to it (or generate a great deal of heat); in addition, when a large interaction volume is involved, the total number of modes available into which the forward-traveling wave may decay also becomes proportionally large. To account for all these, it can be shown[5] that the peak acoustic power should be, for $2\alpha_p \gg I_l\tilde{g}(\bar{\omega}_s) \gg L^{-1}$,

$$P_{ac}(\text{peak}) \cong \left[\frac{I_s(0)}{I_l(0)}\right]\left[\frac{I_l(0)\tilde{g}(\bar{\omega}_s)}{2\alpha_p}\right]\left[\frac{\bar{\omega}_p}{\omega_l}\right] P_l(0) \quad (6)$$

in the steady state, where $P_l(0)$ is the total power of the incident primary light. On the basis of the data shown in Fig. 2(a) on the Stokes light, this formula predicts a peak acoustic power of $3.4\times10^{-3}$ W for the case $I_l(0)=6\times10^7$ W/cm$^2$ as compared with the measured value of $1.6\times10^{-3}$ W. Thus, all the experimental results consistently show a rather surprising agreement with the theoretical estimates made on the basis of the simple steady-state theory. It is possible that the inclusion of the transient effects may further extend the range of agreement.

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[7]The symbols that have the same definitions as in Refs. 2 and 5 are: $L$, interaction length; $k$, Boltzmann's constant; $T$, temperature; $\omega_l$ and $C$ are the angular frequency and speed of the primary light in the medium; $\Delta\Omega$, solid angle subtended by the primary light in the medium $\simeq10^{-4}$ sr; $\bar{\omega}_s$, the Stokes frequency that satisfies the Brillouin frequency condition; $\bar{\omega}_p = \omega_l - \bar{\omega}_s$; $v_p$ and $k_p$ are the acoustic velocity and wave number, respectively; $\mu$, $\epsilon$, and $\kappa$ are the permeability, permitivity, and elastic constant of the medium, respectively; $\gamma$, Pockel's elasto-optic constant multiplied by $\epsilon^2$. Here $\alpha_p$ refers to the acoustic attenuation constant; we have previously (Refs. 2 and 5) used it to designate the attenuation constant for the acoustic intensity or power.

[8]The length of the cylindrical region of volume $V_{ac}$ is $\approx$1 cm and the diameter $d \approx 0.36$ cm; $\Delta\theta_{HN} \approx 4\times10^{-2}$ rad; $A_{HN} \approx 0.2$ cm$^2$.

## HOT-ELECTRON INJECTION INTO HELIUM AND OTHER INSULATING LIQUIDS FROM A TUNNEL JUNCTION*

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Hot electrons have been injected into liquid helium, cyclohexane, and benzene from a metal-oxide-metal cathode. The effective barrier for electron injection into helium from these metalliclike electrodes was found to be between 2 and 3 eV, an extra large value probably arising from the additional energy requirements on the free electron in overcoming energy-loss processes and the image potential. The range of these low-energy free electrons is estimated to be ~70 Å.

The binding energy of an electron was ~0.9 eV in cyclohexane and ~1.4 eV in benzene, their difference being attributed to the electron affinity of the benzene molecule.

This paper reports on measurements on the magnitude of current produced in liquid helium, cyclohexane, and benzene from a thermionic tunnel cathode. It is well known[1] that electron temperatures as high as 15 000°K can be generated with only about 10 V across an Al-$Al_2O_3$-Au diode while the lattice is kept at the temperature of the surroundings. Such diodes

can emit more than $10^{-6}$ A into the vacuum. The emission current follows Richardson's law for over four orders of magnitude with the electron temperature being a linear function of the voltage across the tunnel junction. Thus, these cathodes are excellent thermionic emitters into insulators because the lattice temperature of the cathode and the insulator are not affected. Thermionic emission can be used to determine the energy required to inject electrons into liquids, just as the photoelectric method was used in helium[2] and argon,[3] and the electron-stream method[4] was used in helium. In the thermionic emission case, we determine the energy from the ratio of the slopes of the logarithm of the emission currents into vacuum and the liquid:

$$\epsilon = \varphi_{\mathrm{Au}}(1 - s_i/s_v), \tag{1}$$

where

$$s_i = \frac{d[\ln(j_c/v_d^2)]}{dv_d^{-1}} = \frac{-e(\varphi_{\mathrm{Au}} - \epsilon)}{a}, \tag{2}$$

$$s_v = \frac{d[\ln(j_v/v_d^2)]}{dv_d^{-1}} = \frac{-e\varphi_{\mathrm{Au}}}{a}, \tag{3}$$

and $a$ is the factor which relates the junction voltage to the electron temperature. This method requires no calibration other than the work function of the gold since the same cathode is used in both experiments and the constant $a$ need not be known.

When electrons are injected into dense media such as liquids, there is a large attenuation of the current due to back diffusion which depends upon the mean free path, energy loss processes, and the general features of the electric field near the electrode. Thomson[5] calculated this effect assuming that the electric field was constant and that there were only elastic scattering processes. In this case

$$j_c = j_0 \mu E (6\pi)^{1/2}/v_{\mathrm{th}}, \tag{4}$$

where $j_0$ is the current available from the electrode, $E$ is the electric field, assumed to be constant, $\mu$ is the mobility, and $v_{\mathrm{th}}$ is the thermal velocity of the electron. For inelastic scattering processes one can derive an expression for the net current flow by solving the following equation derived from continuity of current:

$$j_c = j_0 e^{-(x/x_0)} - D(\partial n/\partial x) + ne\mu E, \tag{5}$$

where $x_0$ is the average distance traveled into the liquid by the hot electrons before they are thermalized down to their lowest conducting state (this lowest state may be an electron in a bubble in helium, a negative ion in benzene, or a trapped electron in cyclohexane), $n$ is the density, and $D$ is the diffusion constant of the thermalized electrons. It is assumed that the relaxation of the entire distribution of hot electrons can be represented by the single range $x_0$.

If $E$ is again constant, and $n$ at the electrodes is zero, then

$$j_c = j_0 eEx_0/kT. \tag{6}$$

[Thomson's result (4) can be derived from (6) by making $x_0$ equal to the mean free path for elastic scattering.]

Because helium can be made so pure and because it has a dielectric constant almost equal to unity, there is no distortion of the field very close to the electrodes except due to the image charge. Typically, for applied fields between $10^3$ and $10^4$ V/cm, the maximum of the potential is located from 180 to 600 Å from the electrode. Thus, if the range of the hot electrons is short, most of them will be trapped in the potential well because of the image charge and will finally recombine at the electrode.

If again it is assumed that the injection can be described by an exponential with a single average range, $x_0$, for the distribution of hot electrons, then the solution of (5) for moderately high fields of $10^3$-$10^4$ V/cm is

$$j_c \simeq j_0 \exp(-x_m/x_0), \tag{7}$$

where in (5) $E = E_a - e/4x^2$, $E_a$ is the applied field, and $x_m = (e/4E_a)^{1/2}$ is the position of the maximum of the potential. (Onsager[6] derived a similar expression while considering the question of radiation-induced conductivity in dense gases.)

We do not know how $x_0$ varies with the energy of the injected hot electrons. However, if it is assumed that $x_0$ increases with energy, then the more energetic electrons will penetrate much further into the helium than the lower energy electrons. Because of the sharp

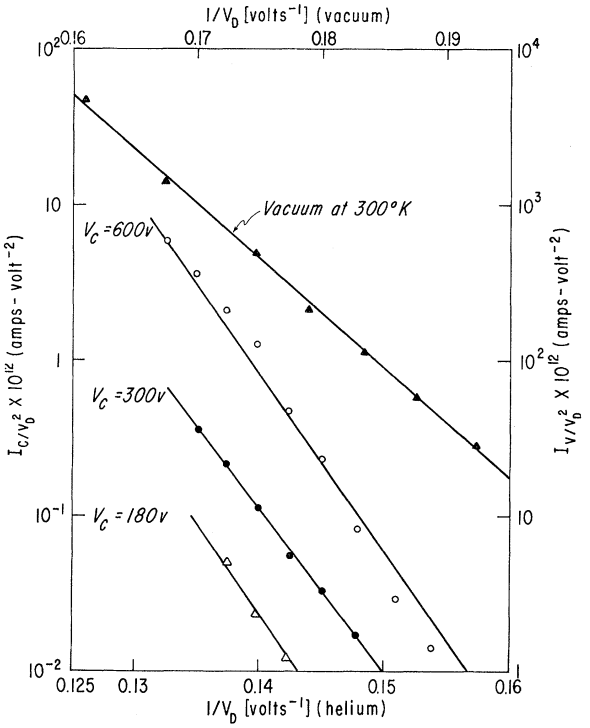


FIG. 1. Semilog plot of the emission current divided by the junction voltage squared versus reciprocal junction voltage for vacuum at 300°K and helium at 1.8°K. Vacuum emission current at 4.2°K is about 50 times smaller than the room-temperature current but has about the same slope. Area $\simeq 0.1$ cm$^2$.

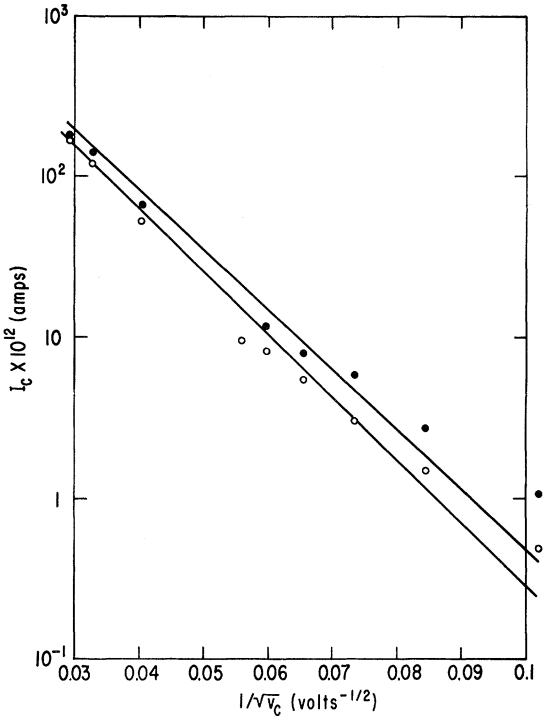


FIG. 2. Semilog plot of the emission current into helium at 1.8°K versus the reciprocal square root of the applied voltage across the helium. The sensitivity of our apparatus was around $10^{-12}$ A; so the deviation from a straight line at low current levels may be experimental. Area $\simeq 0.1$ cm$^2$. Results from two typical separate runs are shown.

dependence of the current on $x_0$, these energetic electrons will contribute much more to the current than the lower energy ones and the observed barrier should be much larger than the adiabatic barrier.[7-9]

Figure 1 shows typical data for emission into vacuum at 300°K and into helium at 1.8°K versus reciprocal junction voltage. Emission into vacuum at 4.2°K, while being 50 times smaller that at 300°K, shows only a slight reduction in the absolute magnitude of the slope. This has the effect of slightly increasing the value of the barrier derived from our data which was between 2 and 3 eV (obtained from over 15 runs from several diodes). Figure 2 shows our results for emission as a function of applied field. As can be seen, a semilog plot of $j$ vs $v^{-1/2}$, where $v$ is the voltage across the helium, gives a reasonably straight line. The slope of this line gives $x_0$ which was found to be 70 Å. It seems to us that this implies a very rapid relaxation down to the bubble state. It is difficult to estimate this relaxation time, but if one assumes that the electron is scattered by every helium atom, then the average relaxation time for the distribution is of the order of magnitude of $10^{-11}$ sec.

The relative straight lines obtained from both Figs. 1 and 2 indicate that (7) is a moderately good approximation for the current characteristics. The true adiabatic barrier should be obtained from a measurement of $j_0$ as a function of electron temperature, and not $j_C$ as we have done. Thus unless saturation currents versus applied field across the helium are observed, a different value for the barrier would be expected.

The very short range raises a question regarding the accuracy of the barrier determinations by Sommer[4] and Woolf and Rayfield[2] and the binding energy determination of Halpern et al.[3] Sommer[4] assumed that all of the incident electrons of sufficient energy are transmitted into the helium. Since $x_0$ is so small, Eq. (6) applies and for field strengths of $10^2$ V/cm there would be an attenuation of more

than 3. If $x_0$ is a function of energy as expected, then the contribution of the higher energy electrons would be underestimated and Sommer's value would be a lower limit on the true barrier. Woolf and Rayfield[2] observed an attenuation of about $10^6$. With such a large deviation from saturation, it is difficult to assess the reliability of these results. Because of the relatively low field strength and the inhomogeneous nature of the cesium-antimony cathodes used by them, the possibility exists that they may have been looking at a photoassisted field emission from isolated points charged to a high negative potential. Halpern's[3] value for the binding of an electron in argon includes the energy needed to overcome the energy-loss processes since he had an attenuation of about 10. The magnitude of this effect is not known and therefore 0.33 eV represents a probable lower limit on the true binding energy.

The electron-emission characteristic into cyclohexane is shown in Fig. 3. The field-strength dependence was linear up to current values limited by space charge. Probably Eq. (4) or (6) governs the injection and the binding energy derived is pretty good, provided $x_0$ is no more than a linear function of energy. The values for the binding energies obtained from Fig. 3 is 0.9 eV. Similar experiments in benzene give a value of 1.4 eV. Because the benzene and cyclohexane molecules are about the same size and these liquids have about the same dielectric constant, the difference is attributed to the electron affinity of the benzene molecule.

The attenuation of the current for both benzene and cyclohexane was about $10^3$ to $10^4$. This implies an $x_0$ of about 10 Å. Since cyclohexane is saturated, it is doubted that the short range is due to electron capture by a single molecule, but it may be a result of the electron being trapped in a cage of two cyclohexane molecules.

The saturation of the current in cyclohexane at low applied fields is due to the onset of space-charge-limited current. The saturated current is approximately proportional to the square of the applied field. From these results, we derive a mobility in good agreement with pulse measurements.[10] We used the geometrical area in order to derive the value of mobility. Since there is good agreement, this means most of the area of the diode is effectively injecting into the liquid. We kept the diode operating voltage less than the critical value needed to create negative resistivity and the resulting emission from pin holes.[11]

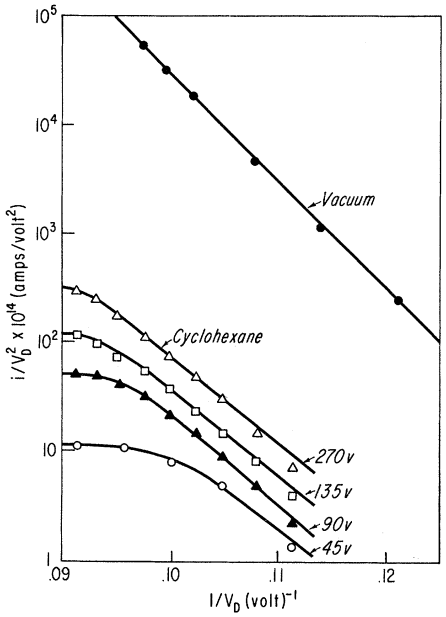


FIG. 3. Semilog plot of the emission current divided by the junction voltage squared versus reciprocal junction voltage for vacuum and cyclohexane at 300°K.

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## LATTICE MODEL FOR THE λ TRANSITION IN A BOSE FLUID*

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The critical properties of a lattice of planar classical "spins," which can be considered as a model for the λ transition in a Bose fluid, are studied using high-temperature series expansions. It is conjectured that in three dimensions the critical exponents ($T > T_c$) for the specific heat and susceptibility of the model are $\alpha = 0$ (corresponding to a logarithmic singularity) and $\gamma = 1\frac{5}{16}$, respectively.

In this Letter we report some results obtained from a relatively simple model of the λ transition in a Bose fluid proposed recently by Vaks and Larkin.[1] These authors have shown that close to the λ point (where the long-wave correlations dominate), the grand partition function of a system of interacting bosons is essentially equivalent to the partition function of a lattice of planar classical "spins" in zero external magnetic field. We can write the Hamiltonian for such a lattice of $N$ sites, in a magnetic field $\vec{H}$, as

$$\mathcal{H} = -2J\sum_{\langle ij\rangle} \vec{s}_i \cdot \vec{s}_j - m\vec{H} \cdot \sum_{i=1}^{N} \vec{s}_i, \qquad (1)$$

where $\vec{s}_i$ is a two-dimensional unit vector, $m$ is the magnetic moment per spin, and the first summation is taken over all nearest-neighbor pairs in the lattice. The direction of the magnetic field $\vec{H}$ is taken parallel to the planes containing the spins. It is interesting to note that in one and two dimensions the planar-spin model has no spontaneous magnetization.[2] The spontaneous magnetization is analogous to the order parameter $|\Psi|$ in a superfluid. This suggests $|\Psi| = 0$ in one and two dimensions.[3] (In two dimensions a phase transition of the type suggested by Stanley and Kaplan[4] would not be excluded.)

The partition function and correlation functions of the planar-spin model can be evaluated exactly in one dimension[1,5] provided $\vec{H} = 0$. To study the properties of the model in higher dimensions we have derived the leading coefficients in the high-temperature series expansions of the zero-field specific heat and susceptibility. The techniques which were used by the present authors to derive the high-temperature series of the classical Heisenberg model[5-7] have been applied to the planar-spin model.

The first eight coefficients of both the susceptibility ($\chi_0$) series and the specific-heat ($C_0$) series have been obtained for a general lattice in zero field. For the three cubic lattices the values of the coefficients $a_r$ and $b_r$, defined by

$$\chi_0 = (Nm^2/2kT)\sum_{r\geq 0} a_r K^r \qquad (2)$$

and

$$C_0 = Nk\sum_{r\geq 2} b_r K^r, \qquad (3)$$

with $K = J/kT$, are presented in Tables I and II, respectively. The susceptibility series

Table I. High-temperature susceptibility coefficients.

| $n$ | $a_n$ (fcc) | $a_n$ (bcc) | $a_n$ (simple cubic) |
|---|---|---|---|
| 0 | 1 | 1 | 1 |
| 1 | 12 | 8 | 6 |
| 2 | 132 | 56 | 30 |
| 3 | 1398 | 388 | 147 |
| 4 | 14 496 | 2592 | 696 |
| 5 | 148 294 | 17 230.667 | 3275 |
| 6 | 1503 063 | 112 843.333 | 15 171.5 |
| 7 | 15 132 379.25 | 736 900.167 | 70 009.125 |
| 8 | 151 568 185.167 | 4773 834.333 | 320 513.25 |