compared with H_{c2}^*, except perhaps for the samples with high spin concentrations.

[20]The nature of the magnetically ordered state cannot be determined from the $H_{c2}^*(T)$ results in Fig. 2 alone; however, the zero-field studies (Ref. 13), in view of Bennemann's theory (Ref. 11), indicate that the magnetically ordered state in $La_{3-x}Gd_xIn$ is ferromagnetic and that the range of order is at least over distances comparable with the coherence length.

[21]The value of τ_{so} determined in this way is in good agreement with Bennemann's value (Ref. 11).

[22]According to the BCS theory, $l_{so}/\xi_0 \simeq kT_{c0}\tau_{so}/0.18\hbar$. For $T_{c0} = 9.1°K$ (Ref. 13) and $\tau_{so} \sim 2\times10^{-13}$ sec (for $La_{3-x}Gd_xIn$ with 1 at.% Gd (Ref. 11), $l_{so}/\xi_0 \sim 1$.

ELECTROMAGNETIC SHEAR-WAVE INTERACTION IN A SUPERCONDUCTOR*

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The purpose of this paper is twofold:

(1) Expressions for the electromagnetic part of the interaction between electrons and shear waves in a superconductor have been developed in the case where the electronic mean free path l is long compared with the wavelength λ. We believe these expressions have a wide range of validity in both frequency and temperature. The lower end of the frequency range corresponding to the extreme anomalous limit is discussed as a special example.

(2) It presents accurate measurements[1] on the turn-off of electromagnetic interaction between electrons and transverse phonons near the superconducting transition temperature of a metal. To our knowledge, these are the first measurements performed with sufficient accuracy to provide a quantitative test of experiment against theory. We believe this also to be the first experiment to measure the penetration depth [in this case the London penetration depth at 0°, $\lambda_L(0)$] of a superconductor at a single wave vector[2] q; because the wave propagates through the sample it probes the penetration depth in the bulk, contrary to the usual methods employed in penetration-depth studies.

It has been known for many years that the shear-wave attenuation in superconductors at low frequencies shows a sharp decrease in a millidegree range below the superconducting-transition temperature T_c when the sample is cooled through this region; cf. Morse[3] and Morse and Bohm.[4] It has been shown by several authors (see below) that this decrease could be explained at least qualitatively as due to screening of the induced fields by the sharply increasing number of "super" electrons in this region. However, the absence of sufficiently accurate measurements and a quantitative theory valid over the entire temperature and frequency range of electromagnetic interaction in the superconductor has prevented resolution of the interesting questions this problem poses.

The theoretical treatment given by Cullen and Ferrell[2] was for high frequencies (typically 1 Gc/sec). It should be noted that the expression for the electromagnetic attenuation takes a different form at lower frequencies owing to the frequency dependence of the terms involved. We find that a straightforward derivation of the electromagnetic part of the electronic attenuation in the superconducting state, α_S, relative to that in the normal state, α_N, leads to the following formula, valid at all temperatures below T_c and at all frequencies, assuming $ql \gg 1$:

$$\left.\frac{\alpha_S}{\alpha_N}\right|_E = \frac{\sigma_{1S}|E_S|^2}{\sigma_{1N}|E_N|^2}$$

$$= \frac{\sigma_{1S}/\sigma_{1N}[(q^2c^2/4\pi\omega\sigma_{1N})^2+1]}{(q^2c^2/4\pi\omega\sigma_{1N}+\sigma_{2S}/\sigma_{1N})^2+(\sigma_{1S}/\sigma_{1N})^2}. \quad (1)$$

In Eq. (1) $\sigma_{1S}+i\sigma_{2S}=\sigma_S$ is the transverse conductivity in the superconducting state, and $\sigma_{1N}+i\sigma_{2N}=\sigma_N$ the transverse conductivity in the normal state. E_S and E_N are the induced self-consistent electronic fields in the superconducting and normal state, respectively; ω is the angular frequency of the shear wave, and c is the velocity of light. The frequency variation of (1) is slow at frequencies up to a few hundred megacycles per second.

In the extreme anomalous limit,[5] σ_N has negligible imaginary part, and σ_{1N} can be rewritten[5] as

$$\sigma_{1N} = \frac{e^2}{4\pi^2\hbar q}\oint R_e \cos^2\varphi d\varphi. \quad (1a)$$

In special cases, this may be approximated by the free-electron expression. In general this should not be done however, because real metal effects may be important. In Eq. (1a) R_e is the product of the principal radii of curvature at a given point on the "effective zone" (i.e., regions where the Fermi velocity is normal to q) along which the integral is evaluated; the location of this point is given by the angle φ. At frequencies below approximately 100 Mc/sec (depending on the metal) the term $q^2c^2/4\pi\omega\sigma_{1N} \ll 1$ and can therefore be neglected. In this frequency range Eq. (1) can therefore be simplified to

$$\left.\frac{\alpha_S}{\alpha_N}\right|_E = \frac{\sigma_{1S}/\sigma_{1N}}{(\sigma_{1S}\sigma_{1N})^2 + (\sigma_{2S}/\sigma_{1N})^2}. \tag{2}$$

The frequency and temperature dependence of σ_{1S}/σ_{1N} has been given in Ref. 2 for higher frequencies but can be used without serious error even at frequencies much below 100 Mc/sec.[6] σ_{1S}/σ_{1N} is very strongly temperature dependent near T_c starting from unity with infinite slope at T_c, and increasing to about 1.5 over approximately 15 mdeg measured from T_c.

An expression similar to (2) has been given by Tsuneto[7] but only for temperatures well below T_c where electromagnetic absorption has nearly vanished. Furthermore, σ_{1S}/σ_{1N} was put equal to 1 in Ref. 7. The expression derived in Ref. 7 for temperatures closer to T_c has a serious divergence that makes it inapplicable in a range of $\simeq 8$ mdeg below T_c at ≈ 100 Mc/sec. We believe that in fact (1) and (2) are valid for all temperatures in the superconducting state. Equation (2) has also been used by Macintosh[8] for very high frequencies, where it does not hold strictly as can be seen in Ref. 2, or by estimating the terms in Eq. (1). Claiborne and Morse[9] have also treated the electromagnetic attenuation near T_c. Their expression has zero slope at T_c, instead of the infinite slope obtained from (1) and (2). It should be noted that while the infinite slope predicted by (1) and (2) does not manifest itself very clearly in an actual experiment because the initial drop is small, it should at least give rise to a very sharp break in the attenuation curve at T_c. This sharp break would not be a result of the onset of screening (Meissner effect) but rather due to the rapid increase in σ_{1S}/σ_{1N}. A few millidegrees below T_c, however, screening is the dominant cause of the fall-off. We see, then, that at low frequencies the increase in σ_{1S}/σ_{1N} at T_c can be expected to have the opposite effect of what is found in Ref. 2 at high frequencies. The crossover from an initial drop to an initial rise at T_c occurs at a frequency corresponding to $q^2c^2/4\pi\omega\sigma_{1N} = 1$. Typically this equality is satisfied in the frequency range 100-500 Mc/sec for most metals. This important term has been omitted in Refs. 7-9. It is worth noticing that when the turn-off of electromagnetic interaction in early work on this problem first was described as "discontinuous" (Ref. 3) and later as a "rapid fall," this was referring to the entire transition region. Claiborne and Morse[9] first showed experimentally that in fact the transition had a finite width.

To write Eq. (1) or (2) in a more explicit form one uses the standard expression for σ_{2S} as given by ordinary Pippard theory (cf. Tinkham[10]). For the experiment reported here, for which $q\xi_0 < 1$, where ξ_0 is the coherence distance at zero temperature, this simply leads to the London result for pure metals, $\sigma_{2S} = c^2/4\pi\lambda_L^2\omega$. (For $q\xi_0 \gg 1$ the Pippard limit would apply so that this expression would have to be multiplied by $3\pi/4q\xi_0$.) Furthermore, using the Bardeen-Cooper-Schrieffer[11] (BCS) temperature dependence for $\lambda_L(T)$ one has at temperature T

$$\sigma_{2S} = \frac{c^2}{2\pi T_c \lambda_L^2(0)\omega}(T_c - T).$$

Therefore, putting $T_c - T = \Delta T$, we have

$$\frac{\sigma_{2S}}{\sigma_{1N}} = \frac{hc^2}{e^2 v_s T_c} \frac{1}{\lambda_L^2(0)\oint R_e \cos^2\varphi d\varphi} \Delta T. \tag{3}$$

Here v_s is the velocity of the shear wave. We adopt the following point of view: Since σ_{1S}/σ_{1N} has been calculated in terms of experimentally known parameters, and v_s (Chandrasekhar and Rayne[12]) and T_c are also known from experiments, we see that the experiment reported here determines the quantity

$$\lambda_L^2(0)\oint R_e \cos^2\varphi d\varphi.$$

Furthermore, in simple metals the integral over the effective zone can be estimated fairly accurately and $\lambda_L(0)$ can therefore be extracted.

In the particular case of indium there are experimental and theoretical reasons[13,14] to believe that the effective zones along edges in the second and third zone do not contribute. While we cannot in this brief note discuss those arguments, they point to a gross reduction of the electronic mean free path along the edges thus eliminating these potentially effective zones from the interaction discussed here. The effective zone integral with $q \parallel [110]$ in indium is therefore close to the free-electron value.

The experiments to be reported here were performed with $q \parallel [110]$ and polarization $\epsilon \parallel [110]$ on a single-crystal indium sample of high purity (estimated mean free path $l \simeq 0.2$ mm) and thickness 1.206 mm. The experimental technique can be described in short as follows: The signal was detected after traversing the sample once. The video pulse from the receiver was peak detected on a Hewlett Packard oscilloscope by a plug-in scanner 1782A and fed to one axis on a Moseley model 7001 AMR recorder. To the other axis the off-balance thermometer voltage from a millidegree-calibrated Ge thermometer was fed after being amplified by an Astrodata Nanovoltmeter 121Z. In this way a continuous recording of attenuation versus temperature was obtained. The sensitivity of the thermometer-amplifier-recorder system was such that 1 mdeg corresponded to 12 mm on the temperature axis. Noise in the thermometer system and instability in the He temperature usually corresponded to a total of $<\pm 0.1$ mdeg in runs where good temperature control was obtained. The runs were made by slow pumping from 4.2°K down to about 30 mdeg above T_c. Here the rate of change of temperature was reduced by adjusting a bleeding valve in the manostat until the rate of change was around 1 mdeg/min. A temperature sweep at this speed was then made from above T_c to $\simeq 25$ mdeg below T_c, where electromagnetic attenuation is only a few percent of the normal state value.

Special care was taken to cancel the earth's magnetic field at the sample to about 0.005 G by means of three pairs of Helmholtz coils. The repetition rate of the pulsed rf signal could be changed by a factor of 100 thus changing the heat input to the sample by the same factor. No heating effects were detected. Also no detectable amplitude effect was present at the signal voltages that were used in the experiment reported here. Figure 1 shows a recording at 75 Mc/sec and Fig. 2 a replot on a linear scale, and a series of points corresponding to calculated values from Eq. (2) using $\sigma_{2S}/\sigma_{1N} = 1.83 \times 10^2 \Delta T$. This choice produces a reasonably good over-all fit between theoretical and experimental results. It also gives the zero level of electromagnetic attenuation, and therefore the total electromagnetic attenuation in a direct way. Notice that the deformation part of the attenuation has been subtracted by using the BCS[11] formula for the temperature dependence of $\alpha_S/\alpha_N|_{\mathrm{def}}$, the deformation part of the electronic attenuation. $\alpha_{N,\mathrm{def}}$ was put equal to $0.5\alpha_{N,E}$ in agreement with experimental results obtained by Liebowitz and Fossheim.[15] It is seen that the slope of the BCS function is relatively small except very near T_c.

By equating the experimental value for σ_{2S}/σ_{1N} given above to the theoretical expression (3), one obtains an experimental value for $\lambda_L(0)$ of 2.5×10^{-6} cm. The free-electron value is 1.56×10^{-6} cm. It is not surprising to find that $\lambda_L(0)$ is higher than the free-electron value since it involves the density of states at the Fermi surface. Since it is well known[16] that the density of states at the Fermi surface is

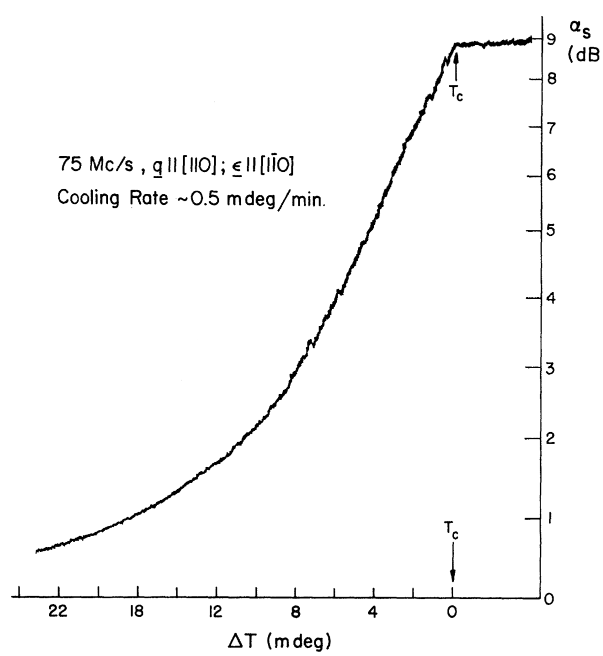

FIG. 1. Recorder tracing of attenuation versus temperature. Its original size was $\simeq 30$ cm on each side. Notice the nonlinear attenuation scale with arbitrarily assigned zero. This curve is one out of a series taken at different power levels, pulse repetition rates, and signal amplitudes during cool-down and warm-up runs. They all appeared identical over a range of low amplitudes.

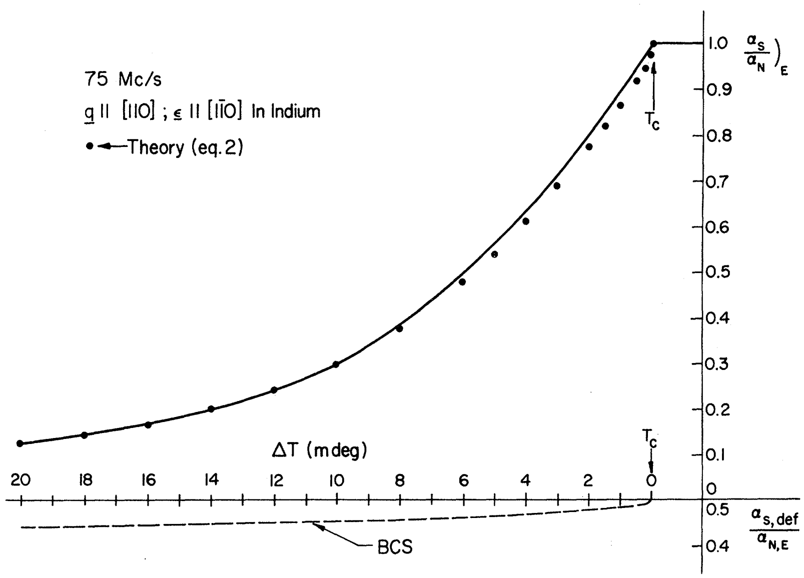

FIG. 2. The full line is a replot of the curve in Fig. 1 after the deformation attenuation has been subtracted, using the BCS function (bottom curve) for this part of the attenuation. The fit is then made by fixing the theoretical value of Eq. (2) to the curve at $\Delta T = 20$ mdeg. The ratio $\alpha_{S,\,\mathrm{def}}/\alpha_{N,E} = 0.50$ at T_c was taken from Ref. 15.

much higher than the free-electron value, an enhancement of $\lambda_L(0)$ has to be expected. In fact, by estimating the value of $\lambda_L(0)$ on the basis of Ref. 16 we find that one would expect it to be $\simeq 2.3 \times 10^{-6}$ cm. With this value for $\lambda_L(0)$, however, the fit is not as good, the theoretical curve falling below the experimental one over nearly the entire temperature range.

It should be noted that there are several possible reasons why a broadening of the transition can occur, for instance strains, temperature instability of the helium bath, possible amplitude effects, or uncancelled external fields. In the present experiment we believe that only the first source of broadening could be present to a noticeable extent. We expect it to be small, however, because of the fact that indium anneals at liquid-nitrogen temperature and in agreement with the observation that T_c is sharply defined by the experiment. The sample was kept at or below liquid-nitrogen temperature for weeks prior to the runs, after it had been mounted in the probe. Although the fit between theory and experiment in Fig. 2 is not perfect, we conclude that the agreement is reasonable considering the experimental and theoretical problems involved.

Measurements of the type presented in Fig. 1 have been done at lower and higher frequencies too. Values for $\lambda_L(0)$ close to 2.5×10^{-6} cm were obtained in these experiments through fits similar to the one shown in Fig. 2.

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†On leave from the University of Oslo, Norway.

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OBSERVATION OF TWO INTRINSIC NUCLEAR RELAXATION RATES IN ANTIFERROMAGNETIC $KMnF_3$

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This paper reports the results of the measurement of nuclear spin-lattice relaxation rates in $KMnF_3$, which indicate that there are at least two distinct relaxation processes in the ordered state. The high-temperature rate has a power-law temperature dependence, while the low-temperature rate has an approximate $\exp(-\alpha/T)$ dependence, indicating the effect of a magnon energy gap.

The first measurement[1] of relaxation times in $CuCl_2 \cdot 2H_2O$ led to the theoretical work of Moriya,[2] Van Kranendonk and Bloom,[3] and Mitchell.[4] These papers treated a magnon-Raman relaxation process and yielded the result (neglecting spin-wave interactions, using the long-wavelength limit, and assuming quantization of the electronic and nuclear spins along different directions in the crystal) that the nuclear relaxation rates should be proportional to T^3 for temperature above T_{AE}, and proportional to $T^2 \exp(-T_{AE}/T)$ for temperatures below T_{AE}, where kT_{AE} is the width of the magnon energy gap. Most experimental data have been in poor agreement with these predictions.

Pincus and Winter[5] have described a mechanism by which thermal phonons can participate directly in the nuclear relaxation process. This mechanism, applicable only when $T \ll T_{AE}$, yields a linear temperature dependence for the direct process and a T^7 dependence for the Raman process. In $CuCl_2 \cdot 2H_2O$, where $T_{AE} \approx 1$°K, an approximate T^7 dependence has been observed[6,7] between 1.25 and 0.95°K.[7] However, in this temperature region the Pincus-Winter theory is not valid, and, even if it were, the magnitude of the coupling is too small to explain the experimental results. In this temperature range for $T > T_{AE}$, a pure spin-wave process should predominate in this crystal. Abkowitz and Lowe[8] also observed a T^7 temperature dependence for protons in $CoCl_2 \cdot 6H_2O$ between 2 and 1.24°K but again these measurements are in the temperature range where a pure spin-wave process is expected.

Recently, Pincus[9] and Narath and Fromhold[10] have described a three-magnon process which has been observed in $CrCl_3$.[10] In this material, the Cr^{53} hyperfine interaction is almost isotropic, resulting in the exclusion of a magnon-Raman process to first order. The three-magnon process in the ferromagnetic state was investigated both theoretically and experimentally as a function of magnetic field for two external magnetic field directions, and there is reasonable agreement between theory and experiment. The temperature dependence for the three-magnon process has been calculated by Pincus[9] and found to be T^5 for $T > T_{AE}$, and to be proportional to a product of terms, one of which is $\exp(-T_{AE}/T)$, below T_{AE}. This exponential dependence is characteristic