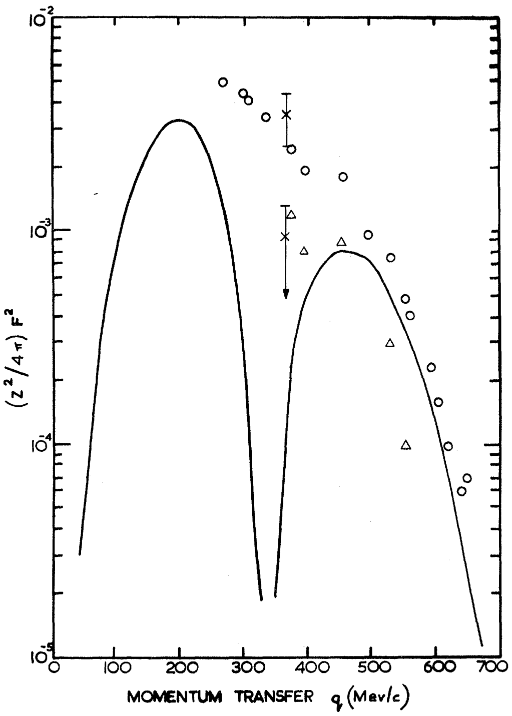

FIG. 2. Form factor for the excitation of a $J=2^-$, $T=1$ level in C^{12} at 19.2-MeV excitation energy. The full line is the square of the transverse matrix element calculated with the wave functions of Ref. 7. The experimental points are plotted as in Fig. 1.

in which F^2 stands for the elastic form factor. This expression may be rearranged to extract the quantity corresponding to the square of the transverse matrix element:

$$|\langle 2^- \| T_2^{\mathrm{mag}}(q) \| 0^+ \rangle|^2 \equiv \frac{1}{8\pi}\frac{F^2Z^2}{MA\epsilon}\left(\frac{\mu_p-\mu_n}{2}\right)^2\frac{q_{\mathrm{in}}^4}{M^2}. \qquad (3)$$

Thus this matrix element has a zero value for the same momentum transfer at which the elastic form factor is 0. A good fit to the elastic form factor for carbon, derived from published measurements and some unpublished Orsay data, is

$$F(q)=[1-0.306q^2]\exp(-0.731q^2). \qquad (4)$$

This form factor has a zero value for $q=357$ MeV/c and thus the 2^- state cannot contribute very much to the measured transverse form factor at $q=368$ MeV/c.

The conclusion must be that despite the various measurements already made, these have not yet the systematic character needed to identify all of the components of all of the transitions to states at 19.5-MeV excitation in C^{12}. It would seem likely that a positive-parity state with a large longitudinal matrix element must be added to the 1^- and 2^- states for which theoretical and experimental evidence appears to concur.

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BREAK-UP OF THE 1^+ (12.71 MeV) STATE OF ^{12}C INTO THREE α PARTICLES

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In the reaction

$$^3\mathrm{He}+{}^{10}\mathrm{B}\to{}^{12}\mathrm{C}(1^+, 12.71\ \mathrm{MeV})+p\to 3\alpha+p, \qquad (1)$$

the energy spectrum of an outgoing α particle exhibits three well-defined peaks, the central one of which is located in the kinematical region where two α particles interact through the first 2^+ excited state of ^{8}Be.[1] This behavior can be understood as a result of the interaction between α particles in the final state.

The cross section for Reaction (1) is

$$\frac{d\sigma}{d\Omega_p dE_\alpha d\Omega_\alpha}=\frac{d\sigma^*}{d\Omega_p}\frac{dP}{dE_\alpha d\Omega_\alpha}, \qquad (2)$$

where $d\sigma^*/d\Omega_p$ is the cross section for the reaction $^{10}B(^3He,p)^{12}C^*$ and $dP/dE_\alpha d\Omega_\alpha$ is the probability for the decay $^{12}C^* \to 3\alpha$.[2] It is necessary only to consider this probability since it determines the energy spectrum of the outgoing α particle up to a constant factor. In the ^{12}C rest system, it is

$$\frac{dP}{dWd\Omega} \propto [W(2E-3W)]^{1/2} \int d\omega\, |(\psi_f, H_{\text{int}}\psi_i)|^2, \qquad (3)$$

where E is the total kinetic energy of the three α particles, $W = \hbar^2 p^2/2m$ is the kinetic energy of the observed α of momentum $\hbar\vec{p}$ in solid angle $d\Omega$, and $d\omega$ is the solid angle associated with the relative momentum $\hbar\vec{k}$ of the unobserved α's. ψ_i is the 1^+, 12.71-MeV excited state of ^{12}C and

$$\psi_f = \underset{123}{\text{sym}}\{\psi_{kp}^{(-)}(123)\},$$

(α particles numbered 1, 2, 3) is the eigenstate of the three α-particle system corresponding to the free motion of all particles. It belongs to the double continuum of the energy-eigenvalue spectrum characterized by the two vectors $\vec{k}, \vec{p}$.

For the calculation of the matrix element in Eq. (3) it must be noticed that angular momentum and parity conservation select the 1^+ component of ψ_f. This component contains, in a first approximation, those terms that describe the motion of two α particles with relative $l=2$ coupled to a third α particle with $l'=2$ relative to the center of mass of the pair. Since relative $l=2$ implies little interpenetration of the two α particles, we can use the asymptotic form for the wave function of their relative motion.

Then, for the calculation of the matrix element in Eq. (3) we shall use

$$\psi_{kp}^{(-)}(123) = \varphi(12,3) + \chi(12,3) + \chi(23,1) + \chi(31,2), \qquad (4)$$

where

$$\varphi(12,3) \equiv e^{i(\vec{k}\cdot\vec{r} + \vec{p}\cdot\vec{\rho})} \qquad (5)$$

and

$$\chi(12,3) \equiv f^{(-)}(\vec{k},\hat{r})(e^{-ikr}/r)e^{i\vec{p}\cdot\vec{\rho}}. \qquad (6)$$

In the definitions (5) and (6) $\hbar\vec{k}$ is the relative momentum of the pair 12, $\hbar\vec{p}$ is the momentum of particle 3, $\vec{r} = \vec{r}_1 - \vec{r}_2$ is the relative position of the pair 12, and $\vec{\rho} = \vec{r}_3 - \frac{1}{2}(\vec{r}_1 + \vec{r}_2)$ the position of particle 3 relative to the center of mass of the pair 12. $\chi(23,1)$ and $\chi(31,2)$ are given by the corresponding expressions with

$$\vec{k}_{23} = -\tfrac{1}{2}\vec{k} - \tfrac{3}{4}\vec{p}, \quad \vec{p}_1 = \vec{k} - \tfrac{1}{2}\vec{p},$$
$$\vec{k}_{31} = -\tfrac{1}{2}\vec{k} + \tfrac{3}{4}\vec{p}, \quad \vec{p}_2 = -\vec{k} - \tfrac{1}{2}\vec{p}, \qquad (7)$$

substituted for $\vec{k}$ and $\vec{p}$.

The amplitude $f^{(-)}(\vec{k},\hat{r})$ of the incoming spherical waves is obtained from the α-α phase shifts by a well-known expression.

Application of Eq. (4) with the definitions (5), (6) after expansion in angular momentum eigenstates and use of conservation of angular momentum and of the symmetry of the initial and final states and of the interaction yields

$$(\psi_f, H_{\text{int}}\psi_i) = (3!)^{1/2} \sum_{mm'} C_{22}(1M; mm')\{b(k,p)Y_{2m}(\hat{k})Y_{2m'}(\hat{p}) + a(k,p)T_2(k)Y_{2m}(\hat{k})Y_{2m'}(\hat{p})$$
$$+ a(k_{23},p_1)T_2(k_{23})Y_{2m}(\hat{k}_{23})Y_{2m'}(\hat{p}_1) + a(k_{31},p_2)T_2(k_{31})Y_{2m}(\hat{k}_{31})Y_{2m'}(\hat{p}_2)\} \qquad (8)$$

with

$$T_2(k) = e^{i\delta_2}\sin\delta_2/k \qquad (9)$$

($\delta_2 = \alpha$-α 2^+ phase shift).

The quantities $b(k,p)$ and $a(k,p)$ represent the spatial overlap of the plane-wave and scattered-wave parts, respectively, of the final state with $H_{\text{int}}\psi_i$. An estimate of the relative magnitude of $|b(k,p)|$ and $|a(k,p)|$, utilizing a harmonic-oscillator wave function for the initial state and a phenomenological Gaussian potential[3] for H_{int}, shows that $|b|/|a| < 10\%$. This is in agreement with the experimentally observed near absence of simultaneous break up of $^{12}C^*$. Accordingly we neglect the term $b(k,p)$.

The $a(k,p)$'s will be slowly varying functions of k and p. On the other hand, $T_2(k)$ has a pole corresponding to the first 2^+ excited state of ^{8}Be. We therefore make the further approximation of taking each $a(k,p)$ to be a constant corresponding to its value at the point (k_r, p_r) for which $T_2(k)$ has the pole. The three a's in Eq. (8) become then the same constant and the matrix element is

$$(\psi_f, H_{\text{int}}\psi_i) = (3!)^{1/2} a \sum_{mm'} C_{22}(1M; mm')\{T_2(k)Y_{2m}(\hat{k})Y_{2m'}(\hat{p}) + T_2(k_{23})Y_{2m}(\hat{k}_{23})Y_{2m'}(\hat{p}_1) + T_2(k_{31})Y_{2m}(\hat{k}_{31})Y_{2m'}(\hat{p}_2)\}. \tag{10}$$

Substitution of (10) into Eq. (3), integration over $d\omega$, use of experimentally determined phase shifts δ_2,[4] and corrections for Coulomb effects (and after transformation to the laboratory system) gives the result shown in Fig. 1 by the solid line for the ^{3}He energy and proton and α-particle angles of Fig. 29 of Ref. 1. The height of the central peak of the theoretical curve has been normalized to the experimental value at the central peak. Other than this, there are no free parameters involved.

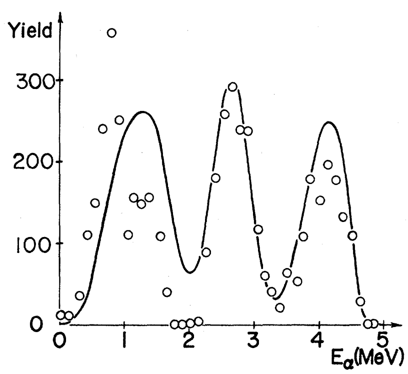

FIG. 1. The solid line is the calculated α-particle energy distribution. The circles are the experimental results of Fig. 29 of Ref. 1.

The theoretical curve is seen to give a good over-all fit to the experimental α-particle energy distribution. (It should be noted that because of other processes occurring in the same kinematical region, the low-energy peak is particularly sensitive to the way in which the background is subtracted.)

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