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COLLISIONLESS DAMPING OF LARGE-AMPLITUDE PLASMA WAVES*

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We report preliminary results of an experiment designed to compare the collisionless damping of small- and large-amplitude plasma waves.

The damping in time or in space of small-amplitude plasma waves in a collisionless plasma was predicted by Landau.[1] In this perturbation theory, the electric field of the wave is assumed to be a small quantity, and the collisionless Boltzmann equation is linearized by neglecting $\partial f_1/\partial v$ compared with $\partial f_0/\partial v$ (where f_0 and f_1 are the "equilibrium" and "perturbed" parts of the electron velocity distribution function). In essence, the theory includes the effect of the time-averaged electron velocity distribution on the waves but not vice versa. Thus the theory does not apply to large-amplitude waves. This linearized theory has been experimentally tested in detail.[2-7]

The damping in time of large-amplitude electron plasma waves has been treated theoretically for the case of very small damping by O'Neil[8] and by Al'tshul' and Karpman.[9] In these theories, the collisionless Boltzmann equation is linearized by assuming that the amplitude of the wave electric field is almost constant in time. Thus the effect of the electric field on the time-averaged electron velocity distribution is included, but not vice versa. The theory of Landau predicts exponential damping of the waves. The theories of O'Neil and Al't-shul' and Karpman differ from each other in detail but both predict oscillations in the wave amplitude on a time scale long compared with the wave period. Physically, these oscillations are associated with energy exchange between the wave and the resonant electrons, which are oscillating back and forth in the troughs of the wave. The present experiment deals with spatial damping of large-amplitude waves at a phase velocity where moderately heavy damping is normally observed. The case of moderately damped waves has been investigated by Knorr,[10] Gary,[11] and Armstrong[12] but these theories consider only temporal, not spatial, damping and in addition do not emphasize the oscillations in the wave amplitude. To the authors' knowledge, the spatial case has not been treated in detail theoretically. However, by analogy with existing theory and from elementary physical arguments we may expect large-amplitude waves to exhibit spatial amplitude oscillations. Defining $k_{\mathrm{osc}} = 2\pi/\lambda_{\mathrm{osc}}$, where λ_{osc} is the distance between amplitude peaks, we expect

$$k_{\mathrm{osc}} = \frac{1}{v_p}\left(\frac{ekE}{m}\right)^{1/2}, \qquad (1)$$

where v_p and k are the phase velocity of the wave and wave number of the wave, respectively, and m and e are the electron mass and charge.

The amplitude oscillations should occur if

$$\frac{k_{\mathrm{osc}}}{k_i} \gtrsim (2\pi)^{1/2}, \tag{2}$$

where k_i is the imaginary part of the wave number of a small-amplitude plasma wave at the same frequency. Exponential damping will occur if Eq. (2) is not satisfied. Finite size effects would be expected to change these results quantitatively but not qualitatively.

The apparatus used for this experiment has been previously described.[4,5,13] For the present purpose it may be considered simply a device which produces a 2-m long uniform column of collisionless plasma bounded at a radius of 5.2 cm by a good conductor. The plasma density is about 5×10^8 electrons/cc at the center and drops smoothly to zero at the wall with a half-maximum radius of about 1 cm. The plasma is immersed in a strong longitudinal magnetic field. The electron velocity distribution is a Maxwellian with a temperature of about 7 eV. The background is $\sim2\times10^{-5}$ Torr (mostly H_2). Hence the Debye length is about 1 mm; the electron mean free path for electron-ion collisions is of the order of 1000 m and for electron-neutral collisions is of the order of 40 m. For the present electron-wave experiments extending over a length of about a meter, the plasma is collisionless in the sense of the theory.

When an rf voltage is applied to a probe inserted into this plasma, electron plasma waves are excited which propagate in both directions along the plasma column. While many radial eigenmodes of the plasma are excited, only the lowest radial eigenmode is observable for appreciable distances from the antenna when the frequency is sufficiently high, since all higher modes are very heavily damped. In previous experiments[5] we have measured the dispersion of these waves. At small wave numbers the dispersion is dominated by the finite radial size of the system. At large wave numbers the dispersion is almost the same as for an infinite-size, finite-temperature plasma, and is accurately predicted by the theory of Landau. The damping of these waves has also been measured[3,4] and it was shown that heavy exponential damping is observed, even though collisional damping is negligible under these circumstances; that the damping is caused by electrons traveling at the phase velocity of the wave; and that the magnitude of the damping, its dependence on phase velocity, and its dependence on plasma temperature are accurately predicted by the theory of Landau. Thus small-amplitude electron plasma waves in this maching are very well understood, and we may limit the present discussion to changes which take place as the wave amplitude is increased.

For wave transmission measurements, two radial probes are used as antennas. The probes may be moved the full length of the plasma in a slot in the surrounding tube. A detector is built into the base of the two probes to monitor the rf voltage level when they are used as transmitters. One probe is connected by a coaxial cable to a chopped signal generator. The other probe is connected to a tuned receiver whose output drives a coherent detector operated at the transmitter chopping frequency. Provision is made to add a reference signal from the transmitter to the receiver rf signal, i.e., we may use the system as an interferometer. The transmitter is set at a series of fixed power levels, and at each, the receiving probe is moved longitudinally. The position of the receiving probe is transduced to the x axis of an x-y recorder, and the received power is applied to the y axis. The receiver output is approximately proportional to the input power.

Typical raw data are shown in Fig. 1. For Curve A of this figure the root-mean-square rf voltage applied to the transmitting probe, V, was 0.9 V, and the usual Landau damping is observed. The sharp spike in received power where the transmitter and receiver probes cross is caused by the "near field" of the transmitter. This near field is shielded out at larger distances by the conducting tube surrounding the plasmas which acts as a waveguide beyond cutoff at this frequency (202 MHz). The

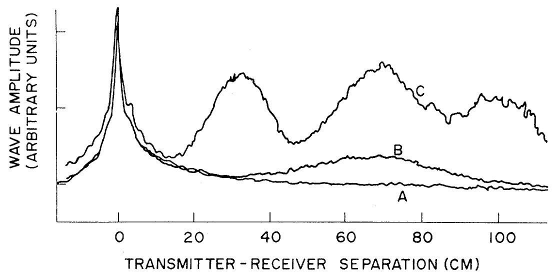

FIG. 1. Wave amplitude versus position. Transmitter voltage 0.9, 2.85, and 9 V, for Curves A, B, and C, respectively.

signal at distances greater than 5 cm from the transmitter is due to a plasma wave of wavelength 2.3 cm (measured with the interferometer setup). Its damping length for amplitude (measured using more amplifier gain and a logarithmic receiver) is 33 cm. For Curve B the wave power was increased 10 dB by increasing the rf probe voltage to 2.85 V and a 10-dB pad was inserted between the receiver probe and the first stage of the receiver, to keep the over-all system gain constant. For Curve C the wave power was increased 20 dB and a 20-dB pad was inserted in the receiver line. For the near field, these substitutions make no difference (within the accuracy to which the power was set) since the increase in power is offset by the reduction in receiver sensitivity. However, the wave no longer damps exponentially to zero; the oscillations expected from the large-amplitude theory are observed instead.

To compare the results with theory we must estimate the wave electric field corresponding to various probe voltages. For this we require the coupling constant between the probe and the plasma wave. It is obtained by measuring the total power loss between the two probes in a transmission experiment, allowing for the wave damping, and dividing by two under the assumption that both probes have the same coupling constant. Combining the coupling constant with the area of the wave eigenfunction and the wave group velocity (obtained from the dispersion curve) gives the electric field of the wave. These calculations give $E = 0.2$, 0.7, and 2.1 V/cm for Curves A, B, and C, respectively, with uncertainties of a factor of 2. The left-hand side of Eq. (2) is 2.3, 4.1, and 7.4 for Curves A, B, and C, respectively. Thus Eq. (2) predicts correctly the power level at which large-amplitude effects will appear. Using this estimate for E we may predict from Eq. (1) the value of k_{osc} for Curve C, obtaining $k_{\text{osc}} = 0.22$, which is in satisfactory agreement with the observed value of 0.17.

Using data similar to Fig. 1, we have measured k_{osc} as a function of rf potential applied to the probe. The result is shown in Fig. 2. If it is assumed that the plasma wave to probe coupling is independent of power, Fig. 2 demonstrates that k_{osc} is accurately proportional to $E^{1/2}$, as expected from Eq. (1).

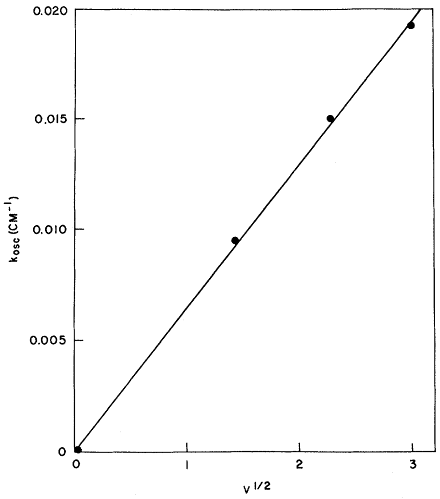

FIG. 2. Oscillation wave number versus transmitter power.

In summary, the damping of electron plasma waves is shown experimentally to be qualitatively different for small- and large-amplitude waves. Small-amplitude waves damp exponentially; large-amplitude waves exhibit amplitude oscillations for large distances. The transition between the two cases occurs at the expected wave amplitude. The wave number associated with the amplitude oscillations is proportional to the square root of the wave electric field and its absolute magnitude is correctly predicted by theory.

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SHELL STRUCTURE OF A HOT-ELECTRON PLASMA

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There have been reports[1-4] on some interesting features of hot-electron plasmas generated by microwave discharge in mirror geometries. It has been believed that the production and heating of the hot-electron plasma takes place mainly in the electron-cyclotron resonance region[5] within the mirror machine. In this Letter we report experimental evidence that, when microwave power is fed across the magnetic flux lines, the hot electrons form a closed-shell structure and that while the plasma is created mostly at the electron-cyclotron fundamental resonance, subsequent electron heating is performed at the electron-cyclotron harmonic resonance.

The hot-electron plasma device at the Institute of Plasma Physics is called TPM[6] for "Test Plasma by Microwave Discharge." The device consists of ten sectionalized coils, which permit mirror ratios from 2.5:1 to 3.5:1, and of a stainless-steel vacuum tank (30 cm diam × 40 cm length) serving as a multimode microwave cavity, which is evacuated by two vacuum pumps connected to the cavity through circular apertures (12 cm diam) in the end walls (Fig. 1).

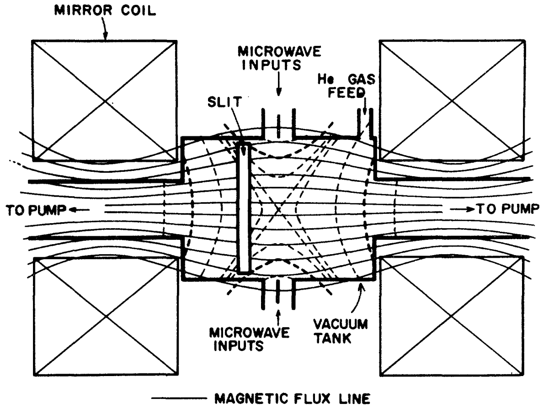

FIG. 1. Schematic diagram of the TPM machine. The bold hyperbolalike dashed lines show the second-harmonic resonance zone and those across the flux lines indicate the fundamental electron-cyclotron resonance region.

The base pressure in the vacuum tank is 6×10^{-7} Torr and relatively high-pressure helium $[(1-6)\times10^{-4}$ Torr$]$ is admitted in steady flow during operation. The 6.4-GHz microwave source for the discharge is a 20-msec pulse, having a power up to 5 kW at a repetition rate of five pulses per second. The microwave power is introduced radially through two opposite pairs of waveguide ports in the cavity wall. The plasmas produced at power levels above 3 kW are observed to be very similar to one another in electron temperature and electron density. The hot-electron temperature is determined to be about 150 keV from x-ray spectra. This temperature value is observed to be almost constant during the first 140 msec of the quiet afterglow. An R-band microwave interferometer indicates an electron density of about 2×10^{12} electrons/cm^3 during the microwave discharge.

In order to determine the radial density profile of the hot electrons of the TPM, the radial emission of x rays from the plasma is observed by scanning with an x-ray telescope along a slit in the cavity wall. Two vertical slits (14 cm × 30 cm) positioned 7 cm off the midplane in the cavity wall, facing each other diametrically, are used for observing the contained plasma along its diameter across the magnetic flux lines. The slits are covered with 5-mm-thick Pyrex glass plates. The total number of photons of the x-ray bremsstrahlung is observed with a 2-cm^2 NaI scintillator. The solid angle subtended by the crystal through the collimator is about 10^{-5} sr and it collects photons from a cylindrical volume of plasma approximately 10 cm long with a cross section of 3 cm^2. The collimation system excludes any reception of unwanted x rays from the cavity walls.

When the x-ray telescope is moved (0.05 cm/sec) along the slit, it is possible to calculate the radial distribution of x-ray emission by using Abel's transformation. This distribution