STUDY OF GIANT RESONANCES THROUGH RADIATIVE PION CAPTURE WITH APPLICATION TO ^{16}O

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Using a generalized Goldhaber-Teller model of the giant multipole resonance in ^{16}O, we have evaluated radiative pion-capture rates from 1s and 2p orbits, leading to various multipole isospin states in ^{16}N, and obtained the corresponding γ spectra. The subsequent neutron emission was considered on the basis of Wigner's R-matrix theory. Whereas in 1s capture, the spectra are dominated by the dipole resonances, in the 2p capture there is an even stronger contribution from the 2^+ and 3^+ quadrupole states. The total radiative capture rate compares favorably with experiment.

On the basis of approximate SU(4) symmetry of light nuclei one expects[1] collective excitations of these nuclei with selection rules (for $A=2Z=2N=4n$) $\Delta S=1$, $\Delta T=0$ (spin resonance); $\Delta S=0$, $\Delta T=1$, $\Delta T_3=\pm1, 0$ (isospin resonance); and $\Delta S=1$, $\Delta T=1$, $\Delta T_3=\pm1, 0$ (spin-isospin resonance). The best-known member of this multiplet is the familiar $\Delta T_3=0$ isospin resonance or giant dipole state seen in photonuclear reactions.[2] It has been pointed out[3,4] that the radiative pion-capture process

$$\pi^- + N(A,Z) \to \gamma + N^*(A,Z-1) \tag{1}$$

should be an even better means than the capture of muons[5] for studying the $\Delta T_3=-1$ analog to the giant resonance state (both dipole and quadrupole) of the capturing nucleus: One may perform a measurement of the photon spectrum,[6,7] or else the photon may be used as a coincidence signal for a time-of-flight energy determination[8] of the neutrons[9,10] emitted in the subsequent decay

$$N^*(A,Z-1) \to N^{(*)}(A-1,Z-1)+n. \tag{2}$$

But whereas muon capture contains both Fermi and Gamow-Teller matrix elements, pion capture depends on Gamow-Teller-type transitions only,[3,4] and hence will excite spin-isospin resonances without exciting the isospin resonance. In this way the two capture processes complement each other. A further significant difference arises from the fact that muons are captured from the 1s Bohr orbit, pions mostly from higher orbits,[11] and it turns out that nuclear quadrupole spin-isospin oscillations ($1^+, 2^+, 3^+$ states), which play a minor role in 1s capture,[12] give very large contributions in 2p radiative pion capture.[4,13] A demonstration in this way of their existence would be of very great interest.

The calculation of the rates of Reaction (1) has been carried out for ^{16}O on the basis of the generalized Goldhaber-Teller model, following our previous procedure.[12,14] The corresponding giant resonance levels of ^{16}O and ^{16}N are given in Fig. 1 of Ref. 12; the quadrupole levels in this figure were based on a calculation of Spicer and Eisenberg[15] using the particle-hole model.

The capture rate of process (1) from a 0^+, $T=0$, $S=0$ state to a definite final state (J,M) is given by[4]

$$\lambda_J = 2\frac{e^2}{4\pi}\frac{f^2}{4\pi}\frac{k}{m_\pi^{\ 3}}\langle|\vec{\epsilon}\cdot\vec{\mathrm{M}}|^2\rangle, \tag{3}$$

where

$$\langle|\vec{\epsilon}\cdot\vec{\mathrm{M}}|^2\rangle = \int d\hat{k}\sum|\langle JM|\sum_i \tau_i^{(-)}\vec{\sigma}_i\cdot\vec{\epsilon}\exp(-i\vec{\mathrm{k}}\cdot\vec{\mathrm{r}}_i)\varphi_\pi(\vec{\mathrm{r}}_i)|00\rangle|^2,$$

with $e^2=4\pi/137$, $f^2/4\pi=0.08$, $\vec{k}$ = photon momentum, $k=m_\pi-\omega_J$ (ω_J = excitation energy of ^{16}N measured from the ^{16}O ground state), $\vec{\epsilon}$ = photon polarization vector, and φ_π = pion wave function. As before,[12] the matrix element is expressed by the transition density of the operator $\tau^{(-)}\vec{\sigma}$, given by the Goldhaber-Teller model. Only the spin-isospin mode ($T=1, L, S=1$) of the collective nuclear vibrations contributes, corresponding to the states $J^\pi=1^+$ (for $L=0$, monopole), $0^-, 1^-, 2^-$ ($L=1$, dipole), 1^+, $2^+, 3^+$ (quadrupole), etc. After summing over M and over photon polarizations and, for the $2p$ state, averaging over its orientation, we find

$$\langle|\vec{\epsilon}\cdot\vec{M}|^2\rangle=|C_L|^2(\hat{J}/\hat{L})^2\sum_{ll'}i^{l'-l}\hat{l}\hat{l}'(l0,\lambda0|L0)(l'0,\lambda0|L0)I_{l\lambda}^{L*}I_{l'\lambda}^{L}$$
$$\times\{\delta_{ll'}-\hat{l}\hat{l}'\hat{L}^2\sum_j(l0,10|j0)(l'0,10|j0)W(\lambda lJ1;Lj)W(\lambda l'J1;Lj)\}, \tag{4}$$

with

$$\hat{J}=(2J+1)^{1/2},$$
$$C_0=(2\pi/r_{\mathrm{rms}})(A/m\omega_J)^{1/2},$$
$$C_1=2\pi(A/3m\omega_J)^{1/2},$$
$$C_2=(2\pi/r_{\mathrm{rms}})(2A/5m\omega_J)^{1/2}, \tag{5}$$

A = mass number = 16, m = proton mass, and

$$I_{l\lambda}^{0}=\int_0^\infty R_{n\lambda}(r)j_l(kr)\frac{d}{dr}[r^3\rho_0(r)]dr,$$
$$I_{l\lambda}^{L}=\int_0^\infty R_{n\lambda}(r)j_l(kr)r^{L+1}\frac{d\rho_0(r)}{dr}dr\quad(L=1,2), \tag{6}$$

where $\varphi_\pi=R_{n\lambda}(r)Y_{\lambda\mu}(\hat{r})$, $\rho_0(r)$ = nuclear ground-state density. In our application to ^{16}O we consider only $1s$ and $2p$ capture ($\lambda=0,1$) and $L\leq2$. The integrals (6) may be expressed by derivatives of the ground-state form factor $F(k)$ with $F(0)=1$; e.g.,

$$I_{01}^{1}=-\frac{1}{8\pi}\left(\frac{f\pi}{6a_\pi^5}\right)^{1/2}\frac{1}{k^2}\frac{d}{dk}[k^3F(k)], \tag{7}$$

with $a_\pi=137/Zm_\pi$, and $f_\pi=\exp(-\bar{r}/a_\pi)=0.84$ representing the deviation of φ_π from its point-charge value. Table I shows the individual capture rates in the third column. They add up to the total radiative capture rates $\lambda_{1s}=5.1\times10^{17}\ \mathrm{sec}^{-1}$, $\lambda_{2p}=1.1\times10^{14}\ \mathrm{sec}^{-1}$. We divide these by the total $1s$ and $2p$ pion absorption rates taken from Ericson[16] and weight the two branching ratios by the probabilities of s- and p-state capture given by Eisenberg and Kessler,[11] to obtain the branching ratio of radiative capture to all pion absorption processes, $R_{\mathrm{th}}\cong1.5\,\%$. This is of the right order of the experimental value, $R_{\mathrm{expt}}\sim1\,\%$, quoted by Anderson and Eisenberg.[4]

The rates λ_J of Table I, together with the

Table I. Radiative pion capture rates to spin-isospin giant resonance states in ^{16}O, photon energies, neutron decay energies, branching ratios, and decay widths.

Spin-isospin state	ω_J (MeV)	$\lambda_J^{(1s)}$ (10^{16} sec^{-1})	$\lambda_J^{(2p)}$ (10^{12} sec^{-1})	k (MeV)	Ground-state decay E_n (MeV)	Ground-state decay Branching ratio (%)	$\frac{3}{2}^-$ state decay E_n (MeV)	$\frac{3}{2}^-$ state decay Branching ratio (%)	Γ_{exp} (MeV)
m 1^+	26.5	5.96	6.54	113.5	13.6	0	7.3	100	5
d 0^-	22.5	0	5.31	117.5	9.6	2	3.3	98	1.5
1^-	22.0	13.61	8.28	118.0	9.1	30	2.8	70	1.5
2^-	17.5	17.88	23.2	122.5	4.6	100	···	···	1.0
q 1^+	29.1	1.00	7.27	110.9	16.1	4	9.8	96	2.0
2^+	28.1	5.34	23.9	111.9	15.2	6	8.9	94	2.0
3^+	20.5	7.40	34.5	119.5	7.6	~100	1.3	~0	2.0

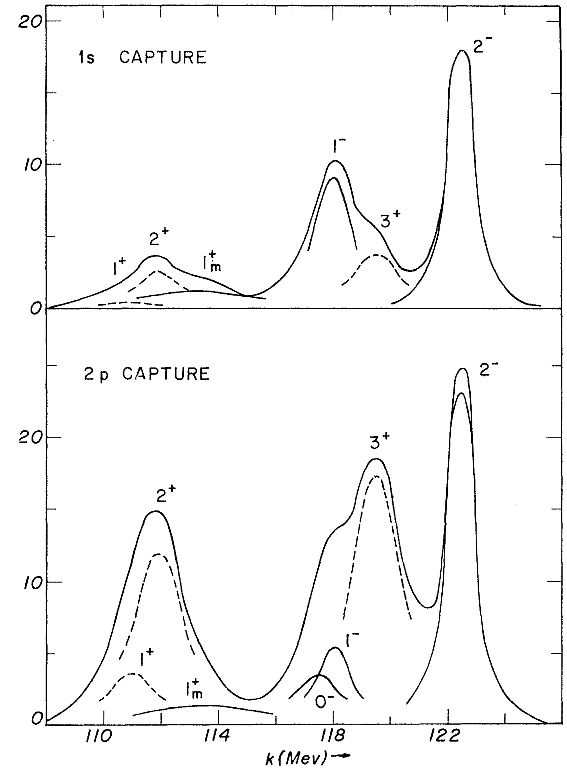

FIG. 1. Photon spectra from radiative pion capture in ^{16}O from $1s$ and $2p$ orbits, leading to the spin-isospin giant resonance states of ^{16}N (arbitrary units). Broken lines are the quadrupole resonances.

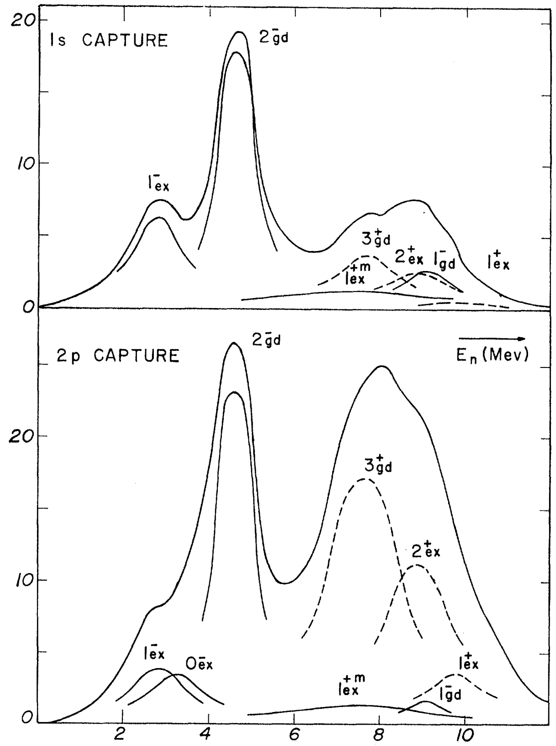

FIG. 2. Neutron spectra from the decay of the spin-isospin giant resonance states in ^{16}N excited by radiative pion capture in ^{16}O (arbitrary units). Decay to ground state and third excited state of ^{15}N is indicated.

experimental widths Γ_{expt} of the last column (obtained from photon absorption[2] and from electron scattering[14]), were used to obtain the photon spectra of Fig. 1. The contributions of the isospin states, present in muon capture,[12] are absent here. As stated before, in the p-state capture (which is quite dominant over s-state capture[11] for ^{16}O), there is a very large contribution from the quadrupole states (broken lines); in $1s$ and muon capture,[12] this contribution is quite small. The same features also appear in the neutron spectra of Fig. 2 (obtained from the remaining columns of Table I) for the subsequent decay $^{16}N^* \to {}^{15}N^{(*)} + n$; one sees that here, a large number of ~8-MeV neutrons should be emitted due to the $2p$ capture and quadrupole dominance, whereas the muon-capture[12] neutrons should be mainly ~5 MeV. A 6.3-MeV de-excitation γ ray following the excited-state neutron decays of $^{16}N^*$ (Fig. 2) may perhaps be observable also, just as in the case of photoexcitation of the giant resonances.[17]

A qualitative agreement of the calculated with the measured photon spectra[6,7] (for nuclei other than ^{16}O, however) is evident. The shape of the latter was shown[7] to be describable by a direct-reaction mechanism in conjunction with the Fermi-gas model. This model, however, turns out to predict much too low rates[18] for the process, whereas the rates from the giant resonance mechanism come out correctly,[3,4] as evidenced above. The similarity of the yields for selected nuclei up to ^{63}Cu suggest that the spin-isospin analog of the isospin resonance occurs throughout the periodic table.

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DOES CHARGE OBEY A SUPERSELECTION RULE?

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It has been suggested[1] that one can test a supposed superselection rule by allowing a particle to interact with two parts of a system under the following circumstances. The incident particle and the other system as a whole may be eigenstates of the additively conserved quantum number in question (charge is considered in Ref. 1). However, if the two parts of the system are not individually eigenstates of the quantum number the particle will not be either, when it is in the region between the two parts (after interacting with one part). The combination of system plus particle will still be an eigenstate of the quantum number (with the same eigenvalue).

I wish to point out that only if the two parts of the system can be separated from one another can we speak of interaction with one part, and only if the particle can be separated from both parts (after the first interaction), so that there is no mutual interaction, can we consider the particle as an isolated system which is not an eigenstate of the quantum number in question. For an operator obeys a superselection rule if and only if the states, of pure isolated systems, in our physical Hilbert space are all eigenstates of that operator. We must know that there is no superselection rule before we can proceed with the experiment, for if there were a superselection rule such a (noninteracting) separation would be impossible, i.e., the construction of the apparatus precludes the superselection rule.

One can prove that angular momentum cannot obey a superselection rule without using the arguments presented in Ref. 1 (which, I claim, have not proved it). Consider J_z. If there exists an eigenstate of J_z with eigenvalue m, namely $|m\rangle$, we write

$$J_z |m\rangle = m |m\rangle. \tag{1}$$

Rotational invariance implies that there exists a state $|m\rangle'$ such that

$$J_z' |m\rangle' = m |m\rangle', \tag{2}$$

where J_z' is the angular-momentum operator in the (arbitrarily chosen) z' direction. But (it is well known for angular momentum that) $|m\rangle'$ is an eigenstate of J_z only if J_z' and J_z commute, therefore since

$$[J_z, J_z'] \neq 0 \tag{3}$$

when the z' and z directions are not parallel, we find

$$J_z |m\rangle' \neq \text{const} |m\rangle'. \tag{2$'$}$$

Furthermore, Eq. (3) implies that

$$J_z' |m\rangle \neq \text{const} |m\rangle. \tag{2$''$}$$

Now consider the symmetry of isospin. If