{-1}.$$

These are in excellent agreement with a recent compilation [A. H. Rosenfeld, et al., Rev. Mod. Phys. 39, 1 (1967)] except for the uncertainties, which have been substantially improved by recent experiments and the consistency requirements of the least-squares analysis.

[12]G. H. Trilling, Argonne National Laboratory Report No. ANL-7130, 1965 (unpublished).

[13]Final-state Coulomb interactions present in Γ_{++-} but absent in Γ_{+00} can distort the amplitudes of τ decay. The effect on the branching ratio in Eq. (8) has been estimated by R. H. Dalitz [Proc. Roy. Soc. (London) A69, 527 (1956)] to be of order 2.6%, i.e., about the same as the statistical accuracy. Similar remarks hold with respect to the branching ratio in Eq. (10). The effects on the branching ratios in Eqs. (7) and (9) tend to cancel.

[14]It should be pointed out that our neglect of quadratic terms in Eqs. (3) and (4) does not affect the conclusions to be drawn from Fig. 1. The quadratic terms change the curves only where they do not act as boundaries of the shaded areas.

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[19]In certain special models the *CP* noninvariance due to an electromagnetic correction might be particularly small in $K_{\pi 3}$ decays. For example, if the *CP* noninvariance is related to a possible large noninvariance in $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\gamma$ through the first vertex in the sequence $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\gamma \rightarrow \pi^{\pm}\pi^{\pm}\pi^{\mp}$, then parity conservation at the second vertex requires that only the magnetic multipoles enter. If these are largely absent [see for example, D. Cline, Phys. Rev. Letters 16, 367 (1966)], *CP* noninvariance would be correspondingly small in $K_{\pi 3}$ decays. On the other hand, the *CP*-noninvariant $E1$ multipole is present in the sequence $K_2^{0} \rightarrow \pi^{+}\pi^{-}\gamma \rightarrow \pi^{+}\pi^{-}$ (or $2\pi^{0}$).

[20]Paris-CERN-Orsay Collaboration, in Proceedings of the Heidelberg Conference on High Energy Physics (to be published). We wish to thank Dr. D. Myatt for permission to use these data prior to their announcement.