parity states and between the singlet and triplet states.

[9]It should be remarked that the analyticity of f_5 at $s = 0$ would require the deuteron residue $r_{12}(s)$ to vanish at $s = 0$. Then by factorization theorem either $r_{11}(0)$ vanishes in which case $f_2{}^0(0,t)$ should be superconvergent, or $r_{22}(0)$ in which case we get no new relation. There seems to be no method of knowing on general ground as to which amplitude the deuteron chooses at $s = 0$.

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QUASIELASTIC ELECTRON-DEUTERON SCATTERING AT FORWARD ANGLES*

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This paper presents a preliminary report of recent measurements of quasielastic electron-deuteron scattering. Data points were taken at a scattered-electron laboratory angle of 20° and for a range of four-momentum transfers from 7 to 70 F^{-2}. Three different measured quantities have been extracted from the data: (1) the ratio of electrons without a coincident proton to electrons with a coincidence, which it is hoped, after small corrections, is equal to the neutron-to-proton cross-section ratio (σ_n/σ_p); (2) the ratio $\sigma_{\mathrm{all}\,e^-}(\mathrm{D})/\sigma_{\mathrm{all}\,e^-}(\mathrm{H})$ of the total electron-deuteron "area method" quasielastic cross section to the elastic e-p cross section from hydrogen, which should equal $(\sigma_n+\sigma_p)/\sigma_p$; (3) the ratio $\sigma_{p\text{-D}}/\sigma_{p\text{-H}}$ of the electron-proton coincidence cross section from deuterium to the same e-p coincidence cross section from hydrogen using the "area method." The measurements were made in conjunction with elastic electron-proton cross-section measurements from hydrogen.[1]

Electrons from the external beam of the Cambridge Electron Accelerator struck a liquid-hydrogen or deuterium target. The scattered electrons were detected in a magnetic spectrometer followed by a Cherenkov and a shower counter.[1] The momentum acceptance was 15% and the momentum resolution was approximately 2.5% (full width at half-maximum).

Protons were detected in a two-counter telescope of large solid angle, protected from the high background fluxes of low-energy particles either by lead absorber or by a sweeping magnet. A 12×12 checkerboard counter hodoscope was used to measure the angular distribution of recoiling protons.

There are three important experimental corrections which can confuse the assignment of an event to the σ_p or σ_n categories: (1) A chance coincidence can occur in the proton telescope (with a probability of between 2 and 5%) when a neutron event is present; (2) a fraction (typically 0.2 to 0.7%) of the neutrons can produce a proton count by charge exchange; (3) protons can be absorbed or scattered out before they count in the telescope. The proton absorption, measured using elastic scattering from hydrogen, was about 5% when lead absorber was used and about 2% without it.

In order to interpret the experimental ratio of noncoincidence to coincidence counts in terms of σ_n/σ_p, it is necessary to correct the ratio for those protons thrown outside of the solid angle of our proton detector. The presence of binding and, hence, of momentum of the nucleons within the deuteron causes the recoiling (quasielastic) particles to emerge with a distribution of angles and momenta around those particles recoiling elastically from free e-p scattering. A detailed calculation must also take into account some other small corrections. The theoretical work of Durand[2] and McGee[3] was used to calculate the full triply differential cross section. The analysis reported here ignores all final-state–interaction effects, although Durand and McGee have written down a theoretical treatment of them. The modified Hulthén wave function[4] with an assumed 5% D-state probability was used throughout the data analysis. The use of better wave functions[5] makes insignificant difference to the analysis.

For electrons at the top of the quasielastic momentum peak, the fraction of protons thrown outside of the counter-telescope acceptance due to the S-state part of the deuteron wave function was between 0.2 and 0.5%. The D-state part introduced another 0.5% loss, which was nearly proportional to the assumed D-state probability. Several other small terms introduced a correction of between 0.2 and 0.5% to the calculated σ_n/σ_p ratios.

The second quantities extracted from the data were the ratios $\sigma_{\mathrm{all}\,e^-}(\mathrm{D})/\sigma_{\mathrm{all}\,e^-}(\mathrm{H})$ and $\sigma_{p\text{-D}}/\sigma_{p\text{-H}}$. In the analysis of the $\sigma_{\mathrm{all}\,e^-}$ ratio, the proton counter telescope was ignored entirely, and the ratios thus extracted are electron-only measurements. Besides the loss of protons due to the finite solid angle described above, several other corrections enter into these ratios. First, 3 and 10% of the electrons are thrown out of the momentum acceptance by the high-momentum components of the wave function. The size of this effect was calculated using the Durand-McGee[2,3] theory, and is a function of the assumed deuteron D-state probability. For assumed D-state probabilities of 3, 5, and 7%, the quoted cross sections should be multiplied by factors of 0.992, 1.000, and 1.008, respectively.

Second, the radiative correction is important. The correction for hydrogen was taken from the work of Meister and Yennie.[6] The deuterium radiative correction assumed that the quasielastic electron peak was comprised of a collection of delta functions, each with its own radiative tail identical to the radiative tail calculated in the equivalent hydrogen case.

Finally, there was contamination due to pion electroproduction. Because the nucleons within the deuteron are in motion, a larger fraction of these events appeared within the momentum acceptance for the deuteron-scattering case than for the hydrogen-scattering case. A theoretical calculation of the N^* excitation was made using the work of Adler.[7] After normalizing the N^* shape to the observed peak excitation, the N^* subtraction itself was a 15% correction to the deuterium ($\sigma_{p\text{-D}}$) data at $q^2 = 70$ F^{-2}, but fell rapidly with decreasing q^2 and was negligible below 20 F^{-2}.

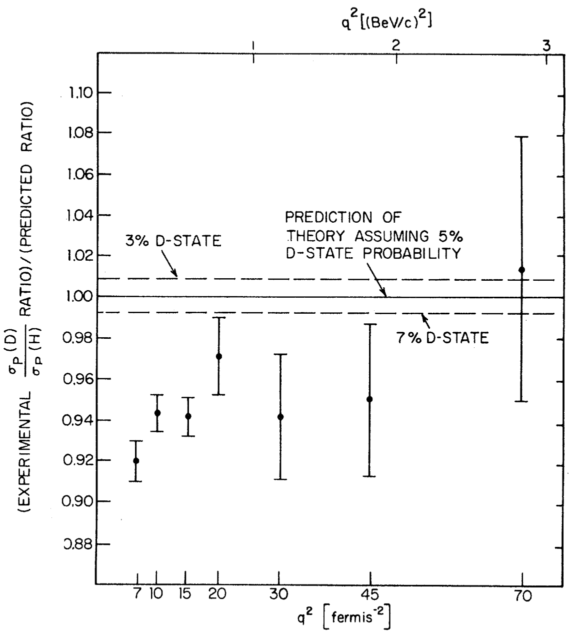

FIG. 1. Ratio $\sigma_{p\text{-}D}/\sigma_{p\text{-}H}$ of deuterium to hydrogen proton coincidence cross sections.

The measured quantities are presented in Table I. Figure 1 shows that $\sigma_{p\text{-D}}$ is found to be systematically lower than $\sigma_{p\text{-H}}$.

Several other comparisons with the theory have been made. First, the recoil-proton angular distributions for electrons at the top of the quasielastic peak were found to agree with the Durand-McGee[2,3] predictions at all momentum transfers except 7 F^{-2}, where there were significantly fewer protons than predicted in the tails of the angular distribution.

Second, the electron quasielastic momentum distributions were found to be very slightly narrower than predicted by the theory for the $q^2 = 7$, 10, and 15 F^{-2} points.

Table I. Summary of the data.

q^2 (F^{-2})	q^2 [(BeV/c)^2]	Electron angle (deg)	σ_n/σ_p coincidence method at top of peak	$\sigma_{\mathrm{all}\,e}(\mathrm{D})/\sigma_{\mathrm{all}\,e}(\mathrm{H})$ Area method	$\sigma_p(\mathrm{D})/\sigma_p(\mathrm{H})$ Area method
7	0.272	20	0.246 ± 0.0063	1.184 ± 0.0102	0.919 ± 0.008
10	0.389	20	0.278 ± 0.0069	1.239 ± 0.0131	0.943 ± 0.010
15	0.583	20	0.303 ± 0.0085	1.263 ± 0.0136	0.941 ± 0.011
20	0.778	20	0.361 ± 0.0130	1.353 ± 0.0258	0.971 ± 0.019
30	1.167	20	0.396 ± 0.0142	1.379 ± 0.0414	0.941 ± 0.030
45	1.751	20	0.435 ± 0.0145	1.395 ± 0.0607	0.950 ± 0.034
70	2.723	20.16	0.390 ± 0.0326	1.458 ± 0.111	1.015 ± 0.066
15	0.583	90	0.418 ± 0.0628	1.637 ± 0.203	1.026 ± 0.135

Third, at $q^2 = 7$ and 10 F^{-2}, a significant excess of (e, not p) electron events was found at the lower q^2 points on the threshold side of the quasielastic peak. This excess was approximately twice that expected from the already known[8] elastic electron-deuteron process.

Fourth, measurements were made of the ratio (e, not p)/($e+p$) for various final electron scattered energies E'; the variation of this ratio with E' should be predicted correctly by the theory. Agreement was found at the higher momentum transfer points, but at $q^2 = 15$ F^{-2} and below, a significant excess was observed in the ratio (e, not p)/($e+p$) in the regions both above and below the peak.

Thus, while we have full confidence in our experimental results, the extraction of neutron cross sections and neutron electromagnetic form factors from the data is questionable until a better theoretical treatment is available to fit the data. The observed anomalies might be due to final-state interactions which have not yet been completely calculated. At q^2 of 20, 30, 45, and 70 F^{-2}, the data are in agreement with the theory, except for the systematically low values for the ratio $\sigma_{p\text{-D}}/\sigma_{p\text{-H}}$.

Despite the anomalies, it is nevertheless important to discuss the implications of these data in terms of nucleon form factors. The data will be compared with the postulated "scaling law" for nucleon form factors, which takes the following form:

$$G_{Ep}(q^2) = G_{Mp}(q^2)/\mu_p = G_{Mn}(q^2)/\mu_n.$$

There is no "scaling law" for $G_{En}(q^2)$. The following possibilities will be discussed:

$$G_{En}(q^2) = 0, \qquad \text{(i)}$$

$$G_{En}(q^2) = -(q^2/4M^2)G_{Mn}(q^2) \qquad \text{(ii)}$$

[i.e., $F_{1n}(q^2) = 0$].

The former disagrees with experiments[9] on the slope of G_{En} just above $q^2 = 0$, while the latter gives very large σ_n/σ_p ratios at high momentum transfers. A reasonable guess about the behavior of $G_{En}(q^2)$ is that it might begin at $q^2 = 0$ like (ii), and go over to (i) at high momentum transfers.

For example, we note that the noncoincidence data are all reasonably consistent with the following ad hoc analytic form for $G_{En}(q^2)$, assuming the scaling law for the other three form factors:

$$G_{En}(q^2) = -[\tau/(1+4\tau)]G_{Mn}(q^2), \qquad \text{(iii)}$$

where $\tau = q^2/4M^2$.

In Fig. 2 the "area-method" ratio $[\sigma_{\text{all}\,e}(\text{D})/\sigma_{\text{all}\,e}(\text{H})]$ is shown together with the quantity $1+\sigma_n/\sigma_p$, where the ratio σ_n/σ_p is taken from the coincidence data at the top of the quasielastic peak. Also shown are the scaling-law predictions with the different assumptions on G_{En}.

The original hope was that the coincidence data at the top of the quasielastic peak would be more reliably interpretable in terms of free-neutron cross sections. Unfortunately, Fig. 2 shows that the coincidence-method ratios σ_n/σ_p are much too high at our lowest momentum transfers. However, the area-method ratios $\sigma_{\text{all}\,e}(\text{D})/\sigma_{\text{all}\,e}(\text{H})$ seem to give values which behave reasonably in the low-q^2 region. As the momentum transfer increases, the two methods give more nearly identical answers, as expected: Problems with the impulse-approximation theory should diminish at high momentum transfers.

We suspect that the problem with the low-q^2 coincidence-data points lies in weaknesses

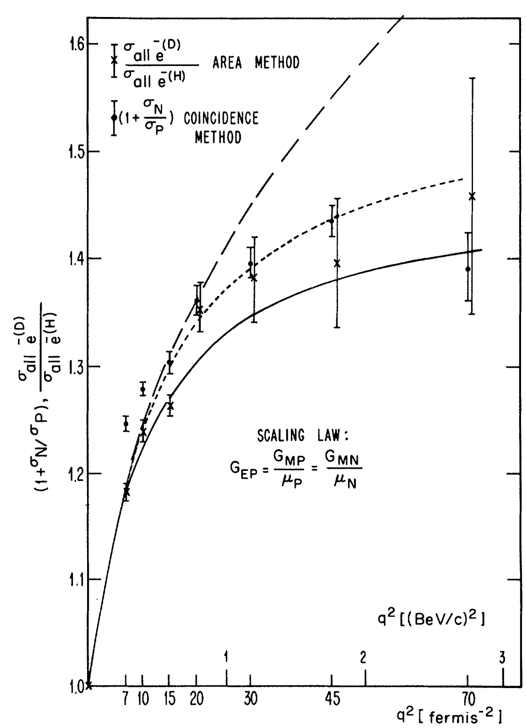

FIG. 2. Comparison of data with scaling law + G_{En} assumption: (i) solid line $G_{En} = 0$; (ii) dashed line $G_{En} = -\tau G_{Mn}$; (iii) dotted line $G_{En} = -[\tau/(1+4\tau)]G_{Mn}$.

in the deuteron theory. The noncoincidence data may be subject to larger or smaller errors. We note, however, that the low-q^2 noncoincidence data agree more closely with our preconceived ideas about a reasonable ratio σ_n/σ_p. The theoretical uncertainties in both measurements should decrease with increasing q^2, and indeed, the measured values of the two techniques do come together at high momentum transfer. Therefore, we think that the high-q^2 points can be trusted, but possibly only to the extent of the (larger) errors on the noncoincidence measurements.

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CURRENT ALGEBRAS AND POLE DOMINANCE APPLIED TO THREE-POINT FUNCTIONS*

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We demonstrate that the "hard-pion" process $A_1 \to \rho\pi$ can be correctly calculated by "soft-pion" techniques. The difficulty in earlier treatments by these methods is shown to be due to the fact that dispersing with an inappropriate invariant fixed omits important pole contributions. We criticize the derivation of the Kawarabayashi-Suzuki relationship.

In this note we show that, when calculating matrix elements via dispersion relations, one must properly include singularities in all variables. Dispersing with an inappropriate invariant fixed (e.g., q^2) can omit important pole contributions, e.g., those arising from terms proportional to $\delta(q^2-M^2)$ in the absorptive part.

By taking account of this fact, we are able to resolve the problem of the calculation of the A_1 width by "soft-pion" methods. The original application of conventional current-algebra and pole-dominance techniques to three-point functions led to a width which was far too large.[1] This has been a difficulty with the interpretation that the A_1 resonance at 1080 MeV is a chiral partner of the ρ meson.[1-3] Using a phenomenological Lagrangian which gives many of the current-algebra results, Schwinger has obtained a more reasonable A_1 width.[3] More recently Schnitzer and Weinberg[4] have obtained similar results by applying the pole-dominance assumption to Ward identities derived from the current algebras.[2] They suggest that the conventional approach (called by them the "ordinary" or "soft-pion" method) does not work because the pion in $A \to \rho\pi$ is not "soft."

It is the purpose of this note to point out (a) that from a careful application of the standard techniques one can obtain the Schwinger, Schnitzer-