# HARMONIC GENERATION AND TURBULENCELIKE SPECTRUM IN A BEAM-PLASMA INTERACTION

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The power spectrum of uhf emission from a pulsed, 3-$\mu$sec beam-plasma interaction has been found to contain peaks at harmonics of the fundamental interaction frequency up to $n=7$, and to fall off with increasing frequency as $f^{-5.5\pm0.5}$. When an approximate correction for finite plasma diameter and probe response is included, the exponent becomes very nearly $-5$. This functional form indicates a turbulent wave-number spectrum going as $k^{-5}$, since the electron plasma waves most strongly excited by a beam have phase and group velocities nearly equal to the beam velocity $v_b$ (here $k=k_z=k_{\|}$). The observations cannot be accounted for using linear theory—say, the hot-plasma dielectric tensor—but suggest that appreciable nonlinear behavior and plasma turbulence occur early in the interaction. The stochastic mechanisms invoked by Stix[1] to explain electron heating in the Massachusetts Institute of Technology[2] and Oak Ridge[3] beam-plasma experiments are especially effective when $\omega_{pe}\approx\omega_{ce}$, as is the case here, and thus these measurements may help to illuminate an intermediate stage in the evolution of the latter experiments.

The apparatus, which has been described elsewhere,[4] consists of (1) a steady-state, 25-eV plasma formed by a P.I.G. discharge; (2) a repetitively pulsed, 400-V electron gun; both are immersed in (3) a uniform magnetic field of 400 G. Beam and plasma are cylindrical in cross section, of 0.9 cm diam, and flow coaxially along the field in the $z$ direction. A convective (complex-$k$) instability[5] takes place over a length of 14 cm from the cathode surface of the electron gun, within which the waves grow exponentially, reach a saturation level, and then decay. Previous experiments with this apparatus[4] have shown that in the region of growth, the wave properties at the fundamental frequency are adequately described by a dispersion relation derived using the linearized Vlasov equation.

In the present experiment, the uhf signals, which are received on an $E_r$-sensitive, axially traveling probe located at the edge of the discharge column, are band-pass filtered, analyzed on a Hewlett-Packard 8551 spectrum analyzer, and then detected and smoothed using a lock-in amplifier synchronized to the pulse repetition frequency. Careful prefiltering keeps the signals presented to the analyzer within the proper frequency interval for unambiguous spectral decomposition. Input power to the beam is 12 W, of which less than 1 W is transferred to the plasma in the form of oscillatory fields during the 3-$\mu$sec pulse; the probe intercepts about 1 % of the latter power. Little subsidiary plasma formation takes place during the pulse.

Table I summarizes the experimental conditions.

Figure 1 shows the power spectral density of the unstable waves, $P(f)$, as measured at $r=0.56$ cm and at three positions along the column which correspond to (1) the region of exponential growth, (2) the saturation point wherein nonlinear effects set in to limit the wave amplitude, and (3) a decaying region where beam energy spread and phase mixing reduce the wave amplitude. It is in this latter region that turbulence might be expected to arise. The data range over five decades in power level and five octaves in frequency and clearly illustrate the fundamental at 460 MHz as well as harmonics up to $n=7$; the eighth is at the noise level. While the harmonic structure is most clearly developed at the saturation point ($z=7$ cm), there are indications of it at $z=5$ cm in the exponentially growing region, and it persists out into the decaying region ($z=10$ cm) with some filling in of the intervals between

Table I. Summary of experimental conditions.

| | |
|---|---|
| Electron plasma frequency | $f_{pe}=1200$ MHz |
| Electron cyclotron frequency | $f_{ce}=1100$ MHz |
| Electron-neutral collision frequency | $\nu\simeq10^7$ sec$^{-1}$ |
| Electron temperature | $T_e\simeq25$ eV |
| He-ion temperature | $T_i\simeq1$ eV |
| Electron beam voltage | $V_b=400$ V |
| Beam current | $i_b=30$ mA |
| Beam plasma frequency | $f_b=160$ MHz |
| Beam pulse length | $\tau_b=3$ $\mu$sec |
| Pulse repetition time | $\tau_r=2$ msec |

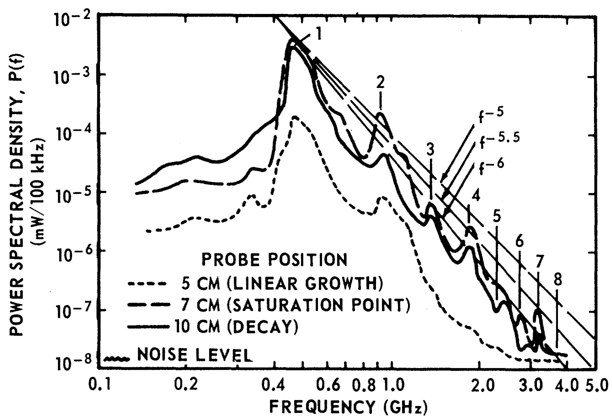


FIG. 1. Power spectral density of emission from a beam-plasma interaction at three axial positions, showing harmonics up to the seventh.

the peaks. When averaged over the peaks, all three spectra follow approximately a power law of $f^{-5.5}$ for three octaves above $f_1$. The data incorporate a calibration of the frequency response of the entire system and we conservatively estimate that the exponent $-5.5$ is good to within about $\pm 0.5$ insofar as equipment is concerned. The only unknown is the response of the probe to the radially nonuniform wave field existing in the free space beyond the plasma column, which, for the fundamental, behaves as $K_1(kr)$, where $K_1(z)$ is the modified Bessel function of complex argument. However, the higher harmonics, being of shorter wavelength, will decay more rapidly outside the beam-plasma column and measurements taken in that region will underestimate the high-frequency content of the spectrum in the main body of the plasma. A rough calculation of the size of this effect leads to a +4-dB correction at $n=7$ as compared with $n=1$, bringing the power law exponent close to $-5.0$ for the internal spectrum. Improved calculations are underway on this point.

Figure 2 is a cross plot of the spectra of Fig. 1 and gives the variation in power level with axial position, at fixed frequencies; an average over a 100-MHz band centered at each of the harmonic peaks has been performed. Wave growth is approximately exponential between the dashed line marked "Start-up Distance" and the one denoted by "Saturation Distance," and the slopes yield $-0.9 \leq k_i \leq -0.6$ cm$^{-1}$ for all seven curves. Here the wave amplitude is assumed to behave as $\exp(ik_rz - i\omega t - k_iz)$ for a convective instability. The vertical lines at

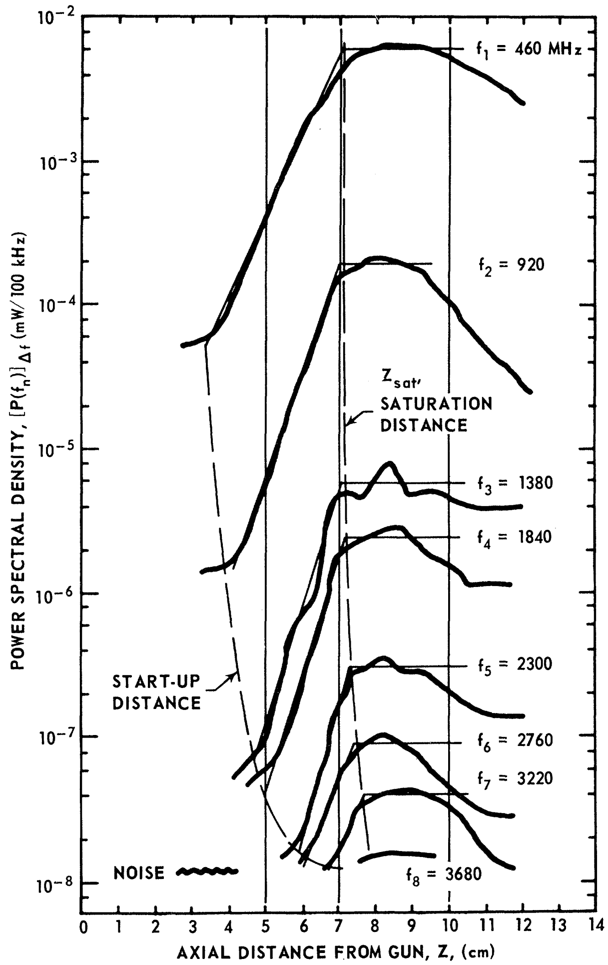


FIG. 2. Power spectral densities at each harmonic versus axial distance from electron gun. The spectra of Fig. 1 were taken at the vertical lines at 5, 7, and 10 cm. These have been averaged over $\Delta f = 100$ MHz centered at $f_n$ to remove dependence of level on details of line shape.

$z = 5$, 7, and 10 cm indicate the positions of the spectra on Fig. 1. It is worth noting that the spatial behavior of all harmonics is similar, but that the higher frequency fields begin their growth farther along the axis than do the lower ones, and all grow more rapidly than does the fundamental; Table II lists the values of $k_i$ for each harmonic. Thus it seems that the harmonic generation is a local effect, and the existence of higher frequencies at a given point apparently relies on the fundamental field driving the plasma to nonlinear behavior there, and not on some modulation spectrum near $z=0$ which excites independently growing modes at the frequencies observed downstream.

Wavelength measurements at the first four

Table II. Harmonic wave properties.

| Harmonic $n$ | Frequency $f_n$ (MHz) | Wavelength $2\pi/k_{rn}=\lambda_n$ (cm) | Axial growth constant $-k_i$ $(\mathrm{cm}^{-1})$ |
|---|---|---|---|
| 1 | 460 | 1.7 | 0.62 |
| 2 | 920 | 1.0 | 0.80 |
| 3 | 1380 | 0.8 | 0.91 |
| 4 | 1840 | 0.6 | 0.89 |
| 5 | 2300 | Not measured | 0.94 |
| 6 | 2760 | Not measured | 0.69 |
| 7 | 3220 | Not measured | 0.65 |

harmonics are listed in Table II and show that $\lambda_n \approx v_b/(f_n+f_b)$, which implies that these waves have phase velocities equal to the slow space-charge wave velocity on the electron beam.[4,5] If this is true for all harmonics, as it should be, then all waves have the same phase velocity and $\omega_n = k_n v_{\mathrm{ph}}$, so that the frequency spectrum transforms directly into a wave-number spectrum, or $P(k) \sim k^{-5.5}$. Such a distribution is close to the turbulent spectrum of $k^{-5}$ derived netized plasma having wavelengths $\lambda_D \ll \lambda \ll L$, where $\lambda_D$ = Debye length and $L$ is a characteristic dimension of the plasma; as discussed above, the corrections for probe response in a finite column bring it even closer. Thus this beam-plasma interaction exhibits a turbulencelike spectrum; furthermore, the spectrum apparently occurs not only where the nonlinear effects are expected to be strong, but where the amplitudes are still small enough so that the linear theory applies. The emission peaks appear to emerge from an underlying frequency continuum, grow, spread out, and then tend toward another continuum at a higher power level, all the while maintaining much the same slope in the frequency domain. This suggests that a low level of background turbulence exists in the warm, 25-eV plasma before the beam pulse enters, and that while the fundamental frequency and wavelength may be determined by a linear beam-wave interaction, the amplitudes are governed at all times by the nonlinearities inherent in plasma turbulence. Thus Chen's remarks[7] on the universality of the phenomenon are reinforced.

Finally, simultaneous measurements have been made of uhf wave-amplitude fluctuations and of electron fluxes to separate probes several millimeters away from the column edge. The former were observed in real time using a 3000-MHz EG&G oscilloscope and a 100-MHz high-pass filter, while the latter were dc coupled to an ordinary 25-MHz Tektronix oscilloscope. During a 5-$\mu$sec interval containing the beam pulse, the wave envelope is in almost perfect time correlation with fluctuations in probe electron current, showing that the oscillating electric fields lead to an enhanced electron diffusion across the magnetic field.

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