in the deuteron theory. The noncoincidence data may be subject to larger or smaller errors. We note, however, that the low-q^2 noncoincidence data agree more closely with our preconceived ideas about a reasonable ratio σ_n/σ_p. The theoretical uncertainties in both measurements should decrease with increasing q^2, and indeed, the measured values of the two techniques do come together at high momentum transfer. Therefore, we think that the high-q^2 points can be trusted, but possibly only to the extent of the (larger) errors on the noncoincidence measurements.

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CURRENT ALGEBRAS AND POLE DOMINANCE APPLIED TO THREE-POINT FUNCTIONS*

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We demonstrate that the "hard-pion" process $A_1 \to \rho\pi$ can be correctly calculated by "soft-pion" techniques. The difficulty in earlier treatments by these methods is shown to be due to the fact that dispersing with an inappropriate invariant fixed omits important pole contributions. We criticize the derivation of the Kawarabayashi-Suzuki relationship.

In this note we show that, when calculating matrix elements via dispersion relations, one must properly include singularities in all variables. Dispersing with an inappropriate invariant fixed (e.g., q^2) can omit important pole contributions, e.g., those arising from terms proportional to $\delta(q^2-M^2)$ in the absorptive part.

By taking account of this fact, we are able to resolve the problem of the calculation of the A_1 width by "soft-pion" methods. The original application of conventional current-algebra and pole-dominance techniques to three-point functions led to a width which was far too large.[1] This has been a difficulty with the interpretation that the A_1 resonance at 1080 MeV is a chiral partner of the ρ meson.[1-3] Using a phenomenological Lagrangian which gives many of the current-algebra results, Schwinger has obtained a more reasonable A_1 width.[3] More recently Schnitzer and Weinberg[4] have obtained similar results by applying the pole-dominance assumption to Ward identities derived from the current algebras.[2] They suggest that the conventional approach (called by them the "ordinary" or "soft-pion" method) does not work because the pion in $A \to \rho\pi$ is not "soft."

It is the purpose of this note to point out (a) that from a careful application of the standard techniques one can obtain the Schwinger, Schnitzer-

Weinberg results, and (b) that there is, as yet, no valid current-algebra derivation of the Kawarabayashi-Suzuki (KS) relation[5] [Eq. (14)].

From partial conservation of axial-vector current (PCAC) and the current algebras one can obtain,[1] in the limit $q^\mu \to 0$,

$$f_\pi m_\pi^{\ 2} W_\mu = \langle A^+(p) | A_\mu^{\ +}(0) | 0\rangle$$
$$\equiv \sqrt{2} f_A \epsilon_\mu^{\ A}, \tag{1}$$

where we have defined[6]

$$W_\mu \equiv i \int d^4x\, e^{-iq\cdot x} \theta(x_0) \langle A^+(p) | [\varphi_\pi^{\ +}(x), V_\mu^{\ 3}(0)] | 0\rangle. \tag{2}$$

The most general form for W_μ is

$$W_\mu = F_1 \epsilon_\mu^{\ A} + (\epsilon^A \cdot q)[F_2 p_\mu + F_3 \Delta_\mu], \tag{3}$$

where $\Delta \equiv p - q$ and the F_i are $F_i(q^2, \Delta^2)$. From Eq. (1) we obtain

$$f_\pi m_\pi^{\ 2} F_1(0, m_A^{\ 2}) = \sqrt{2} f_A. \tag{4}$$

We calculate F_1 via dispersion relations. In a pole-dominance approximation, the absorptive part of W_μ is proportional to

$$\langle A(p) | \varphi_\pi(0) | \rho(\Delta)\rangle \langle \rho(\Delta) | V_\mu(0) | 0\rangle \delta(\Delta^2 - m_\rho^{\ 2}) - \langle A(p) | V_\mu(0) | \pi(q)\rangle \langle \pi(q) | \varphi_\pi(0) | 0\rangle \delta(q^2 - m_\pi^{\ 2}). \tag{5}$$

If one now assumes that the F_i satisfy unsubtracted dispersion relations (UDR) at fixed q^2, the second term in Eq. (5) is entirely omitted. This, in fact, is the source of the difficulties in previous work.[1] Since we wish to be sure to include both pole contributions we do not fix either Δ^2 or q^2 but some linear combination thereof. We now assume UDR for the F_i in the remaining variable. We thereby relate the F_i to the $A\pi\rho$ coupling constants and to form factors $\mathfrak{F}_i(\Delta^2)$:

$$\langle A^+(p) | \varphi_\pi^{\ +}(0) | \rho^0(\Delta)\rangle = \frac{(-1)}{q^2 - m_\pi^{\ 2}} [g_{A\rho\pi}(\epsilon^A \cdot \epsilon^\rho) + h_{A\rho\pi}(\epsilon^A \cdot q)(\epsilon^\rho \cdot p)], \tag{6}$$

$$\langle A^+(p) | V_\mu^{\ 3}(0) | \pi^+(q)\rangle = \mathfrak{F}_1(\Delta^2) \epsilon_\mu^{\ A} + (\epsilon^A \cdot q)[\mathfrak{F}_2(\Delta^2) p_\mu + \mathfrak{F}_3(\Delta^2) \Delta_\mu]. \tag{7}$$

We now make two further assumptions: (i) $g_{A\rho\pi}$ and $h_{A\rho\pi}$ are constants (this, of course, is in the spirit of PCAC and pole dominance), and (ii) the $\mathfrak{F}_i$ have at most one subtraction. (As mentioned hereafter, these assumptions are not entirely necessary and can be put on a different footing.)

Current conservation as applied to Eq. (7) tells us that

$$-\mathfrak{F}_1 + \tfrac{1}{2}(m_A^{\ 2} - m_\pi^{\ 2}) \mathfrak{F}_2 + \Delta^2(\mathfrak{F}_3 + \tfrac{1}{2}\mathfrak{F}_2) = 0. \tag{8}$$

With our assumption (ii), this implies that (a) the combination $(\mathfrak{F}_3 + \frac{1}{2}\mathfrak{F}_2)$ must be unsubtracted, and (b) its residue at the ρ pole is, in the pole-dominance approximation, related to the subtraction constant in $-\mathfrak{F}_1 + \frac{1}{2}(m_A^{\ 2} - m_\pi^{\ 2})\mathfrak{F}_2$. The ρ-pole residues can, of course, be directly calculated in terms of $g_{A\rho\pi}$ and $h_{A\rho\pi}$.

We now know the F_i in terms of $g_{A\rho\pi}$, $h_{A\rho\pi}$, and the one subtraction constant in the $\mathfrak{F}_i$ thus far undetermined. The latter can be eliminated by taking the divergence of Eq. (3): $\Delta_\mu W^\mu = 0$. We now use Eq. (4) to obtain

$$g_{A\rho\pi} = \frac{\sqrt{2} f_A}{f_\pi f_\rho} \frac{m_\rho^{\ 2}}{m_A^{\ 2}} (m_A^{\ 2} - m_\rho^{\ 2}). \tag{9}$$

This differs by the factor $(m_\rho^{\ 2}/m_A^{\ 2})$ from earlier results based on similar methods.[1]

An analogous treatment can be applied to the matrix element of an axial current and a pion field between a ρ meson and the vacuum. Here the relevant absorptive part is proportional to

$$\langle\rho(p)|A_\mu(0)|\pi(q)\rangle\langle\pi(q)|\varphi_\pi(0)|0\rangle\delta(q^2-m_\pi^2)-\langle\rho(p)|\varphi_\pi(0)|\pi(\Delta)\rangle\langle\pi(\Delta)|A_\mu(0)|0\rangle\delta(\Delta^2-m_\pi^2)$$
$$-\langle\rho(p)|\varphi_\pi(0)|A(\Delta)\rangle\langle A(\Delta)|A_\mu(0)|0\rangle\delta(\Delta^2-m_A^2). \tag{10}$$

We obtain

$$f_\rho=g_{\rho\pi\pi}f_\pi^2+\frac{f_Af_\pi g_{A\rho\pi}}{\sqrt{2}(m_A^2-m_\rho^2)}\frac{m_\rho^2}{m_A^2}. \tag{11}$$

If one disperses the F_i with an arbitrary combination of Δ^2 and q^2 [e.g., $\alpha q^2+(1-\alpha)\Delta^2$] fixed and demands that the result be independent of α, then assumption (ii) can be dropped. Further, if one takes the limit $\alpha\to1$ (i.e., fixed q^2), one finds that F_1 must now be subtracted in such a way as to include the effects of the pion contribution.[7]

It is also possible to apply the Fubini[8] procedure to the full $T_{\mu\nu}$ (the matrix element of two currents); in this case one relates integrals over absorptive parts to quantities like the right-hand side of Eq. (1) and no $q_\mu\to0$ (or $q^2\to0$) limiting procedure is necessary. The foregoing results [Eqs. (9) and (11)] are again obtained with assumption (ii) replaced by an unsubtraction assumption on the form factors in $T_{\mu\nu}$. In this way one can also derive[7,9]

$$h_{A\rho\pi}=0 \tag{12}$$

and the Weinberg[2] sum rule

$$\frac{f_\rho^2}{m_\rho^2}=\frac{f_A^2}{m_A^2}+\tfrac{1}{2}f_\pi^2. \tag{13}$$

Equations (9) and (11)-(13) are the results obtained by Schnitzer and Weinberg using the Ward-identity approach[4] for the case in which their parameter $\delta=0$.[10] Schwinger's Lagrangian contains the foregoing ($\delta=0$) results and the relations

$$\xi\equiv f_\rho/f_\pi m_p=1 \tag{14}$$

and

$$f_A=f_p. \tag{15}$$

Equation (14) is the KS relation[5]; Eq. (15) is obtained from the algebra of fields, but not from the algebra of currents.[2,11]

We would like to point out that although (14) may in fact be true, it cannot at present be regarded as a result derived from current algebras. The original derivation,[5] which was based on the same matrix element that led us to Eq. (11), gave the equation

$$f_\rho=g_{\rho\pi\pi}f_\pi^2. \tag{16}$$

This is precisely Eq. (11) with the A contribution omitted. This is not surprising since no such contribution was considered by the authors of Ref. 5. However, if one assumes the existence of the A meson, Eq. (16) must be regarded with the greatest suspicion. Furthermore Eq. (14) is obtained from (16) only by assuming

$$\lambda\equiv f_\rho g_{\rho\pi\pi}/m_\rho^2=1. \tag{17}$$

However, the combination of (9) and (11)-(14) implies

$$\lambda=1-m_\rho^2/2m_A^2\simeq\tfrac{3}{4}, \tag{18}$$

a result first obtained by Schwinger.[2,12] Thus, the logic leading to (14) is not impeccable! There is not a clear case for (14) as an empirical result either, since the comparison is based upon the $\rho\to2\pi$ decay rate which measures not ξ but

$$g_{\rho\pi\pi}=\lambda m_\rho^2/f_\rho=(m_\rho/f_\pi)(\lambda/\xi)$$

and the experimental situation on λ is not clear.[13,14] If ξ should indeed turn out to be 1, we would view this as an indication of coupling-constant relationships beyond those implied by current algebras.[15,16]

Finally, let us conclude with some comments on the existence of the A. The original argument[1] for the existence of the A meson rested upon the observation that, without the A, there is no term in Eq. (10) proportional to $\epsilon_\mu{}^\rho$ at fixed q^2 (i.e., neglecting the first term). Hence, according to this argument, we would have $f_\rho=0$. As we have seen, fixing q^2 misses important contributions and this argument is not correct.[17] A more reasonable argument in favor of the A[18] is that if it does not exist then Eqs. (11) and (13) imply $\lambda=\frac{1}{2}$,[19] a result clearly excluded by the data.[13]

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Note added in proof.—After completing our work, we saw the paper of Geffen.[20] He makes the assumption of fixed-q^2 UDR for $F_{2,3}$. He obtains somewhat different sum rules from ours which, however, become the same when our additional result[10] $h_{A\rho\pi}+2r=0$ is used. (In this case, it can be shown that our assumptions imply his.)

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[6]f is defined analogously to f_A; f_π is defined by $\langle\pi^+(q)|A_\mu{}^+(0)|0\rangle=q_\mu f_\pi$. Here as throughout we omit the usual covariant normalization factors.

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[9]If one dispersed at fixed q^2, the results of Ref. 1 would instead be obtained; of course, no $q^\mu\to 0$ limit would be involved. This fact, together with the fact that with the $q^\mu\to 0$ limit, we obtain the results of Ref. 4, shows that what matters is not whether one uses "soft-pion" ($q^\mu\to 0$) techniques, but whether one disperses correctly.

[10]The general results can be obtained by the present methods if assumption (i) is relaxed, i.e., we allow $g_{A\rho\pi}$ and $h_{A\rho\pi}$ to be functions of q^2. One can then show that, in the pole approximation, $h_{A\rho\pi}$ is still independent of q^2 (strict PCAC) but that $g_{A\rho\pi}$ must be linear in q^2: $g_{A\rho\pi}(q^2)=g_{A\rho\pi}(0)+rq^2$. $g_{A\rho\pi}$ is then replaced by $g_{A\rho\pi}(0)+(m_A{}^2-m_A{}^2)r$ in Eq. (9) and by $g_{A\rho\pi}(0)-(m_A{}^2-m_\rho{}^2)r$ in Eq. (11). Equation (12) now becomes $h_{A\rho\pi}+2r=0$. The $r=0$ results quoted in the text correspond to $\delta=0$ of Ref. 4. Details of the general case will be given in Ref. 7.

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[12]The additional parameter δ in Ref. 3 permits one to have both ξ and $\lambda=1$, for $\delta=-1$. See, however, Ref. 14.

[13]Compare, for example, A. Wehmann et al., Phys. Rev. Letters 18, 929 (1967) with B. D. Hyams et al., Phys. Letters 24B, 634 (1967).

[14]Even if one were to argue that, notwithstanding the ambiguity in the $\rho\to 2\mu$ data, the most reasonable interpretation is $\lambda=\xi=1$, one would still have to reckon with the fact that this implies an A_1 width of ~60 MeV, compared with the experimental value of 130 ± 40 MeV [A. H. Rosenfeld et al., Rev. Mod. Phys. 39, 1 (1967)]. We feel that the data at present leave considerable uncertainty in the values of ξ and λ.

[15]For example, those implied by the additional assumptions of Schwinger (Ref. 3) and J. J. Sakurai, Phys. Rev. Letters 17, 552 (1966).

[16]The derivation by F. J. Gilman and H. J. Schnitzer [Phys. Rev. 150, 1362 (1966)], while different from that of Ref. 5 (they examine a four-point function), appears also to assume $\lambda=1$ and the absence of A contributions.

[17]It is amusing to note that, as pointed out by Geffen (private communication), the same matrix element was used, in mutually contradictory ways, to argue the existence of the A (Ref. 1) and to obtain the KS relation (Ref. 5).

[18]This is not necessarily to be identified with the A_1 at 1080 MeV, however. The derivation of the mass relation $m_A{}^2=2m_\rho{}^2$ in Ref. 2, which provides support for this identification, is based on combining (13), (14), and (15). As we have indicated, Eq. (14) is rather dubious. It may be well to bear this in mind in view of the experimental uncertainty as to whether the A_1 is a true resonance. See N. M. Cason et al., Phys. Rev. Letters 18, 880 (1967), and references contained therein. Of course, the algebra of fields, via Eqs. (13) and (15), provides a relationship between the value of ξ and the mass ratio. In this case, one must regard at least one of these numbers as purely empirical.

[19]If one applies the above techniques to the matrix element between a π and the vacuum, one can show that the form factor of the pion is subtracted unless $\lambda=1$. In other words current algebras do not exclude $\lambda\neq 1$.

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