IMPLICATIONS OF THE DOUBLE-POLE CHARACTER OF THE A_2 MESON*

K. E. Lassila and P. V. Ruuskanen
Institute for Atomic Research and Department of Physics, Iowa State University, Ames, Iowa
(Received 26 July 1967; revised manuscript received 5 September 1967)

We explain the observed double-pole characteristics of the $A_2^-(1300)$ by a nondiagonalizable degenerate mass matrix for two mesons one of which decays predominantly into $\pi+\rho$. The presence of the other, possibly already observed meson would cause the A_2 characteristics to change considerably from one production process to another. From available limited statistics experiments (except for the CERN work discussed here), this appears likely.

Recent very precise measurements by the CERN missing-mass spectrometer group[1] have established that the $A_2^-(1300)$ mesons has a two-peak resonance structure which is extremely well fitted by an unusual double-pole[2] excitation function,

$$\rho(E)\propto(E-E_0)^2/[(E-E_0)^2+\Gamma^2/16]^2. \qquad (1)$$

This particular double-pole form was shown to be physically realizable in nature by the present authors in a considerably different context.[3] In this paper we study some of the consequences which follow from this special type of double-pole mass distribution for an "elementary" particle.

In the missing-mass experiment, high-energy π^- mesons bombard a hydrogen target and the angle and momentum of the recoil proton are measured. We represent this process by the general diagram in Fig. 1, where the presence of the A_2 (two lines labeled by M) is deduced from measurements on the final-state proton p_2. The observed A_2 ($I^GJ^P=1^-2^+$) decays by strong interactions predominantly[4] (93 %) into $\pi+\rho$, but a general decay state d (with only two decay products for convenience) is shown. The four-momenta and polarization label each line ($\hbar=c=1$). The transition amplitude for the process in Fig. 1 will be written as

$$\langle k_1\lambda_{d_1};k_2\lambda_{d_2};p_2\lambda_2|T|q;p_1\lambda_1\rangle\equiv T_{fi}(s,t)=[\bar{F}_{dM}(k_1\lambda_{d_1};k_2\lambda_{d_2})]^{\alpha}[D_M(\Delta^2)]_{\alpha\alpha'}[\bar{G}_{Mp_2;i}(p_2\lambda_2;p_1\lambda_1;q)]^{\alpha'}. \qquad (2)$$

The particle helicity labels λ_j and α, α' for the A_2 components are written explicitly but will be subsequently suppressed as they do not play any role in what follows. Here $\bar{G}_{Mp_2;i}(p_2;p_1;q)$ is the amplitude for producing a meson M of four-momentum $\Delta=q+p_1-p_2$ in a polarization state α' from the π^-p_1 initial state i, $D_M(\Delta^2)$ is the propagator for M, and $\bar{F}_{dM}(k_1;k_2)$ is the decay vertex function for meson M in a polarization state characterized by α.

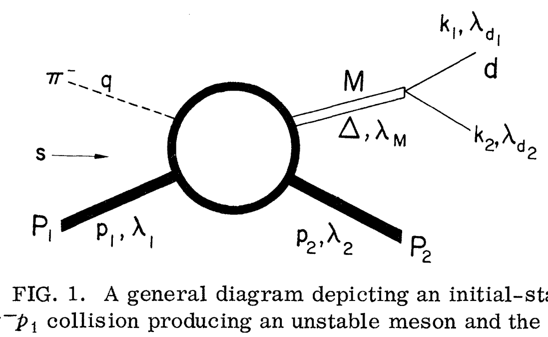

FIG. 1. A general diagram depicting an initial-state π^-p_1 collision producing an unstable meson and the recoil proton p_2. The lines are labeled by four-momenta and polarization, or z-axis spin-projection, quantum number.

The propagator for a stable high-spin meson given, e.g., by Weinberg[5] has a matrix numerator (projection operator) that operates in the space of spin components of the wave function. Following Coleman and Schnitzer's[6] study of spin-one mesons, one can make the necessary modifications for particles with mass distributions and include mixing so that the mass term in the propagator becomes a mass matrix. Rivers[7] does just this for spin-2 mesons and includes also the effects of renormalization due to the mixing. Using these authors' results, we can readily incorporate the spin-matrix numerator of the propagator into the vertices on either side of D_M (indicated by dropping the bars on F and G) and make the pole approximation[6] since we are interested in a very limited energy range of the mass distribution.

In S-matrix or field theory the diagram of Fig. 1 will give a pole in the transition amplitude at the intermediate-state meson's (complex) mass value. For a double pole, especially one giving rise to the mass distribution in Eq. (1), it is not sufficient simply to have two such poles at mass values M_1 and M_2 superimposed or separated as this will give two Breit-Wigner amplitudes as far as the mass distribution of M is concerned. However, that at least two particles or states are required to form a double pole is generally accepted because of arguments based on potential scattering by Eden and Landshoff,[8] a number of model calculations,[9] and a physical example.[3]

We introduce a double pole into T_{fi} by requiring that 2×2 mass matrix with complex diagonal elements and nonzero off-diagonal coupling elements cannot be diagonalized. This condition means that there is only one linear combination of states $|M_1\rangle$ and $|M_2\rangle$ with exponential decay. All other superpositions decay nonexponentially. In terms of $|M_1\rangle$ and $|M_2\rangle$, the denominator D_D of the propagator matrix is

$$D_D = \Delta^2 - \begin{pmatrix} M_1^2 & M_{12} \\ M_{21} & M_2^2 \end{pmatrix}. \tag{3}$$

With the assumption of time-reversal invariance and Hermiticity of the Hamiltonian the off-diagonal elements of the mass matrix are real and equal provided that the usual assumption in particle mixing of direct coupling between $|M_1\rangle$ and $|M_2\rangle$ is made (which we assume in the following). The situation in which the two particles are coupled because of common decay modes is more complicated, though not different basically from what follows, and will be included in a more expanded presentation of this problem.

Inserting Eq. (3) into Eq. (2) we get

$$T_{fi} = (F_{dM_1}, F_{dM_2}) \begin{pmatrix} \Delta^2 - M_1^2 & -M_{12} \\ -M_{21} & \Delta^2 - M_2^2 \end{pmatrix}^{-1} \begin{pmatrix} G_{M_1p_2;i} \\ G_{M_2p_2;i} \end{pmatrix}$$

$$= [F_{dM_1}(\Delta^2 - M_2^2)G_{M_1p_2;i} + F_{dM_2}(\Delta^2 - M_1^2)G_{M_2p_2;i} + F_{dM_2}M_{21}G_{M_1p_2;i} + F_{dM_1}M_{12}G_{M_2p_2;i}]/\det D_D. \tag{4}$$

The mass distribution observed in the missing-mass experiments is given by the Δ^2 dependence, studied through the recoil proton, of $|T_{fi}|^2$.

For the jth particle, we assume a Breit-Wigner distribution in squared mass so that $M_j^2 = m_j^2 - i\gamma_j m_j$ with m_j and γ_j real ($m_j\gamma_j$ is the half-width at half maximum in squared mass). Next, we study $|T_{fi}|^2$ to see how the double-pole form Eq. (1) that fits the mass distribution can be obtained. From $|\det D_D|^2$ one finds straightforwardly $m_1^2 = m_2^2$ and $(\gamma_2 m_2 - \gamma_1 m_1)^2 = 4M_{12}^2$ or $(\gamma_2 - \gamma_1)^2 = 4(M_{12}/m_2)^2$ as the double-pole condition; and the numerator of 16 terms in $|T_{fi}|^2$ must be reduced to the form of the numerator in Eq. (1). The only way to meet this requirement appears to be to demand that one of the intermediate states, $|M_1\rangle$ say, does not communicate with the initial π^-p state or with the final $p_2 d$ state but only with $|M_2\rangle$ (i.e., $M_{12} \neq 0$, $F_{dM_1} = G_{M_1p_2;i} = 0$ which in turn implies $\gamma_1 \equiv 0$). Then, the mass distribution predicted from $|T_{fi}|^2$ will be exactly that of Eq. (1).

This particular situation is furthermore physically quite reasonable. If we assume that d consists only of $\pi\rho$ states (at least[4] 93% true), then M_1 does not decay into $\pi\rho$ and its production is also inhibited if the A_2 is produced peripherally (which appears to be indicated by some data[10]) via a ρ, or ρ Regge trajectory, exchange mechanism. This appropriate ρ-exchange diagram will have only $\pi\rho$ states coupling to M_i. With M_2 taken as the particle decaying predominantly to $\pi\rho$, then $F_{dM_1} = G_{M_1p_2;i} = 0$, and the double pole in question occurs in the $\pi\rho$ elastic-scattering channel. However, this need not be the only production mechanism; perhaps the s-channel energy is such as to excite a baryon resonance which decays into M_2p_2 but not into M_1p_2. Then $G_{M_1p_2;i}$ still is 0 and it would not be necessary for the double pole to occur in the $\pi\rho$ elastic-scattering channel in order that the mass distribution be as in Eq. (1).

What kind of a particle would M_1 have to be? Its partner M_2 would be the spin-2 meson which should be grouped together with the $f^0(1254)$, $K^*(1415)$, and $f^*(1500)$ mesons to form the $J^P = 2^+$ nonet often pictured as a quark-antiquark ($q\bar{q}$) bound system in the $^3P_2$ state. There seems to be rather strong concurrence among theoretical papers[11] that these mesons should belong in the 405-dimensional representation of SU(6). In analogy with the ω-φ mixing situation in the 35-dimensional representation of SU(6), M_1 could logically belong to the 27-dimensional representation of SU(3), which along with the nonet, are contained in the 405 of SU(6). Essentially nothing in the earlier analyses of the nonet itself would be changed. As is well known, the simple quark model with mesons given as $q\bar{q}$ and baryons as qqq does not tolerate higher dimensional SU(3) multiplets, such as the 27. However, there now appears to exist evidence[12] indicating the existence of such more complicated structures; and theoretical investigations[13] have used $qq\bar{q}\bar{q}$ quark structures to describe higher spin, isospin, or hypercharge mesons. In such a model the quark structure of M_1^- could be $\bar{q}_p\bar{q}_n q_\Lambda q_\Lambda$ with exactly the same external quantum numbers as the M_2^- (and A_2^-). The $\pi\rho$ decay mode of M_1^- (also its production) would be inhibited as required for the double-pole mass distribution by the invariance of three-point couplings under the $SU(6)_W$ transformations involving the SU(2) subgroups of λ-quark W spin. The natural decay mode for this particle, in view of its quark content and rearrangement arguments,[13] is K^-K^0.

In conclusion, we would like to emphasize that, attractive as the above quark model speculations are but independently of them, we feel that another particle with a very narrow width compared with the tabulated[4] A_2 width must exist. The experimentally measured[1] double-humped resonance curve of ~4000 events can be used to obtain an estimate of this width. As indicated by the fitting function used, the two bumps observed are symmetric in both height and width; but, because of the experimental uncertainty in the measured curve, we can relax the double pole conditions as given below Eq. (4) somewhat. Therefore, taking $|T_{fi}|^2$ and using the relation between γ_1 and F_{dM_1}, we estimate that M_1^- can have a total width of at most 2 or 3 MeV with mass not different from the observed midpoint, 1297 MeV, by more than the CERN experimental resolution, $\lesssim 15$ MeV. To emphasize, these numbers are rather stringent limits away from the double-pole condition based on experimental uncertainty in the observed symmetry of the two humps. If Eq. (1) is indeed the correct shape, then increasing statistics will reduce these allowed deviations. To get information about this particle (M_1) one could study the way the mass distribution changes from reaction to reaction as the production mechanism changes and as different decay states are detected. This has the effect of changing the relative importance of the four terms (the F's and G's vary) in the numerator of Eq. (4), so the double-humped distribution should be readily made to disappear; but the denominator remains the same, a Breit-Wigner factor squared. Thus, it is quite possible that the shapes observed for A_2^0 and A_2^+ could be different from that for A_2^- in Eq. (1) depending on how G can be varied, which should be determinable from future experiments. Also, independently of the above quark discussion, it would be very interesting to have data on the $K\bar{K}$ decay mode of the precision of the CERN experiment. Existing data on the A_2 do appear to show considerable width fluctuations, but statistics are poor.

It seems to us that this double-humped mass distribution can now be taken fairly seriously as it has been observed by the missing-mass group under several different circumstances. For this reason and also because of the observed symmetry of the double-humped distribution we feel it to be quite unlikely that two separate, unrelated particles or resonances could be involved. If it could be shown experimentally that a double-pole explanation for the CERN experiment[1] is correct and that the same excitation curve always characterizes the A_2 (that is, the double-pole predictions based on the particle-pole correspondence ideas in the present paper are wrong) then one would conclude that only one particle (a "fundamental" double pole) is involved. This would have interesting consequences for our usual particle-pole, particle-field, or single Regge trajectory ideas.[14]

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*Work was performed at the Ames Laboratory of the U. S. Atomic Energy Commission.

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CERN Report No. 67/830/5-TH. 798, 1967 (to be published) from which it appears that the "missing-mass" data are perfectly compatible with bubble-chamber results. No questions such as the above have existed with the A_2 meson; the original experiments indicated a two-bump signal which became amplified when the events were increased and it now appears to us as an established effect to study.

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[14]The authors would like to thank Professor A. Barut and Dr. L. Sertorio for useful discussions on this point. The double-pole ideas can probably be readily incorporated into the Regge-pole picture and appropriate tests devised.

PROTON-NUCLEUS TOTAL CROSS SECTIONS AT 19.3 BeV/c AND HIGH-ENERGY CURRENTS

L. P. Horwitz
Department of Physics and Department of Mathematics, University of Denver, Denver, Colorado

and

H. Neumann
Department of Physics, University of Denver, Denver, Colorado

(Received 11 August 1967)

It has been suggested recently that relations among high-energy total cross sections obtained from the quark model[1-5] and related algebraic formulations[6-9] be applied to scattering on complex nuclei. In particular, Levinson, Wall, and Lipkin[1] note that as long as multiple-scattering effects are negligible, there are certain cross-section difference relations that should be obeyed for all nuclear targets with the same isospin. Okubo[10] has formulated a generalized current algebra in which the matrix elements of the commutator expressions for arbitrary initial and final states result in relations among scattering matrices that are direct extensions of elementary-particle scattering relations; they should be valid for nuclei.

It is possible to derive these difference relations among total cross sections from the algebraic approach of Cabibbo, Horwitz, and Ne'eman,[8] and we shall use this treatment below. Data for comparison of the difference relations with experiment seem to be unavailable as yet, except for deuterium,[11] where the data agree with these relations. However, proton-nucleus cross sections for nuclei ranging from Li^6 to $Pb^{207.2}$ have been measured by Bellettini et al.[12] at a proton laboratory momentum of 19.3 BeV/c. These data allow us to test speculative extensions of the approach of CHN to complex nuclei.

In the formulation of CHN, the matrix elements of the charges of strong-interaction currents are calculated in $[U(6)\otimes U(6)]_\beta$, the "good" rest algebra of Dashen and Gell-Mann.[13] The resulting amplitude for the elastic scattering of hadrons A and B can be written in general as the sum of six terms:

$$T_{AB}(s,t)=\sum_{j=0,3,8}\{\gamma_{sj}^{A}(t)\gamma_{sj}^{B}(t)g(\alpha_j^{s})\xi_j^{+}(t)t_j^{s}(\nu)+\gamma_{vj}^{A}(t)\nu_{vj}^{B}(t)\xi_j^{-}(t)t_j^{v}(t)\},\tag{1}$$