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LEVELS EXCITED IN ^{206}Pb WITH BOTH TWO-NEUTRON STRIPPING AND PICKUP REACTIONS*

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Single-nucleon stripping and pickup reactions to the same final nucleus have long been used to gain information about the amplitudes of a few configurations in the wave functions of each final state. Now that reliable two-nucleon stripping calculations^{1,2} are available, it is possible to test both the phases and amplitudes of many configurations in a proposed wave function for the final nucleus. To demonstrate this technique, a high-resolution study of levels in ^{206}Pb excited in the (t, p) and (p, t) reactions is reported here. The observed cross sections are compared with the predictions of two-nucleon stripping calculations using the wave functions calculated for ^{206}Pb by True and Ford,³ by Miranda,² and by Broglia and Riedel.²

The experimental data were obtained using a magnetic spectrograph of the Elbek design

in conjunction with 20-MeV triton and 22-MeV proton beams from the Los Alamos tandem Van de Graaff facility. The energy resolution of the experiments was 0.06%, or 10 keV for the tritons and 19 keV for the protons. Exposures were made at spectrograph angles of 20°, 35°, and 65° and the results were averaged to decrease the effects of angular variations in the magnitudes of the relative cross sections. Absolute cross sections were not measured, and for purposes of comparison the ground-state yields of the two reactions were normalized to each other after the angular averaging. The result of this procedure produces a rough estimate of the relative total cross sections to the various observed levels. The energy assignments from the two reactions agree within ± 3 keV and are listed in Table I.

Table I. List of experimental energy levels as measured in either of the two reactions discussed. These are compared with energies and J^π values calculated in Ref. 3 and with known J^π values [Nuclear Data Sheets, compiled by K. Way et al. (Printing and Publishing Office, National Academy of Sciences—National Research Council, Washington, D. C.)].

Level No.	E_x from (p,t) and (t,p) (MeV)	J^π	True and Ford Wave Functions		Level No.	E_x from (p,t) and (t,p) (MeV)	J^π	True and Ford Wave Functions	
			J^π	Energy				J^π	Energy
0	0	0^+	0_1^+	0.000	15	2.776	5^-	5_1^-	2.809
1	0.803	2^+	2_1^+	0.725	16	2.800			
2	1.165	(0^+)	0_2^+	1.363	17	2.924		4_3^+	3.010
3	1.338	3^+			18	3.010		5_2^-	3.063
4	1.464	(2^+)	2_2^+	1.391	19	3.116			
5	1.577				20	3.128	(6^+)		
6	1.682	4^+	4_1^+	1.675	21	3.191			
7	1.785		2_3^+	1.767	22	3.253		6_1^+	3.154
8	1.997	(4^+)	4_2^+	2.013	23	3.383			
9	2.149		2_4^+	2.190	24	3.445			
10	2.197	(7^-)	7_1^-	2.166	25	3.511			
11	2.314		0_3^+	2.056	26	3.595			
12	2.421		2_5^+	2.527	27	3.610			
13	2.650		9_1^-	2.600	28	3.755			
14	2.653	3^-			29	3.805			

A dramatic difference in the spectra obtained in the two reactions is shown in Fig. 1. Since ^{206}Pb is only two neutron holes outside of the doubly closed shells of ^{208}Pb , its levels are strongly populated in the pickup reaction $^{208}\text{Pb}(p, t)$. However, in the reaction $^{204}\text{Pb}(t, p)$ the target ground state must be described as four neutron holes in the closed shell with the $(3p_{1/2})^{-2} \times (2f_{5/2})^{-2}$ configuration predominating.² This limits considerably the variety of two-hole states in ^{206}Pb that can be excited in the (t, p) reaction as is clearly demonstrated in Fig. 1. Only a few states below 2.653 are strongly populated in contrast to the results of the pickup reaction illustrated in Fig. 1. Broglia and Riedel² have calculated the $^{204}\text{Pb}(t, p)^{206}\text{Pb}$ cross sections at 12 MeV from the 0_1^+ , 0_2^+ , and 2_1^+ states, which have previously been reported,⁴ and strength factors G_N for seven other states using two-hole wave functions of Miranda, and

these compared with the data in Fig. 2(a). The calculated cross sections have not been corrected for the difference in bombarding energy since the ratios of low-spin cross sections are expected to be relatively energy independent. The intensities plotted in Fig. 2(a), which are based on strength factors, must be considered as crude estimates of the cross sections, but they predict small cross sections in agreement with experiment.

The majority of states excited in the (t, p) reaction will be one-particle, three-hole states; each such state will have an analog in the ^{208}Pb scheme. Broglia and Riedel² have also calculated cross sections at 12 MeV for the 3^- state at 2.653 MeV and the 5^- state at 3.805 MeV using their one-particle, three-hole wave functions. Although the agreement is not as good as with the two-hole states, the strong excitation is qualitatively predicted. The apparent

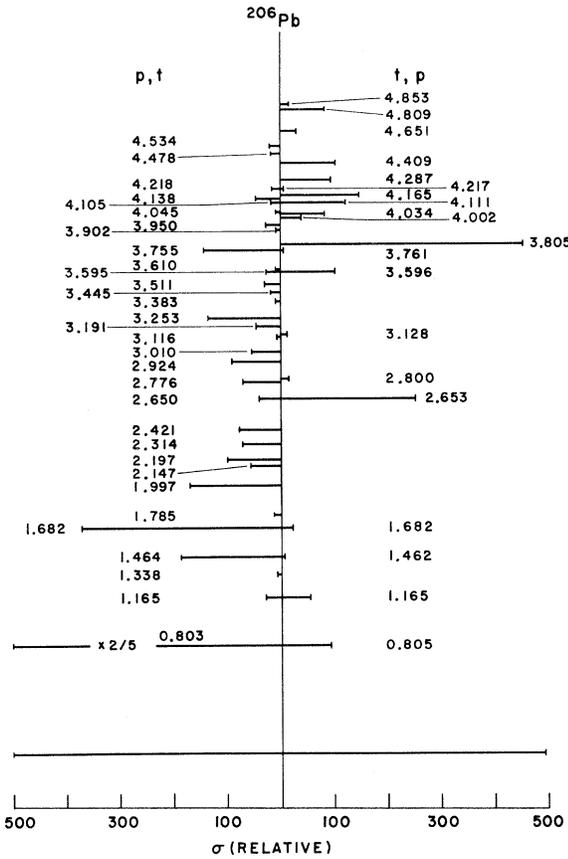


FIG. 1. Comparison of relative cross sections for the reactions $^{204}(t,p)^{206}\text{Pb}$ and $^{208}\text{Pb}(p,t)^{206}\text{Pb}$. The length of each line is proportional to the average of experimental data taken at 20°, 35°, and 65°. The ground-state magnitudes for the two reactions have been set equal to each other.

disagreement with the one-particle, three-hole state at 3.805 MeV is in part attributed to the expected increase in cross section of a 5^- level due to the increased momentum transfer available at 22 MeV. Most of the levels shown in Fig. 2(a) were previously unreported.

Similar calculations for the (p,t) reaction have been carried out by one of us (N.K.G.) for $E_p = 22$ MeV using the True and Ford³ wave function for ^{206}Pb . Data at $E_p = 40$ MeV have been reported⁵ and the ground state and five excited states were observed. Many additional levels have recently been observed at $E_p = 22$ MeV in the present spectrograph measurements with 10-keV resolution⁶ and with 60-keV resolution.⁷ We have compared the calculations and experimental spectra in Fig. 2(b) using known spin assignments where possible and otherwise inferring the spin from the work of

Ref. 3. Table I lists the observed levels and gives the identification with the True and Ford wave functions used here.

Quite reasonable agreement with the calculations is obtained. Of some particular interest are the states at 1.338 and 2.650 MeV. The former is a known 3^+ state and the present observation of the weak excitation of that state indicates that the usual selection rule of $S=0$ or $\Delta\pi = (-1)^L$ for the (p,t) reaction¹ is not absolutely held. Here L and S are the orbital and spin angular momentum of the transferred pair of neutrons. The latter state occurs within 3 keV of the 3^- state seen in the (t,p) reaction. The 9^- assignment is suggested by the calculations of True and Ford only. If this should be the same 3^- level seen in the (t,p) reaction, it suggests that a $(2g_{9/2})^2$ component is present in the ground state of ^{208}Pb or that the $[(1i_{1/2})^{-1} \times (2f_{7/2})^{-1}]$ two-hole state of ^{206}Pb is admixed with the predominantly three-particle, one-

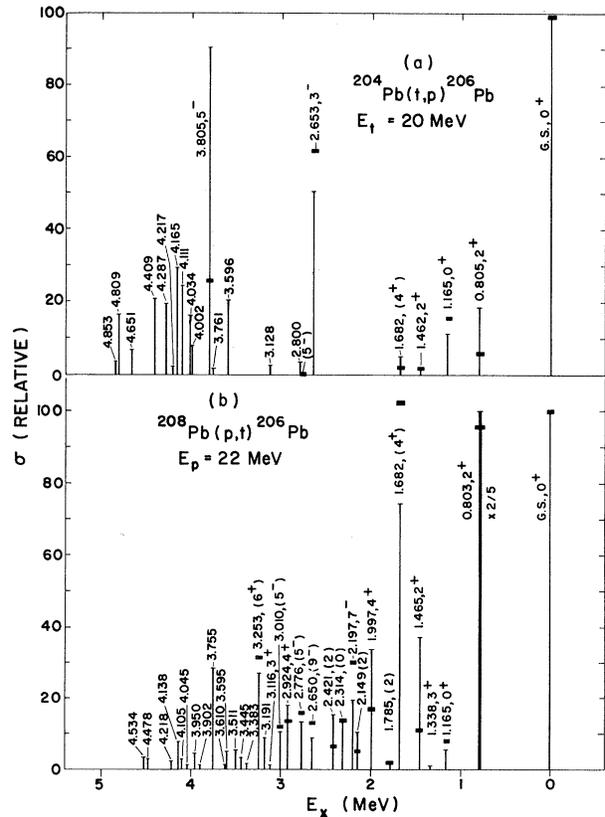


FIG. 2. Comparison of distorted-wave theory with experimental data. The data are as discussed in Fig. 1. The distorted-wave magnitudes are shown as heavy black lines and have been normalized to the ground-state transitions for each case.

hole 3^- state.

The over-all agreement of the predicted relative yields of the five two-hole excited states compared with both the (p, t) and (t, p) data [see Figs. 2(a) and 2(b)] suggests that the intensities and phases of the relevant configuration mixing in the True and Ford and the Miranda wave functions is for the most part correct. Furthermore, the two-nucleon stripping reaction appears to be adequately treated and is a useful tool for the study of nuclear structure.

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INEQUIVALENCE OF $\Phi_C = \Phi$ AND $Z_3 = 0$ †

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It is pointed out that the vanishing of the wave-function renormalization constant associated with a field operator Φ is not implied by the existence of a representation of Φ as a weak limit of a nonlocal product of other fields.

The question of the value of the renormalization constant Z_3 associated with a field Φ has been considered recently in a certain class of field theories. These theories are characterized by the possibility of obtaining Φ as a (weak) limit of a product of other fields in the theory,¹ e.g., the identification of Φ with Φ_C , defined by

$$\Phi_C(x) = \lim_{\xi \rightarrow 0} \Phi_\xi(x),$$

$$\Phi_\xi(x) \equiv \frac{\varphi(x+\xi)\varphi(x) - \langle 0 | \varphi(x+\xi)\varphi(x) | 0 \rangle}{\langle 0 | \varphi(\xi)\varphi(0) | 1 \rangle}, \quad (1)$$

where Φ and φ are scalar fields, $|1\rangle$ denotes a physical Φ -particle state, and the limit is taken in a spacelike direction. Several authors²⁻⁴ have recently attempted to demonstrate that the condition $Z_3 = 0$ is a consequence of the relation

$$\Phi_C(x) = \Phi(x). \quad (2)$$

The purpose of this note is to point out that the manipulations required in these arguments are unjustified, and that in general no conclu-

sion can be drawn regarding $Z_3 = 0$ by the use of Eq. (2) alone.

We first note that there are models where $Z_3 \neq 0$ but Eq. (2) nevertheless holds. The difficulty with the proof that (2) implies $Z_3 = 0$ lies in an interchange in the order of taking limits, which is in general not valid, as can be seen from simple counterexamples.

One recalls that Nishijima proved that a sufficient condition for Eq. (2) to hold in a wide class of theories is for the self-energy associated with the Φ field to diverge more strongly than any possible divergence of the vertex.⁵ This condition clearly does not demand that the corresponding wave-function renormalization constant must vanish. In fact, in a super-renormalizable theory such as the $\Phi\varphi\varphi$ model, in every order, Z_3^{-1} is finite but the self-energy of Φ diverges more strongly than the vertex, so that Eq. (2) still holds, at least in perturbation theory.

In Ref. 2 the vacuum expectation value of the equal-time commutator of Φ_C with $\dot{\Phi}_C$ is evaluated by using Eq. (1) before taking the limits indicated in Eq. (1). The limits were taken at the end, and the following formal expression