In Table I we also show the temperature dependence predicted from Eq. (5) together with the experimental data taken from Fig. 1 at $\Delta\omega = 0$. The calculated Raman intensities were obtained using values of ω_{TO} and Γ listed in Table I and values of P_s^2 determined by the Devonshire thermodynamic formalism. The agreement is quite good over the temperature range shown.

That the lowest frequency TO lattice mode is overdamped is especially important when relating the frequency of this mode to dielectric-constant data through the LST relation. For a damped-classical-oscillator model this relation can be shown[4] to apply specifically to the undamped natural frequency ω_{TO} and not to peaks in ϵ'', Raman spectra, or certain other response functions. These peak frequencies are generally reduced below ω_{TO} by the presence of damping. In the case of overdamping ($\Gamma > 1$) the reduction is most significant and in fact results in an apparent mode frequency of zero in the Raman spectrum. Using the values of ω_{TO} listed in Table I and frequencies for all the other modes determined by Raman measurements, we find that our dielectric-constant data agree with the LST relation. Details will be published elsewhere.[8]

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HIGH-ENERGY, SMALL-ANGLE, p-p AND $\bar{p}$-p SCATTERING, AND p-p TOTAL CROSS SECTIONS*

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We have measured small-angle p-p differential elastic scattering cross sections from 8 to 26 BeV/c and $\bar{p}$-p cross sections at 12 BeV/c, and deduced the real part of the nuclear amplitude from Coulomb-nuclear interference under the assumption of spin independence. We have also measured the p-p total cross section from 8 to 26 BeV/c with errors of 0.3%.

The experimental setups used were similar to those previously described[1,2] except that a high-pressure gas differential Cherenkov counter was used in place of threshold counters to identify protons or antiprotons. Contamination due to π^- in the $\bar{p}$ beam was measured to be <0.3% and has a negligible effect on the results; all other contaminations are negligibly small. Data handling and analysis for the differential cross-section measurement were also similar to those previously described.[1] However, unlike the pion case, at $t=0$ the p-p interaction is complicated by the existence of a singlet and two triplet amplitudes. We here assume spin independence which allows us to make the analysis in terms of a single scattering amplitude. Thus, using the optical theorem, we can deduce the imaginary part of the nuclear amplitude at $t=0$ from the total cross sections which were measured in a separate part of this experiment. The analysis of the total cross sections was similar to that described in Ref. 2 and the results are given in Table I. The momenta were known to 0.2% except for the highest three momenta which were determined to only ~1% because of the failure of an analyzing magnet.

Table I. p-p total cross sections. Measurements within 100 MeV of each other have been combined.

Momentum (BeV/c)	$\sigma_{p\text{-}p}$ (mb)
7.82	40.34 ± 0.12
9.80	39.84 ± 0.12
11.90	39.62 ± 0.12
14.01	39.42 ± 0.12
16.03	39.23 ± 0.12
17.91	39.18 ± 0.12
20.22	39.05 ± 0.12
20.46	39.09 ± 0.12
22.0	38.88 ± 0.12
24.0	38.89 ± 0.12
26.0	38.90 ± 0.12

Table II. The ratio of the real part of the nuclear amplitude to the imaginary part. An additional systematic error of ±0.02 should be applied.

Momentum (BeV/c)	α
7.81[a]	−0.331 ± 0.014
9.86	−0.345 ± 0.018
9.86[a]	−0.343 ± 0.009
11.94[a]	−0.290 ± 0.013
14.03	−0.272 ± 0.013
20.24	−0.205 ± 0.013
24.12	−0.157 ± 0.018
26.12	−0.154 ± 0.025

[a]Measured with the apparatus set to cover a larger angular range.

However, due to the very slow momentum dependence of the total cross section at these energies, the effect of this momentum uncertainty is negligible. Our results are in good agreement with those of Bellettini et al.,[3] but our lowest-momentum measurement at 8 BeV/c is higher than the highest-momentum point of Bugg et al.[4] by approximately twice our error. The results of Galbraith et al.[5] with errors of ±~1.5% overlap our measurements in all cases.

The results for α (the ratio of the real to the imaginary part of the nuclear amplitude) are given in Table II and are shown in Fig. 1 along with the results of other investigations.[3,6] The errors shown on each point are those obtained from the least-squares fits to the data, and do not include the systematic error of ±0.02. It is clear that there is reasonable agreement with all of the previous data except the earlier measurements of Bellettini et al. Their later measurement at 10 BeV/c gave a lower magnitude of α in agreement with our result. When our early runs showed the different energy dependence, additional points were measured at energies close to those of Bellettini et al. These measurements confirmed our conclusion on the energy dependence. The dotted lines in Fig. 1 indicate the range of the p-p forward dispersion relation predictions calculated by Levintov and Adelson-Velsky[7] including the uncertainty in the contributions from the nonphysical region. The solid line represents the calculation made by Söding[8] who made definite assumptions about these contributions. Considering the uncertainty in the theoretical calculations, the agreement between the data and both calculations is good.

For $\bar{p}$-p at 11.9 BeV/c we find $\alpha = -0.006 \pm 0.034$ with an additional systematic error of ±0.06 due primarily to uncertainty in the $\bar{p}$-p total cross section.[5] This is in agreement, within the error, with the dispersion-relation prediction[8] of −0.06.

The assumption of spin independence gives a good fit to the experimental data. If we allow a real part but simply relax the constraint that the magnitudes of the singlet and triplet amplitudes be equal, we find the best fit when the amplitudes are equal, but the data are consistent with a difference as large as ~30% at which point α was larger by ~0.02. We were unable to obtain a good fit to the data under the assumption of purely imaginary amplitudes with dif-

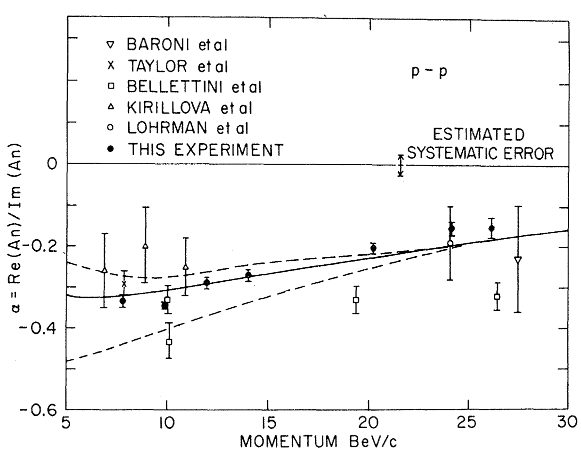

FIG. 1. The ratio of the real part of the nuclear amplitude to the imaginary part versus momentum. The systematic error on this experiment of ±0.02 is indicated. The curves are described in the text.

ferent magnitudes and exponential slopes for singlet and triplet states.

In contrast to the $\pi^{\pm}$-p case, the many simplifying assumptions of the p-p case prevent a critical verification of the forward dispersion relations. However, the agreement between the calculations and the data under the stated assumption is good. This experiment indicates that in p-p scattering, α decreases with increasing momentum above 10 BeV/c, and that for $\bar{p}$-p, α is very small at 12 BeV/c.

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LOW-ENERGY THEOREM IN THE RADIATIVE DECAYS OF CHARGED PIONS*

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A low-energy theorem is derived for the structure-dependent axial-vector form factor in the radiative decay $\pi \to l\nu\gamma$ using current-algebra techniques. By saturation of the sum rule with a few low-lying resonances, an estimate is made which is compared with experiments.

The radiative decays of the charged pions have been of considerable interest[1] especially with reference to the structure of weak interactions. Although the vector contribution can be related[2] to the matrix element of the decay $\pi^0 \to 2\gamma$ by means of the conserved vector-current hypothesis, it has not been possible to calculate the contribution of the axial-vector part. The purpose of the present note is to derive a low-energy theorem [Eq. (22)] for the structure-dependent axial-vector part of the radiative decay $\pi \to l\nu\gamma$ using the techniques of the current algebra. Saturating the sum rule by a few low-lying resonances, we compute the ratio γ of the structure-dependent axial-vector to the vector contribution and compare it with the value obtained by Depommier et al.[3]

The T-matrix element of $\pi^+(k) \to l^+(p_1) + \nu(p_2) + \gamma(q)$ is given by

$$T = -\frac{ieG\cos\theta}{\sqrt{2}}\left(\frac{m_l m_\nu}{4k_0 q_0 p_{10} p_{20} V^4}\right)^{1/2}\left[\epsilon_\mu M_{\mu\nu} l_\nu + F_\pi \bar{u}^{(l)}(p_1)(\gamma\cdot\epsilon)\frac{i\gamma\cdot(p_1+q)-m_l}{2p_1\cdot q}(\gamma\cdot k)(1+\gamma_5)v^{(\nu)}(p_2)\right], \quad (1)$$