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[10]In this case, the denominator in Eq. (6) is finite and interchange of limits is permissible. However, one is still unable to determine Z_3^{-1} from the anticommutator of ψ_V and $\psi_V{}^\dagger$ by use of Eq. (6) for a different reason, namely ψ_V can no longer be treated as an independent field variable in the Lagrangian, as discussed in Ref. 8.

VECTOR CURRENTS AND SPECTRAL-FUNCTION SUM RULES*

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It is argued, on experimental grounds, that Weinberg's second sum rule must be abandoned for vector-current spectral functions as long as the vector-meson dominance approximation is qualitatively valid.

Motivated by the initial successes[1,2] of Weinberg's two sum rules for the spectral functions of chiral SU(2)$\otimes$SU(2) symmetry, attempts have been made to extend the same techniques to the spectral functions of vector currents with different isospins and hypercharges.[3,4] We wish to show that, if the vector-meson dominance approximation is qualitatively valid, there are good experimental reasons to reject Weinberg's second sum rule applied to comparisons among the various vector currents of the broken eightfold way.

We define the spectral functions $\rho^{(\alpha)}(m^2)$ via [1-5]

$$\Delta_{\mu\nu}{}^{\alpha}(q) = i\int d^4x\, e^{-iq\cdot x}\langle 0|T[j_\mu{}^\alpha(x) j_\nu{}^\alpha(0)]|0\rangle \quad (\alpha \text{ not summed})$$
$$= \int dm^2 \frac{\rho^{(\alpha)}(m^2)[\delta_{\mu\nu} + (q_\mu q_\nu/m^2)]}{(q^2+m^2-i\epsilon)} - \delta_{\mu 4}\delta_{\nu 4}\int dm^2 \frac{\rho^{(\alpha)}(m^2)}{m^2}, \tag{1}$$

where α is a unitary spin index that may run from 1 to 8. The above representation is valid as long as the relevant current is conserved, hence for $\alpha = 1$, 2, 3, and 8; for the strangeness-changing currents ($\alpha = 4$, 5, 6, and 7) we expect, in general, an additional contribution due to scalar excitations which we have not written down explicitly. Applied to the isospin and ($\frac{1}{2}\sqrt{3}$ times) hypercharge currents, the two sum rules of Weinberg[1] read

$$\int dm^2 \frac{\rho^{(3)}(m^2)}{m^2} = \int dm^2 \frac{\rho^{(8)}(m^2)}{m^2} \tag{2}$$

and

$$\int dm^2 \rho^{(3)}(m^2) = \int dm^2 \rho^{(8)}(m^2). \tag{3}$$

In the vector-meson dominance approximation (in which only ρ, ω, and φ are kept), these sum rules can be written as follows:

$$(m_\rho/f_\rho)^2 = \tfrac{3}{4}[(m_\varphi/f_Y)^2\cos^2\theta_Y + (m_\omega/f_Y)^2\sin^2\theta_Y], \tag{4}$$

$$(m_\rho{}^2/f_\rho)^2 = \tfrac{3}{4}[(m_\varphi{}^2/f_Y)^2\cos^2\theta_Y + (m_\omega{}^2/f_Y)^2\sin^2\theta_Y], \tag{5}$$

where the constants $1/f_\rho$, $\cos\theta_Y/f_Y$, $\sin\theta_Y/f_Y$ (which are all measurable from $\rho, \varphi, \omega \to e^+ + e^-$) are defined through the current-field identities[6]

$$j_\mu{}^3 = (m_\rho{}^2/f_\rho)\rho_\mu{}^3,$$

$$j_\mu{}^8 = (\sqrt{3}/2f_Y)(\cos\theta_Y m_\varphi{}^2\varphi_\mu - \sin\theta_Y m_\omega{}^2\omega_\mu). \quad (6)$$

Equations (4) and (5) lead to

$$\tan^2\theta_Y = \left(\frac{m_\varphi{}^2}{m_\omega{}^2}\right)\frac{(m_\rho{}^2 - m_\varphi{}^2)}{(m_\omega{}^2 - m_\rho{}^2)}. \quad (7)$$

This clearly contradicts the experimental spectrum

$$m_\varphi > m_\omega > m_\rho. \quad (8)$$

We are therefore inclined to the view that as long as ρ, ω, φ dominance holds, Weinberg's second sum rule must be rejected.[7]

We may, however, be generous at this point and admit the possibility $m_\omega = m_\rho$ since experimentally ω and ρ are nearly degenerate. This leads to

$$\theta_Y = \tfrac{1}{2}\pi \text{ (or } -\tfrac{1}{2}\pi) \quad (9)$$

which means that there is no $\omega\varphi$ mixing, ω being the "pure" $T=0$ member of the vector meson octet. Now in the approximation that the isoscalar electromagnetic and the "baryonic charge" form factors of the K meson are dominated by ω and φ, the φ decay width is given by[8]

$$\Gamma(\varphi \to K + \bar{K}) = \tfrac{4}{3}(f_Y{}^2/4\pi)[\cos\theta_B/\cos(\theta_Y - \theta_B)]^2 \times (p_{K\bar{K}}{}^3/m_\varphi{}^2), \quad (10)$$

where θ_B is defined through the current-field identity for the baryon current[6]

$$j_\mu{}^{(B)} = (1/f_B)(\sin\theta_B m_\varphi{}^2\varphi_\mu + \cos\theta_B m_\omega{}^2\omega_\mu). \quad (11)$$

Regardless of whether $\omega\varphi$ mixing is representable by a phenomenological Lagrangian of the form $\omega_\mu{}^{(B)}\omega_\mu{}^{(Y)}$ (the "mass-mixing model"[9] which requires $\theta_Y = \theta_B$) or by a term of the form $\omega_{\mu\nu}{}^{(B)}\omega_{\mu\nu}{}^{(Y)}$ (the "current-mixing model"[10] which requires $m_\omega{}^2\tan\theta_Y = m_\varphi{}^2\tan\theta_B$), Eq. (9) demands that $\Gamma(\varphi \to K + \bar{K})$ be zero. Experimentally φ decay into a $K\bar{K}$ pair is fully allowed; the observed φ width is completely consistent with the usual theory of $\omega\varphi$ mixing (the mass-mixing model or the current-mixing model).

Further evidence against $\theta_Y = \frac{1}{2}\pi$ (or $-\frac{1}{2}\pi$) comes from the leptonic decay of the φ meson. Recently two independent experiments[11,12] have conclusively established the existence of the decay mode

$$\varphi \to e^+ + e^-, \quad \mu^+ + \mu^-. \quad (12)$$

Even though neither experimental group was able to establish a firm branching ratio for this decay mode (because the φ production cross section at the relevant energy is not known), very crude estimates[13] indicate that the φ meson contribution to (4), or equivalently to the sum rule[14]

$$\tfrac{1}{3}m_\rho\Gamma(\rho \to l^+l^-) = m_\omega\Gamma(\omega \to l^+l^-) + m_\varphi\Gamma(\varphi \to l^+l^-), \quad (13)$$

is not negligible. It therefore appears unreasonable to leave out the φ meson in the spectral function $\rho^{(8)}$, as done in Ref. 3.

Let us now turn our attention to the strangeness-changing currents and compare $\rho^{(3)}$ with $\rho^{(4,\,5,\,6,\text{ or }7)}$. It has already been noted[3,5] that the two sum rules of Weinberg cannot be simultaneously satisfied in the pole approximation unless there is a scalar (κ) meson, or an important scalar enhancement in the $T = \frac{1}{2}$, $Y = \pm 1$ channel (otherwise K^* would be degenerate with ρ). If we take the point of view that both sum rules are valid, we must relate the K^* width to the ρ width using Weinberg's second sum rule (which does not involve the κ contribution). The result is

$$\Gamma(K^* \to K + \pi)/\Gamma(\rho \to \pi + \pi) = (\tfrac{3}{4})(m_{K^*}/m_\rho)^2(p_{K\pi}/p_{\pi\pi})^3, \quad (14)$$

where we assumed the nonrenormalization of the strangeness-changing currents[15] and the vector-meson dominance of the $\pi\pi$ and $K\pi$ form factors.[16] On the other hand, if we abandon Weinberg's second sum rule, there is no need to introduce κ, and we can establish a relation between the K^* width and the ρ width using Weinberg's first sum rule as follows[17]:

$$\Gamma(K^* \to K + \pi)/\Gamma(\rho \to \pi + \pi) = \tfrac{3}{4}(p_{K\pi}/p_{\pi\pi})^3. \quad (15)$$

Numerically, for $\Gamma(\rho) = 128 \pm 5$ MeV[18] we get $\Gamma(K^*) = 69 \pm 3$ and 50 ± 2 MeV from (14) and (15), respectively. The experimental value 49.8 ± 1.7 MeV appears to be in excellent agreement with

(15) and differs significantly from (14) derived from Weinberg's second sum rule.

In closing we would like to make a few comments.

(i) As long as we saturate the vector-current spectral functions by the experimentally known vector mesons, we are unable to avoid the conclusion that Weinberg's second sum rule is violated. (To save the sum rule we may perhaps introduce another vector meson octet or abandon vector-meson dominance altogether; in such a case, however, there would be no reason to believe in the derivation of Weinberg's mass relation[1] $m_{A_1}=\sqrt{2}m_\rho$ which sparked the whole series of investigations.)

(ii) The convergent calculation of $K\rightarrow 2\pi$ (that appeared recently in this journal[4]) rests heavily on the validity of Weinberg's second sum rule applied to the broken eightfold way (as well as on the validity of $\rho, K^*, A_1, K_A^*, \kappa, \pi, K$ dominance). Even if future experiments at Serpukhov and Weston indeed establish the existence of an intermediate boson at $m=8$ GeV, there is no reason to take seriously the reasoning presented in Ref. 4.

(iii) If the success of Weinberg's mass relation[1] is not accidental, we may be inclined to argue that there is a profound difference in the manners in which chiral SU(2) ⊗ SU(2) and the eightfold way are broken.

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Note added in proof.—After this paper was submitted for publication, the authors of Refs. 3 and 4 have pointed out that the deviation of Weinberg's second sum rule from the gauge field theory is on completely secure grounds for chiral SU(2)⊗SU(2) whereas the same cannot be said about Weinberg's second sum rule as applied to the broken eightfold way. The difference arises because in the SU(3) case the derivation must rely on the vanishing of the vacuum expectation value of the last term of Eq. (11) of Lee, Weinberg, and Zumino[19] (which may perhaps be justified by appealing to the ξ limiting process) while this question does not arise in the chiral SU(2)⊗ SU(2) case. The author is indebted to Professor H. Schnitzer and Professor S. Weinberg for informative discussions.

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