mentally $g_A{}^\Lambda = 0.79$. However, the value of $g_{NK\Lambda}$ is uncertain. We therefore prefer to use the superconvergence result of H. T. Nieh, Phys. Rev. Letters 19, 43 (1967). We have also used $g_\rho{}^2 = g_{K^*}{}^2$ which is essential to remove the divergence. The numerical value of the mass difference is damped to a large extent by the presence of the Cabibbo angle.

NEW LOW-ENERGY THEOREM FOR COMPTON SCATTERING

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(Received 29 May 1967)

We derive a low-energy theorem for pion Compton scattering giving terms of second order in photon frequency. This is then used to derive the value of the low-energy parameters. The low-energy theorem is also converted into a sum rule and a resonance saturation is attempted.

The low-energy theorems for Compton scattering giving terms up to first order in photon frequency have been discussed in the literature.[1] These theorems have been further extended to the case of Compton scattering when the photons also carry a "charge" label and thus describe the "isovector or unitary octet photons."[2] These low-energy theorems directly give information about the low-energy parameters. They can also be converted into sum rules if the relevant amplitudes satisfy unsubtracted dispersion relations.[3] On saturation with low-energy bound states and resonances these sum rules can lead to important understandings of the low-energy dynamical symmetries[4] and to various interesting relations between certain coupling constants which, in general, may not have followed from a symmetry approach.

The purpose of the present note is to show that some low-energy theorems can be obtained to second order in the photon frequency. These theorems are exact to all orders in the strong interactions and to second order in electromagnetism. In order to illustrate the method we discuss the simple case of pion Compton scattering. The method used, however, generalizes in a straightforward way for Compton scattering on targets with spin and is thus applicable to nucleon Compton scattering.

Let the pion Compton T-matrix amplitude be given by

$$T^{\alpha\beta}(s,u,t) = [A^{\alpha\beta}(s,u,t)a_{\mu\nu} + B^{\alpha\beta}(s,u,t)b_{\mu\nu}]\epsilon'^{*\mu}\epsilon^{\nu},$$

where

$$a_{\mu\nu} = tP_\mu P_\nu + (s-u)(k_\mu P_\nu + P_\mu k_\nu{}') + (t-4m^2)k_\mu k_\nu{}' + 2[(s-m^2)(u-m^2)-m^2t]g_{\mu\nu},$$
$$b_{\mu\nu} = k_\mu k_\nu{}' + (\tfrac{1}{2}t)g_{\mu\nu},$$
$$P_\mu = p_\mu + p_\mu{}',$$
$$s=(p+k)^2,\quad u=(p-k')^2,\quad t=(k-k')^2,$$
$$p^2 = p'^2 = m^2,\quad k^2 = k'^2 = 0, \qquad (1)$$

and p, k, ϵ (p', k', ϵ') are incident (final) pion momentum, photon momentum and polarization four vector, respectively. The α, β are "charge" labels pertaining to the final and initial photons. The amplitudes $A(s,u,t)$ and $B(s,u,t)$ are the kinematical singularity-free Mandelstam amplitudes. The new low-energy theorem we obtain is given by

$$\lim_{s\to m^2}\lim_{t\to 0}\left[\frac{dF^{\{\alpha\beta\}}(s,t)}{dt}\right] = 0, \qquad (2)$$

where

$$2F^{\{\alpha\beta\}}(s,t) = (s-m^2)(u-m^2)[A^{\beta\alpha}(s,u,t)+A^{\alpha\beta}(s,u,t)].$$

The low-energy theorem (2) can be used to derive the relations between the low-energy parameters. We find

$$f_2{}^{\{\alpha\beta\}}(m^2) = -(3/20m^2)f_1{}^{\{\alpha\beta\}}(m^2)$$

and

$$f_3^{\{\alpha\beta\}}(m^2)+\frac{5}{14m^2}f_2^{\{\alpha\beta\}}(m^2)+\frac{1}{7}\left[2\frac{df_2^{\{\alpha\beta\}}}{ds}+\frac{3}{10m^2}\frac{df_1^{\{\alpha\beta\}}}{ds}\right]_{s=m^2}=0, \quad (3)$$

where

$$f_J^{\{\alpha\beta\}}(s)=\frac{\sqrt{s}}{\nu}\frac{\langle +0|T_J(s)^{\{\alpha\beta\}}|+0\rangle}{(s-m^2)^{J-1}}$$

are the reduced partial-wave amplitudes for total angular momentum J between initial and final photon helicities equal to 1. $\nu^2=(s-m^2)^2/4s$ is the c.m. momentum squared. We can combine (3) with the value

$$f_1^{\{\alpha\beta\}}(m^2)=-(e^2/12\pi)\{F^\alpha(0),F^\beta(0)\} \quad (4)$$

given by the Thomson limit. We then obtain

$$f_2^{\{\alpha\beta\}}(m^2)=(e^2/40\pi m^2)\tfrac{1}{2}\{F^\alpha(0),F^\beta(0)\}$$

and

$$f_3^{\{\alpha\beta\}}(m^2)+\frac{1}{7}\left[2\frac{df_2^{\{\alpha\beta\}}(s)}{ds}+\frac{3}{10m^2}\frac{df_1^{\{\alpha\beta\}}(s)}{ds}\right]_{s=m^2}=-\frac{e^2}{112\pi m^4}\tfrac{1}{2}\{F^\alpha(0),F^\beta(0)\}. \quad (5)$$

As the above theorems are for amplitudes symmetric in charge indices α,β, the low-energy theorems (2) and (5) are also applicable to the scattering of physical photons on pions. To obtain the corresponding results for $\gamma\pi^+$ and $\gamma\pi^0$ scattering all one has to do is to replace $\frac{1}{2}e^2\{F^\alpha(0),F^\beta(0)\}$ by e^2 and 0, respectively. Thus, e.g.,

$$f_2^{\pi^+\gamma\to\pi^+\gamma}(m^2)=e^2/40\pi m^2,$$
$$f_2^{\pi^0\gamma\to\pi^0\gamma}(m^2)=0. \quad (6)$$

The low-energy theorem (2) for that $F(s,t)$ amplitude which is pure $I=2$ in the t channel $(\pi+\pi\to\gamma_\alpha+\gamma_\beta)$ can also be converted into a sum rule [see Eq. (33)] by use of an unsubtracted dispersion relation. For the corresponding t-channel $I=0$ amplitude we certainly cannot use unsubtracted dispersion relations as the Pomeranchuk Regge pole will contribute to this amplitude. On saturating the $I=2$ sum rule with ω,φ,A_1, and A_2 mesons we obtain

$$\frac{\Gamma(\omega\to\pi^0\gamma)}{m_\omega^3}+\frac{\Gamma(\varphi\to\pi^0\gamma)}{m_\varphi^3}-\frac{\Gamma(A_1^+\to\pi^+\gamma)}{m_{A_1}^3}-\frac{15\Gamma(A_2^+\to\pi^+\gamma)}{m_{A_2}^3}=0. \quad (7)$$

It is interesting to combine this relation (7) with those obtained from the Pagels-Harari and Cabibbo-Radicati sum rules on saturating with the same set of states. Using $\frac{1}{6}\langle r_\pi^2\rangle=1/m_\rho^2$, which is the ρ-pole model value for the pion charge radius,[5] and $\Gamma(\varphi\to\pi^0\gamma)=0$, we obtain

$$\Gamma(\omega\to\pi^0\gamma)=1.6\text{ MeV},$$
$$\Gamma(A_1^+\to\pi^+\gamma)=0.3\text{ MeV},$$
$$\Gamma(A_2^+\to\pi^+\gamma)=0.4\text{ MeV}. \quad (8)$$

The calculated $\Gamma(\omega^0\to\pi^0\gamma)$ seems in reasonable agreement with the measured value 1.15 ± 0.24 MeV.[6] The $\pi\gamma$ widths of A_1^+ and A_2^+ have not yet been measured.

Derivation of the low-energy theorem.—We now briefly sketch the derivation of the low-energy theorem (2). We have the gauge conditions

$$T_{\mu\nu}^{\{\alpha\beta\}}k^\nu=k'^\nu T_{\nu\mu}^{\{\alpha\beta\}}=0,$$

where

$$T_{\mu\nu}^{\{\alpha\beta\}}=\tfrac{1}{2}[T_{\mu\nu}^{\alpha\beta}+T_{\mu\nu}^{\beta\alpha}], \quad (9)$$

which leads to

$$\omega\omega' T_{00}^{\{\alpha\beta\}}=k'_m k_n T_{mn}^{\{\alpha\beta\}}, \quad (10)$$

where $\omega=k_0$, $\omega'=k_0'$. We shall be working in the laboratory frame ($\vec{p}=0$). The angle of the scattering in the laboratory frame shall be denoted by θ, i.e., $\vec{k}'\cdot\vec{k}=\omega\omega'\cos\theta$.

It is obvious from (10) that in order to calculate $T_{mn}^{\{\alpha\beta\}}$ to the order ω^2 one has to know $T_{00}^{\{\alpha\beta\}}$ also to order ω^2. This, however, involves looking explicitly at the contribution to $T_{00}^{\alpha\beta}$ arising from intermediate states which are not degenerate in energy with the pion, i.e., "excited state contributions." Let us write

$$T_{00}^{\alpha\beta}=P_{00}^{\alpha\beta}+E_{00}^{\alpha\beta}, \quad (11)$$

where $P_{00}^{\alpha\beta}$ is the contribution of the pion intermediate state and $E_{00}^{\alpha\beta}$ is the excited-state contribution. As is well known the contribution $E_{00}{}^{\alpha\beta}$ is of order ω^2. We have

$$\frac{-E_{00}{}^{\alpha\beta}}{(2\pi)^3[4E(\vec{p})E(\vec{p}\,')]^{1/2}}=\sum_{n,\zeta_n}\frac{\langle\vec{p}\,'|J_0{}^{\alpha}(0)|\vec{p}+\vec{k},E_n(\vec{p}+\vec{k}),\zeta_n\rangle\langle\vec{p}+\vec{k},E_n(\vec{p}+\vec{k}),\zeta_n|J_0{}^{\beta}(0)|\vec{p}\rangle}{\omega+E(\vec{p})-E_n(\vec{p}+\vec{k})}$$

$$+\sum_{n,\zeta_n}\frac{\langle\vec{p}\,'|J_0{}^{\beta}(0)|\vec{p}-\vec{k}',E_n(\vec{p}-\vec{k}'),\zeta_n\rangle\langle\vec{p}-\vec{k}',E_n(\vec{p}-\vec{k}'),\zeta_n|J_0{}^{\alpha}(0)|\vec{p}\rangle}{-\omega+E(\vec{p})-E_n(\vec{p}-\vec{k}')},\tag{12}$$

where $E(\vec{q})$, $E_n(\vec{q})$ are, respectively, the energy of the pion and of the nth intermediate state with total three-momentum $\vec{q}$. The label ζ_n stands for other quantum numbers like spin needed to specify the intermediate states. Using current conservation we can rewrite (12) as

$$E_{00}{}^{\alpha\beta}/(2\pi)^6[4E(\vec{p})E(\vec{p}\,')]^{\frac{1}{2}}$$

$$=k_m{}'k_l\sum_{n,\zeta_n}\frac{\langle p'|J_m{}^{\alpha}(0)|\vec{p}+\vec{k},E_n(\vec{p}+\vec{k}),\zeta_n\rangle\langle\vec{p}+\vec{k},E_n(\vec{p}+\vec{k}),\zeta_n|J_l{}^{\beta}(0)|\vec{p}\rangle}{[E(\vec{p}\,'),-E_n(\vec{p}+\vec{k})][E_n(\vec{p}+\vec{k})-E(\vec{p})][\omega+E(\vec{p})-E_n(\vec{p}+\vec{k})]}$$

$$+k_mk_l{}'\sum_{n,\zeta_n}\frac{\langle p'|J_m{}^{\beta}(0)|\vec{p}-\vec{k}',E_n(\vec{p}-\vec{k}'),\zeta_n\rangle\langle\vec{p}-\vec{k}',E_n(\vec{p}-\vec{k}'),\zeta_n|J_l{}^{\alpha}(0)|\vec{p}\rangle}{[E(\vec{p}\,')-E_n(\vec{p}-\vec{k}')][E_n(\vec{p}-\vec{k}')-E(\vec{p})][-\omega'+E(\vec{p})-E_n(\vec{p}-\vec{k}')]}$$

$$\equiv k_m{}'k_l\Lambda_{ml}{}^{\alpha\beta}(\omega',\vec{k}',\omega,\vec{k})+k_mk_l{}'\Lambda_{ml}{}^{\beta\alpha}(-\omega,-\vec{k};-\omega',-\vec{k}')$$

$$=k_m{}'k_l\Lambda_{ml}{}^{\alpha\beta}(0,0;0,0)+k_mk_l{}'\Lambda_{ml}{}^{\beta\alpha}(0,0;0,0)+O(\omega^3).\tag{13}$$

Now $\Lambda_{ml}{}^{\alpha\beta}(0,0;0,0)$ is a pure numerical three-tensor.[7] Therefore

$$\Lambda_{ml}{}^{\alpha\beta}(0,0;0,0)=\lambda^{\alpha\beta}\delta_{ml}/(2\pi)^3(2m),\tag{14}$$

where $\lambda^{\alpha\beta}$ is related to the zero-frequency polarizability of the pion. Hence

$$E_{00}{}^{\alpha\beta}=\vec{k}'\cdot\vec{k}(\lambda^{\alpha\beta}+\lambda^{\beta\alpha})+O(\omega^3).\tag{15}$$

Now in general we can write

$$T_{mn}=P_{mn}+T_1\delta_{mn}+T_2k_mk_n{}'+T_3k_m{}'k_n+T_4(k_mk_n+k_m{}'k_n{}')+T_5(k_mk_n-k_m{}'k_n{}'),\tag{16}$$

where P_{mn} is the explicit pion-intermediate state contribution to T_{mn} and $T_i=T_i(\omega',\omega)$ $[i=1,\cdots,5]$ are nonsingular at $\omega=0$. Using (11) and (16) in (9) we obtain

$$T_1\cos\theta+(\omega\omega'\cos^2\theta)T_2+\omega\omega'T_3+(\omega^2+\omega'^2)\cos\theta T_4+(\omega^2-\omega'^2)\cos\theta T_5$$

$$=[P_{00}-k_m{}'k_nP_{mn}/\omega\omega']+E_{00}.\tag{17}$$

Now

$$P_{00}{}^{\{\alpha\beta\}}-\frac{k_m{}'k_nP_{mn}{}^{\{\alpha\beta\}}}{\omega\omega'}=-e^2\cos\theta\{F^{\alpha}(0),F^{\beta}(0)\}+\frac{e^2\omega^2}{4m^2}(2\cos\theta-1)\{F^{\alpha}(0),F^{\beta}(0)\}$$

$$+e^2\omega^2\cos\theta[\{F^{\alpha}(0),F^{\beta\prime}(0)+F^{\alpha\prime}(0),F^{\beta}(0)\}]+O(\omega^3),\tag{18}$$

where $F^{\alpha}(t)$ is the form factor of the pion with charge index α. We have $F^{\alpha}(0)=I^{\alpha}$, the αth component of isospin.

The crossing symmetry gives

$$T_i^{\{\alpha\beta\}}(\omega'\omega)=\eta_i T_i^{\{\alpha\beta\}}(-\omega,-\omega'), \tag{19}$$

where $\eta_i=1$ for $i=1,2,3,4$ and $\eta_5=-1$. Therefore

$$T_1^{\{\alpha\beta\}}(\omega'\omega)=T_1^{\alpha\beta}(0,0)+2m(\omega'-\omega)a_0^{\{\alpha\beta\}}+4m^2(\omega+\omega')^2a_1^{\{\alpha\beta\}}+O(\omega^3),$$

$$T_i^{\{\alpha\beta\}}(\omega',\omega)=T_i^{\{\alpha\beta\}}(0,0)+O(\omega^2) \quad (i=2,3,4),$$

$$T_5^{\{\alpha\beta\}}(\omega',\omega)=O(\omega). \tag{20}$$

Using (15), (18), and (20) in (17) and only comparing the coefficient of 1 and $\omega^2\cos^2\theta$ on both sides, we obtain

$$T_1^{\{\alpha\beta\}}(0,0)=-e^2\{F^{\alpha}(0),F^{\beta}(0)\}, \tag{21}$$

$$T_2^{\{\alpha\beta\}}(0,0)+2a_0^{\{\alpha\beta\}}=0. \tag{22}$$

We do not get anything useful by comparing the coefficient of the ω^2 and $\omega^2\cos\theta$ terms.

We would like to re-express the two low-energy theorems (21) and (22) as referring to the invariant amplitudes. By going to the laboratory frame and using (1) we find

$$T_1^{\alpha\beta}(\omega',\omega)=-2\hat{F}^{\alpha\beta}(\omega',\omega)+\omega\omega'(1-\cos\theta)T_2^{\alpha\beta}(\omega',\omega), \tag{23}$$

$$T_2^{\alpha\beta}(\omega',\omega)=\hat{B}^{\alpha\beta}(\omega',\omega)-4m^2\hat{A}^{\alpha\beta}(\omega',\omega), \tag{24}$$

where $\hat{F}(\omega',\omega)$, $\hat{A}(\omega',\omega)$, and $\hat{B}(\omega',\omega)$, are, respectively, $F(s,t)$, $A(s,t)$, and $B(s,t)$ re-expressed as functions of ω and ω'. Using (23) we get

$$T_1^{\{\alpha\beta\}}(0,0)=-2\hat{F}^{\alpha\beta}(0,0), \tag{25}$$

$$T_2^{\{\alpha\beta\}}(0,0)+2a_0^{\{\alpha\beta\}}=-2f_0^{\{\alpha\beta\}}, \tag{26}$$

where

$$\hat{F}^{\alpha\beta}(\omega',\omega)=\hat{F}^{\alpha\beta}(0,0)+2m(\omega'-\omega)f_0^{\{\alpha\beta\}}+4m^2(\omega+\omega')^2f_1^{\{\alpha\beta\}}+O(\omega^3).$$

We further have in the laboratory frame

$$s=m^2+2m\omega,\quad u=m^2-2m\omega',\quad t=2m(\omega'-\omega),$$

i.e.,

$$2m(\omega'-\omega)=t,$$

$$2m(\omega'+\omega)=2(s-m^2)+t.$$

Therefore combining (21), (22), (25), and (26), we finally get

$$\lim_{s\to m^2}\lim_{t\to 0}F^{\{\alpha\beta\}}(s,t)=\tfrac{1}{2}e^2\{F^{\alpha}(0),F^{\beta}(0)\}, \tag{27}$$

$$\lim_{s\to m^2}\lim_{t\to 0}\frac{dF^{\{\alpha\beta\}}(s,t)}{dt}=0. \tag{2}$$

The low-energy theorem (27) gives the usual Thomson limit. The theorem (2) is new.[8]

Low-energy parameters.—The low-energy theorems (27) and (2) can be used to give information about the low-energy parameters of pion Compton scattering. We have the partial-wave expansion

$$F^{\{\alpha\beta\}}(s,t)=-4\pi\left(1+\frac{t}{s-m^2}\right)\sum_{J=1}^{\infty}(2J+1)f_J^{\{\alpha\beta\}}(s)(s-m^2)^{J-1}\times\left[\frac{P_{J-1}''(\cos\theta_s)-P_J''(\cos\theta_s)+JP_J'(\cos\theta_s)}{J(J+1)}\right], \tag{28}$$

where $\cos\theta_s=1+t/2\nu^2$. Using the expansion (28) together with (27) and (2) we obtain the results given earlier in Eqs. (3)-(6).

Sum rule.—In order to convert the low-energy theorem (2) into a sum we have to discuss the asymptotic behavior of the amplitude $F(s,t)$. By doing the usual Regge analysis in the t channel it can be shown that

$$A^{(I)}(s,u,t)\underset{\substack{s\to\infty\\ t\text{ fixed}}}{\longrightarrow}-a^{(I)}(t)s^{\alpha_I(t)-2}, \tag{29}$$

where the superscript (I) denotes that the amplitude is pure isospin I in the t channel, and $\alpha_I(t)$ is the position of the leading singularity in angular momentum for the isospin-I t-channel amplitude. Using (29) we have

$$F^{(I)}(s,t)\underset{\substack{s\to\infty\\ t\text{ fixed}}}{\longrightarrow}a^{(I)}(t)s^{\alpha_I(t)}. \tag{30}$$

We are only interested in $I=0,2$ amplitudes as the theorem (2) holds only for amplitudes symmetric under α, β. For $I=0$ we certainly have the Pomeranchuk Regge trajectory for which $\alpha(0)=1$. We therefore cannot write unsubtracted dispersion relation for $F^{(0)}(s,t)$. If we assume that $\alpha_2(t)<0$ for a neighborhood in t around $t=0$ we can convert the Thomson theorem (27) and the theorem (2) into sum rules by using the unsubtracted dispersion relation

$$F^{(2)}(s,t)=\frac{1}{\pi}\int \mathrm{Im}F^{(2)}(s',t)ds'\left[\frac{1}{s'-s}+\frac{1}{s'-2m^2+s+t}\right]$$
$$=\frac{-1}{\pi}\int ds'\,\mathrm{Im}A^{(2)}(s',t)(s'-m^2)(s'+t-m^2)\left[\frac{1}{s'-s}+\frac{1}{s'-2m^2+s+t}\right]. \quad (31)$$

We then obtain, using the Thomson theorem,

$$e^2=\frac{2}{\pi}\int ds'(s'-m^2)\,\mathrm{Im}A^{(2)}(s,0) \quad (32)$$

which is the Pagels-Harari sum rule. Using theorem (2), we get the sum rule

$$\frac{2}{\pi}\int ds'(s'-m^2)\left(\frac{d\,\mathrm{Im}A^{(2)}(s',t)}{dt}\right)_{t=0}+\frac{1}{\pi}\int ds'\,\mathrm{Im}A^{(2)}(s',0)=0. \quad (33)$$

If we saturate the sum rule (32) or (33) with direct-channel $\pi\gamma$ resonances then only G-parity -1 mesons can contribute. The possible candidates are ω, φ, A_1, and A_2, mesons. On saturating the sum rule (33) we get the relation (7) discussed earlier.

It is interesting to have an alternative derivation of the sum rules (32) and (33). If we have the asymptotic behavior (29) and $\alpha_2(t)<0$, then the amplitude $(s-u)A^{(2)}(s,u,t)$ must satisfy a superconvergence relation. This is given by

$$-e^2+\frac{1}{\pi}\int ds'(2s'-2m^2+t)\,\mathrm{Im}A^{(2)}(s',t)=0 \quad (34)$$

for t such that $\alpha_2(t)<0$. By evaluating the superconvergence sum rule at $t=0$ and by taking its first derivative with respect to t at $t=0$ we obtain the sum rules (32) and (33).

We must, however, emphasize that if there is no region of t around and including $t=0$ for which $\alpha_2(t)<0$, then neither the sum rule (32) nor the sum rule (33) is valid. Such, for example, would be the case if there is a fixed $J=0$, $I=2$ Regge pole in the t channel.[9] The low-energy theorem and the information about the low-energy parameters, of course, still remain true.

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[5]The only available experimental number for the charge radius of the pion gives $\langle r_\pi \rangle = 0.7 \pm 0.2$ F [C. W. Akerlof et al., Phys. Rev. Letters 16, 147 (1966)] whereas the ρ-pole model gives $\langle r_\pi \rangle = 0.63$ F.

[6]A. H. Rosenfeld et al., University of California Radiation Laboratory Report No. UCRL-8030 (revised), 1966 (unpublished).

[7]The numerical three-tensor $\Lambda_{mn}(0,0;0,0)$ obviously does not depend on $|\vec{k}|$ and $|\vec{k}'|$. It also cannot depend upon the unit vectors in the direction of $\vec{k}$ and $\vec{k}'$ since none of the energy denominators is singular at $\omega=0$ and all the factors in the numerator of the defining expression for $\Lambda_{mn}(0,0;0,0)$ are rational functions of $\vec{k}$, $\vec{k}'$.

[8]Some remarks may be in order on why we have chosen this particular derivation of the low-energy theorems. The present method generalizes with ease to the case of Compton scattering of the physical as well as the charged photons on higher spin targets. It is conceivable that it may be possible, at least for the case of the physical photons, to give a pure S-matrix derivation of the new theorems provided that one is able to write down a set of linearly independent invariant amplitudes which are free from both these kinematic singularities and zeros. This, however, is a highly nontrivial problem and is still unsolved in general.

[9]For such a possibility see A. H. Mueller and T. L. Trueman, Brookhaven National Laboratory Report No. BNL-11308, 1967 (unpublished).