SOLUBILITY CURVE AND MOLAR DENSITY OF DILUTE He^3-He^4 MIXTURES*

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(Received 14 August 1967)

The solubility curve has been measured at the saturated vapor pressure down to 0.025°K, and the limiting solubility at absolute zero has been determined to be $(6.37 \pm 0.05)\%$ He^3. Measurements of the density show that at 0°K and small concentration the volume occupied by a He^3 atom in solution is 1.284 ± 0.005 times the volume occupied by He^4.

We have measured the solubility curve and also the number density relative to pure He^4 of dilute solutions of He^3 in He^4 at the saturated vapor pressure. The measurements extend from 0.025 to 1.25°K in temperature, and from $X = 0.392$ to 0.1538 in He^3 molar concentration. The results give several quantities of interest both in the theory of dilute solutions, which has received a great deal of attention recently,[1-4] and in the practical application of solutions, particularly to dilution refrigerators.[5] The quantities determined are the following:

(a) X_0, the limiting solubility of He^3 in He^4 at 0°K. In conjunction with the theory of Bardeen, Baym, and Pines (BBP),[2] the value of X_0 allows a determination of the binding energy E_0 of a single He^3 atom dissolved in He^4 at 0°K.

(b) $\alpha(X,T)$ defined by the equation for the molar volume of solutions,

$$v(X,T) = v_4(T)[1 + X\alpha(X,T)], \quad (1)$$

where $v_4(T)$ is the molar volume of pure He^4. According to BBP,[2,4] α_0, the value of α for both X and T very small, is related to the long-wavelength limit of the effective interaction $V(q)$ at $q = 0$ by the relation $V(q=0) = -\alpha_0^2 m_4 s^2/n_4$, where s is the velocity of sound, m_4 is the mass of the He^4 atom, and n_4 is the number density of pure He^4.

(c) The derivative of the He^3 effective mass m^* with respect to density. This is obtained from the thermal expansion of the solutions.

The data were obtained from measurements of the capacitance C of a 5-pF parallel-plate capacitor placed at the bottom of a nylon cell containing about 2.0 cm^3 of helium mixture and 2.6 g of cerium magnesium nitrate (CMN) used as a thermometer. The capacitance was measured by a General Radio 1615-A bridge to a precision of 1×10^{-5} pF.

The data were analyzed using an equation based on the Clausius-Mosotti equation, $C = C_0(T) + \Gamma/[v(X,T) - 0.5169\ cm^3]$. Here the small quantity 0.5169 cm^3 is $\frac{4}{3}\pi$ times the molar polarizability of helium. The capacitance of the empty cell $C_0(T)$ and the constant Γ were determined in runs using pure He^4 whose molar volume is accurately known.[6] There was usually a small zero shift in $C_0(T)$ which was observed in each run. Two different sorts of (periment were carried out: (i) To determine molar volume as a function of concentration, at a constant temperature of 0.69°K, successive amounts of He^3 were added to pure He^4 in the cell, changing X from zero to about 0.15, and (ii) to measure the phase-separation curve and thermal expansion, with a constant amount of mixture, the temperature was reduced to 0.02°K and then increased in steps to 1.25°K. To eliminate the zero shift between runs, the experiments in which phase separation occurred were normalized together at 0.07°K, while one of them, the $X = 0.0992$ run, was adjusted to agree with the isothermal experiment at 0.69°K. The uncertainty in the data is about ± 0.002 cm^3/mole except that as the temperature is increased above 0.7°K, an additional uncertainty develops because of evaporation and refluxing effects in the filling tube, equivalent to about 1.0% of X at 1.2°K.

The results for $v(X,T)$ are shown in Fig. 1. In the phase-separation region, all the results lie on one curve which gives the molar volume of the lower, dilute phase. In the single-phase, constant-concentration region there is a pronounced thermal contraction which is approximately proportional to X and T. Assuming that the molar entropy of a solution is given by $s = Xs_F + s_4$, where s_4 is the entropy of pure He^4 and s_F is the entropy of 1 mole of an ideal Fermi gas of the same number density as the solution and with an effective mass m^*, the change in molar volume with temperature at constant pressure can be expressed as

$$v(X,T) - v(X,0) = X\kappa_4[u_F(T) - u_F(0)] \times [\tfrac{2}{3}(1 + \alpha_\kappa X) + \beta^*] + v_4(T) - v_4(0). \quad (2)$$

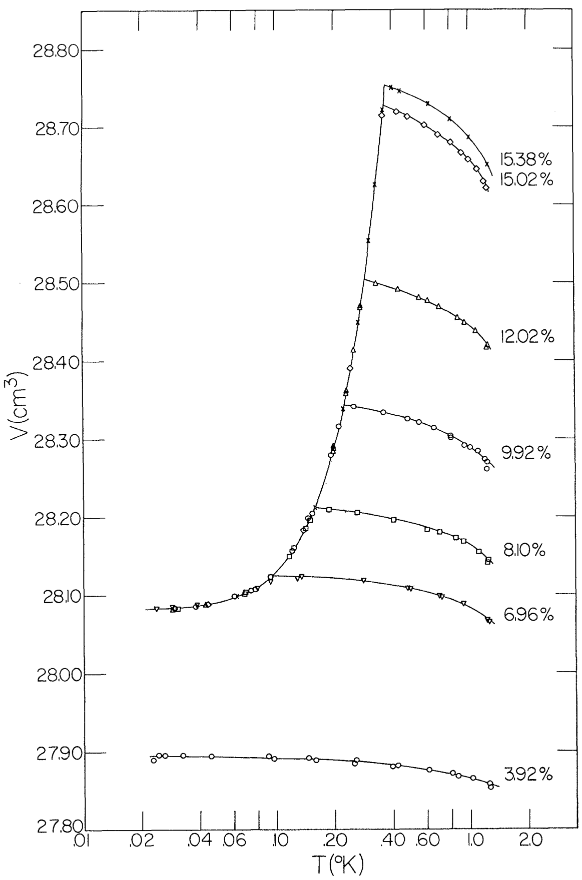

FIG. 1. The molar volume of dilute solutions of He^3 in He^4 at the saturated vapor pressure.

Here κ_4 is the isothermal compressibility of pure He^4, $u_F(T)$ is the molar energy of an ideal Fermi gas of the same number density, $\alpha_K = \kappa_4^{-1}\partial\kappa(X)/\partial X = v_4(0)[\partial\alpha_0/\partial v_4(0)] = 1.4$,[7] and $\beta^* = [v_4(0)/m^*(X)][\partial m^*(X)/\partial v_4(0)]$. The data when corrected to zero pressure agree well with this equation using $m^* = 2.5m_3$ and give $\beta^* = -(1.22 \pm 0.05)$, in excellent agreement with the recent second-sound data of Sandiford and Fairbank.[8]

With the aid of Eq. (2), the molar volume at 0°K, $v(X, 0)$, has been obtained. Within the accuracy of the experiment,

$$v(X,0) = v_4(0)[1 + X\alpha(X,0)] = v_4(0)[1 + \alpha_0 X + \alpha_0' X^2]. \quad (3)$$

The experimental data are not sufficiently accurate to determine α_0 and α_0' separately. Equally good fits are obtained with $\alpha_0 = (0.284 \pm 0.05)$, $\alpha_0' = 0$ or with $\alpha_0 = (0.280 \pm 0.05)$ and $\alpha_0' = 0.55$, but the second pair of values also gives a good fit with data for $X = 0.273$, measured by Kerr,[9] and with the volume of pure He^3.[10] Both values of α_0 are considerably below that recently found by Boghosian and Meyer,[7] $\alpha_0 = 0.308 \pm 0.010$.

The solubility curve, i.e., values of $X(T)$ in the lower phase of the two-phase system, has been derived from the experimental data using Eqs. (2) and (3) to convert $v(X, T)$ to $X(T)$. The results are shown in Fig. 2. Below 0.15°K the solubility curve fits the empirical equation

$$X = X_0[1 + 10.8(°\mathrm{K})^{-2}T^2],$$

where the solubility X_0 at 0°K is $(6.37 \pm 0.05)\%$, in good agreement with previous estimates based on the specific heat.[11]

An analysis of the present data in terms of the chemical potential of He^3 in solution will be postponed to a more detailed publication; we only mention here that fitting the data with the calculations of either BBP[2] or Ebner[3] shows the binding energy for one He^3 atom at 0°K, E_0, to be given by $E_0/R = L_3^0/R + (0.287 \pm 0.007)$°K, where L_3^0 is the latent heat of pure He^3 at 0°K,

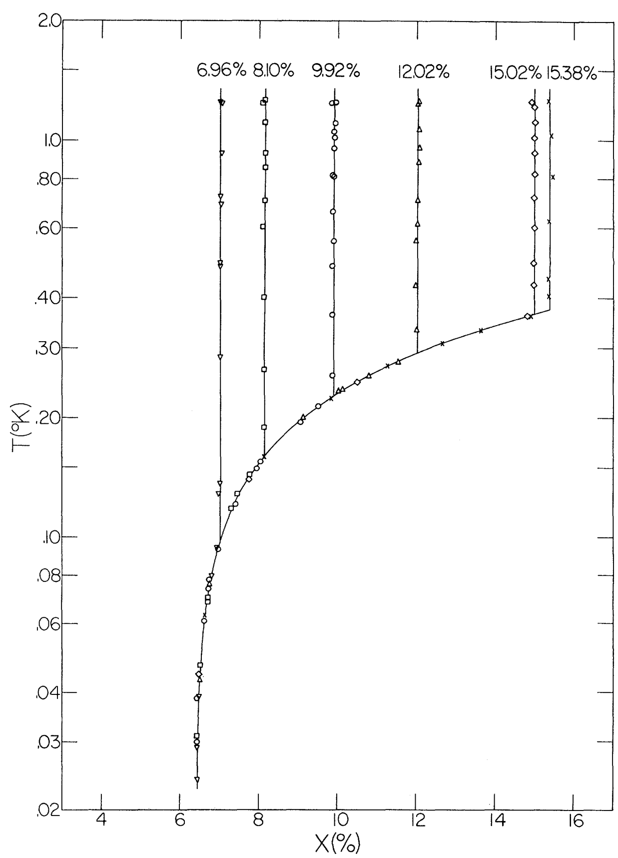

FIG. 2. The solubility curve for He^3 in He^4 at the saturated vapor pressure.

$L_3{}^0/R = (2.47 \pm 0.01)°K.$[12]

The authors gratefully acknowledge some interesting discussions with Professor Gordon Baym and Professor Horst Meyer.

*Work supported by the National Science Foundation.

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DYNAMIC STABILIZATION OF THE THETA PINCH

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(Received 11 September 1967)

Previous work has shown the periodic $\beta = 1$ theta pinch to be marginally stable to the mode $m = 1$ and unstable for all $m > 1$. We show that such a pinch can be dynamically stabilized for all $m \geq 1$.

As a result of the unavoidable end losses from a straight theta pinch, estimates of the length of a possible thermonuclear power reactor are of the order of hundreds of meters.[1] It is possible to remove this loss and to reduce the length by forming a toroidal pinch. Conditions for such toroidal equilibria were described by Meyer and Schmidt.[2] In such a system the plasma surface has a corrugated form and, therefore, stabilizing and destabilizing regions alternate. This configuration is rather complicated theoretically and it is useful to consider a linear analog. This is provided by a straight axisymmetric pinch in which the pinch radius varies periodically along its length thus introducing regions of favorable and unfavorable curvature.

This linear configuration has been studied theoretically in some detail. The most important result[3] is that such a configuration is unstable to a given mode m (>0) if $\beta < [1 + (R/R_w)^{2m}]^{-1}$ everywhere along its length where R and R_w are the radii of the plasma and the surrounding conducting coil. In practice it is almost impossible to avoid the instability regime for $m > 1$ and the condition is very stringent even for $m = 1$.

The purpose of this Letter is to describe a method of completely stabilizing all $m \geq 1$ for a $\beta = 1$ plasma without recourse to the stabilizing effect of a wall. The method may be described as follows: Consider a linear pinch with a periodic surface profile and corresponding periodic external magnetic field produced by a coil having the appropriate current distribution. In the absence of wall stabilization this system is unstable for $m > 1$ and marginally stable for $m = 1$.[4] Now consider a system in which this magnetic field configuration and surface profile are made to propagate along the pinch with a velocity V_w by suitably alternating part of the current in the coil. It is found that if $|V_w| > |V_A|$, then all $m \geq 1$ are stable, where $V_A{}^2 = B^2/\rho$, B being the mean value of the magnetic field at the surface of the plasma and ρ the plasma density.

The dynamic stabilization of the theta pinch has been previously studied by Weibel.[5] In the model he used the plasma consists of noncolliding particles which are specularly reflected at the plasma surface. The basic configuration he considers is marginally stable. The application of an azimuthal magnetic field periodic in time, but constant along the pinch, results in a change to positive stability, all perturbations decaying in time. The present calculation differs in that a fluid model is used, the dynamic stabilization currents are perpen-