ALTERNATIVE CAUSE OF THE SOLAR OBLATENESS

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We show that the turbulent viscosity forces acting on photospheric meridional currents could cause the solar photosphere to be oblate by more than ±70 km. Although the sign of the effect is undetermined, this might remove the alleged discrepancy in Einstein's prediction for the perihelion advance of Mercury.

Dicke and Goldenberg[1] have recently reported that they have observed the solar photosphere to be oblate by a fraction $\Delta=(r_{\rm eq}-r_{\rm pole})/r=5\times10^{-5}$. Since the observed surface rotation can produce an oblateness of only 1.0×10^{-5}, they interpret their measurements as revealing an internal mass quadrupole moment caused by a rapid rotation of the solar interior below the convection zone (CZ).

However, this interpretation depends crucially on the assumption that the stresses caused by agents other than the pressure and the effective gravity may be neglected. It is the purpose of this Letter to point out that there is in the photosphere a turbulent viscosity which is high enough to cause a great deal of distortion when acting on large-scale photospheric meridional currents, which are defined as the projection of the fluid velocity into the local plane of constant longitude.

One of the standard ways[2-6] of treating the interaction of a small-scale turbulent velocity field with a large-scale velocity is to introduce a phenomenological turbulent viscosity $\eta=\frac{1}{3}\rho v_t l$, where ρ is the mass density, v_t is the root-mean-square turbulent velocity, and l is the scale of the turbulence, or "mixing length." Table I gives typical values for these and other parameters in the solar photosphere. The values in the last two columns (except for v_t at $\tau=1.3$; see below) are taken from a CZ model by Baker and Temesvary[7] using the mixing-length theory of Böhm-Vitense, with the mixing length l set equal to $\frac{3}{2}h_p$, where h_p is the pressure scale height. D is the depth in km below the level optical depth $\equiv\tau=\frac{2}{3}$.

The turbulence at the limb optical depth $\tau=0.005$, which is where the oblateness observations actually were made, is the so-called "microturbulence"[8] of scale $l\ll(\kappa\rho)^{-1}$, where κ is the absorption coefficient per unit mass. Since $(\kappa\rho)^{-1}=dD/d\tau$, we can use the photospheric model of Heintze, Hubenet, and de Jager[9] to find that $dD/d\tau\approx1.5\times10^9$ cm at $\tau=0.005$. No direct evidence of the value of l at this height is available, but it is consistent with $l\ll(\kappa\rho)^{-1}$ and with the usual behavior of turbulence to take again $l\approx$ density scale height $\approx\frac{3}{2}h_p\approx3\times10^7$ cm.

The turbulent velocities in the first two columns are values representative of the models discussed by Nissen[10] and by Ref. 9, whereas that of the last column is the convective turbulent velocity from Ref. 7.

Plaskett[11] has observed photospheric meridional currents of about $v_m\approx5\times10^3$ cm/sec, running predominantly from pole to equator. The present author[6] has derived $v_m\approx8\times10^3$ cm/sec in the upper CZ for high latitudes from a high-Reynolds-number approximation based on a theory of anisotropic convective viscosity, the surface circulation being again from pole to equator. However, the approximation used breaks down in the layers considered here.

There is no firm evidence as to whether or not the photospheric centers of magnetic activity move with the meridional currents.[12] However, the magnetic field energy densities

Table I. Solar photosphere parameters ($l=\frac{3}{2}h_p$, $v_m=3\times10^3$ cm/sec).

τ	D (km)	ρ (g/cm^3)	l (cm)	v_t (cm/sec)	$\eta=\frac{1}{3}\rho v_t l$	$v_m\eta l^{-2}$ (dyn/cm^3)	$\frac{1}{2}\rho R_\odot\Omega^2$ (dyn/cm^3)
0.005	−270	5×10^{-8}	3×10^7	1.5×10^5	7.5×10^4	2.5×10^{-7}	1.6×10^{-8}
1.3	+34	4.0×10^{-7}	2.3×10^7	1.5×10^5	4.6×10^5	2.6×10^{-6}	1.5×10^{-7}
8.3	+130	4.6×10^{-7}	3.3×10^7	2.2×10^5	1.1×10^6	3×10^{-6}	1.5×10^{-7}

are considerably greater than the meridional kinetic-energy densities, and the photospheric fields are presumably tied to sources deeper in the CZ, where the meridional currents are slower.

The turbulent-viscosity force density is roughly $\vec{f}_{vis} \approx \nabla \cdot (\eta \nabla \vec{v})$ and contains terms of order $\eta v_m L^{-2}$, where L is the "scale height" of the velocity. Equation (15) of Ref. 6 shows that in the upper CZ, $v_m \approx |(1-s)d\eta/dr|\rho^{-1}$, where s is the anisotropy parameter. Therefore, in general, $L \approx$ density scale height $\gtrsim h_p = \frac{2}{3}l$, and we can thus set $L \approx l$. Note that this implies that we are at the limit of validity of our viscosity theory, which is accurate only for velocity scales > the mean-free-path length. However, order-or-magnitude estimates should still be meaningful.

In Table I we have also tabulated $\eta v_m l^{-2}$, using $v_m = 3\times 10^3$ cm/sec, together with $\frac{1}{2}\rho R_\odot \Omega^2$, which is the average centrifugal-force density. The above value of v_m is to be regarded only as suggestive, since the theory and observation of the large-scale photospheric currents are in a rudimentary state. Note that the level $\tau = 0.005$ is only a scale height above $\tau = \frac{2}{3}$, where Plaskett's observations were made. With this value of v_m we see that the viscous forces are more than 10 times higher than the centrifugal forces, and thus one can conclude that if these viscous forces act in the proper sense over the solar surface, they are capable of producing more than 10 times as much distortion as the centrifugal forces; i.e., 70 km as opposed to 7 km. The oblateness observations showed an excess distortion of 28 km.

In cylindrical coordinates R, Z, φ the R component of the equation of motion with $\partial/\partial t = \partial/\partial\varphi = 0$ is

$$\rho\left(v_R\frac{\partial v_R}{\partial R} + v_Z\frac{\partial v_R}{\partial Z} - R\Omega^2\right) = -\frac{\partial P}{\partial R} - \rho\frac{\partial \psi}{\partial R} + f_R(\eta, v_R, v_Z),$$

where f_R is the radial part of the viscous force. We can neglect a possible anisotropic part of the viscosity. There is no centrifugal term in the Z component of the equation of motion. Note that the centrifugal term above is negative; thus, if f_R is positive, it will have a distorting effect in the same direction. Likewise, a negative f_Z (for $Z > 0$) would tend to push material towards the equatorial regions and produce a positive oblateness.

It is now convenient to go over into a locally Cartesian coordinate system, with the new Z axis vertically upwards from the solar surface and the X axis horizontal and directed along the meridian line toward the equator. Then over most of the solar surface we will have $v_m = (v_R^2 + v_Z^2)^{1/2} \approx |v_x| \gg |v_z|$. Since the velocity stratification is predominantly in the z direction, it follows that $|\partial v/\partial z| \gg |\partial v/\partial x|$. By axial symmetry $\partial v/\partial y = 0$. Further, $\partial\eta/\partial x \approx 0$, and hence the dominant component of the viscous force may be shown to be simply $f_x \approx \eta \partial^2 v_x/\partial z^2 + (\partial\eta/\partial z)(\partial v_x/\partial z)$.

Now if $f_x > 0$, then $f_R > 0 > f_Z$ (for $Z > 0$), as required for positive oblateness. Since $\partial\eta/\partial z < 0$, then $\partial v_x/\partial z < 0$ would contribute toward a positive f_x. We probably have $v_x > 0$ as discussed before, and thus the observed oblateness might perhaps be taken as evidence of a meridional current which decreases with height in the upper photosphere, rather than as evidence of an internal rotation rate rapid enough to conflict with general relativity. Further, Goldreich and Schubert[13] have indicated that such a rapid internal rotation would be quickly damped by an interchange instability.

Using this coordinate system, we can show how the actual distortion occurs. Let us integrate horizontally the x component of the equation of motion, neglecting the acceleration terms, which contribute a force density of

$$\rho|v_x(\partial v_x/\partial x) + v_z(\partial v_x/\partial z)| < \rho v^2 l^{-1} \approx 1\times 10^{-8}$$

at $\tau = 0.005$, which is about the same as the centrifugal forces, but much less than the viscous forces. Since the horizontal gravity force is likewise negligible, the integration gives simply

$$P_{eq} - P_{pole} \approx \int_0^R f_x dx \approx 1.4\times 10^4,$$

which is roughly the same as the total unperturbed pressure at this depth! Since we have integrated along a spherical surface of constant gravitational potential, the equatorial density and temperature on this surface will be correspondingly larger (for $f_x > 0$), and the optical depth greater, than at the poles. This shows the distortion away from spherical symmetry.

However, we have seen that we cannot reach any firm a priori conclusions about the sign of the viscous forces, although they are in the proper direction to affect the oblateness in one way or the other. Hence, these forces must be studied in more detail before any conclusions about the magnitude of the solar quadrupole moment can be drawn. The probable error associated with Einstein's prediction for the perihelion advance of Mercury might, therefore, be considerably changed.

We have indicated that photospheric magnetic fields could retard the meridional currents there. It will be interesting to see whether or not the observed oblateness fluctuates with changing magnetic activity over the present solar cycle.

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GAUGE-FIELD ALGEBRA, SOFT-MESON METHODS, AND DIVERGENCES*

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Using Bjorken's methods in the context of the recently proposed gauge-field algebra of currents, we discuss the problem of divergences in electromagnetic mass differences ($m_{\pi^+}{}^2-m_{\pi^0}{}^2, m_{K^+}{}^2-m_{K^0}{}^2$) and decays, weak decays (e.g., $K_1{}^0\to 2\pi$), and asymptotic behavior of electromagnetic processes.

It has recently been shown by Bjorken[1] that knowledge of equal-time commutators of currents with themselves and with the Hamiltonian allows one to determine the most singular part of radiative corrections, electromagnetic mass shifts, and various other processes. He calculated the most singular parts within the context of a U(6)⊗U(6) current-algebra model. We would like to make use of some of his techniques to examine a similar class of phenomena, assuming however the equal-time commutators of the recently proposed algebra of gauge fields.[2] We assume then, with the authors of Ref. 2, the existence of a generalized Yang-Mills[3] Lagrangian coupled to pseudoscalar mesons as in the σ model[4] and, if desired, elementary fermions (as these additional particles may be introduced into the Lagrangian without altering the commutation relations). We find that all electromagnetic mass differences and decays are logarithmically divergent, and weak decays are quadratically divergent. Moreover, we will make use of partially conserved axial-vector current[5] in several calculations to see that these divergences are neglected in the limit of vanishing pseudoscalar-meson mass.

As a first example, consider the recent evaluation of the $\pi^+-\pi^0$ mass difference by Das et al.[6] They start with the standard form

$$m_{\pi^+}{}^2-m_{\pi^0}{}^2=-\frac{e^2}{4\pi}2m_\pi \operatorname{Re}\int\frac{d^4q}{q^2-i\epsilon}D_{\mu\nu}(q)\int d^4x\,e^{iq\cdot x}$$
$$\times[\langle\pi^+|T\{j_\mu{}^{\mathrm{em}}(x)j_\nu{}^{\mathrm{em}}(0)\}|\pi^+\rangle-\langle\pi^0|T\{j_\mu{}^{\mathrm{em}}(x)j_\nu{}^{\mathrm{em}}(0)\}|\pi^0\rangle], \tag{1}$$