background will be relatively strongly collinear (i.e., in $\theta_{\pi\pi'}$) and not so strongly peaked in t_{NN}. We will fully consider in a future publication such "unnatural grouping" diffraction-dissociation backgrounds and their implications for extracting scattering cross sections on virtual pions.

[9]If the πN system has low mass ($\lesssim$1.5 BeV), for process (B), vacuon exchange does not apply. It would also be completely wrong to substitute empirical πN scattering (for $Bp \to \gamma\alpha$) since a high-energy approximation would need to be applied to line B. Instead, diagrams we have neglected would be dominant, e.g., a diagram of topology (A), $\pi N \to \rho N^* \to N\rho\pi$, via π exchange. Of course, restricting events to πN masses in the diffraction region, as originally suggested by Deck (Ref. 6), avoids this problem.

[10]L. Stodolsky, Phys. Rev. Letters 18, 973 (1967).

[11]When the four-momentum squared of a virtual particle is $\gtrsim 1$ BeV^2 off the mass shell, the form factor effect might be expected to be significant. For form factors associated with timelike momentum and high spin particles see, for example, H. P. Dürr and H. Pilkuhn, Nuovo Cimento 40A, 899 (1965).

[12]For ρN scattering see Ross and Stodolsky, Ref. 2.

[13]This is because of the fact that the appropriate diffraction amplitude depends only on momentum transfer at sufficiently high energy as illustrated by the fact that the parenthetical expressions in f_B, f_C go to constants.

[14]The same results apply if the incident particle has spin but is unpolarized.

[15]Illinois-Argonne-Northwestern Collaboration, to be published. The K^*N scattering parameters were taken the same as πN and no $K\rho$ production was considered.

[16]The distribution with respect to momentum transfer t_{NN} is steeper than calculated and reminds one of experimental results of E. W. Anderson et al., Phys. Rev. Letters 16, 855 (1966), for $p \to p^*$ processes. This steepness may be associated with the resonance production part of the process discussed in the following. We would like to thank J. Leitner for bringing our attention to this point.

[17]J. Berlinghieri et al., Phys. Rev. Letters 18, 1087 (1967) and references therein; K. Lai, private communication.

REGGE PARAMETERS FROM LOW-ENERGY SCATTERING*

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Two superconvergent sum rules are used to calculate the forward Regge rho-exchange parameters using only experimental data below the asymptotic region. The residue and intercept found in this manner are consistent with those determined by the asymptotic data.

It is well known that dispersion relations are useful in the calculation of the scattering amplitude in one energy region provided that the amplitude is relatively well known at other energies (e.g., knowledge of the pion-nucleon total cross sections can be used to find the scattering lengths and coupling constant[1,2]). In principle, dispersion relations can also be used to investigate the asymptotic region of the scattering amplitude. An attempt in this direction was made by Höhler, Baacke, and Strauss,[3] who were able to determine the Regge rho parameters by requiring consistency of the real part of the odd pion-nucleon amplitude when calculated either from the forward pion-nucleon charge exchange differential cross section (denote by $d\sigma/d\Omega$) or by the usual dispersion relation using $\Delta_{\pi p} \equiv \sigma_{\rm tot}(\pi^- p) - \sigma_{\rm tot}(\pi^+ p)$. However, because of the implicit nature of the Regge parameters in such a consistency scheme it is difficult to assign errors.

A method due to Igi[4] can be used to provide a cleaner separation of the asymptotic region and its associated parameters. Recently a superconvergent form of Igi's sum rule has been used to exhibit a very simple consistency condition on the Regge amplitude.[5] In its simplest form the condition is[6]

$$\frac{\gamma}{\pi}\left(\frac{\overline{\omega}}{M}\right)\frac{1}{1+\alpha}\left(\frac{\overline{\omega}}{\omega_0}\right)^{\alpha} = -f^2 + \frac{1}{8\pi^2\hbar^2}\int_{\mu}^{\overline{\omega}} k\,d\omega\,\Delta_{\pi p}, \qquad (1)$$

where $\overline{\omega}$ is a pion energy in the asymptotic scattering region, and γ and α are the forward Regge rho-exchange parameters[7] (residue and intercept, respectively). Evaluation of this integral[8] with $\overline{\omega} = 5$ BeV gives

$$[\gamma/(\alpha+1)](9.38)^{\alpha} = 0.72 \pm 0.08. \qquad (2)$$

Of course this relation by itself does not determine either γ or α, and thus high-energy data should still be needed to define the Regge parameters. Furthermore, a considerable amount of data exists (namely $d\sigma/d\Omega$) which has yet to be exploited. We shall now describe a new sum rule, similar in spirit to (1), which isolates a different combination of Regge parameters and hence when used in conjunction with (1) allows both γ and α to be calculated. To accomplish this we use the "inverse" dispersion relation of Gilbert[9] for the forward, odd, pion-nucleon amplitude[10] $F^{(-)}(\omega)$:

$$\mathrm{Im}F^{(-)}(\omega)=\frac{-4f^2k}{\omega\mu}-\frac{2\omega k}{\pi}\mathrm{P}\int_\mu^\infty\frac{d\omega'}{\omega'^2-\omega^2}\frac{\mathrm{Re}F^{(-)}(\omega')}{k'}.$$

For $\omega>\overline{\omega}$ we assume[11] that $F^{(-)}=F_{\mathrm{asy}}{}^{(-)}$, the asymptotic amplitude; then by adding and subtracting $F_{\mathrm{asy}}{}^{(-)}$, we obtain

$$\mathrm{Im}F^{(-)}(\omega)-\mathrm{Im}F_{\mathrm{asy}}{}^{(-)}(\omega)=-\frac{4f^2k}{\omega\mu}-\frac{2\omega k}{\pi}\mathrm{P}\int_\mu^{\overline{\omega}}\frac{d\omega'}{\omega'^2-\omega^2}\frac{\mathrm{Re}F^{(-)}(\omega')}{k'}+\frac{2\omega k}{\pi}\mathrm{P}\int_0^{\overline{\omega}}\frac{d\omega'\,\mathrm{Re}F_{\mathrm{asy}}{}^{(-)}(\omega')}{(\omega'^2-\omega^2)k'}.$$

If the asymptotic amplitude is dominated by a single pole with $\alpha>0$ we can take the limit $\omega\to\infty$ and obtain the superconvergence relation[12]

$$\frac{\gamma}{\pi\alpha}\tan\left(\frac{1}{2}\pi\alpha\right)\left(\frac{\overline{\omega}}{\omega_0}\right)^\alpha=-\frac{M}{\mu}f^2+\frac{M}{2\pi}\int_\mu^{\overline{\omega}}\frac{d\omega}{k}\mathrm{Re}F^{(-)}(\omega),\quad(3)$$

where $\mathrm{Re}F^{(-)}$ is related to the experimental data by

$$|\mathrm{Re}F^{(-)}|=\frac{k}{q\hbar}\left[2\frac{d\sigma}{d\Omega}-\left(\frac{q\Delta_{\pi p}}{4\pi\hbar}\right)^2\right]^{1/2}.\quad(4)$$

Thus the magnitude of $\mathrm{Re}F^{(-)}$ is determined by the forward pi-charge-exchange cross section and by $\Delta_{\pi p}$. From phase-shift analyses,[13] $\mathrm{Re}F^{(-)}$ is known to be positive at threshold, becoming negative for a short interval near 200 MeV/c. Other than this small negative region $\mathrm{Re}F^{(-)}$ seems to remain positive.[14] Evaluation[15] of the relation (3) gives

$$\frac{\gamma}{\pi\alpha}\tan\left(\frac{1}{2}\pi\alpha\right)\left(\frac{\overline{\omega}}{\omega_0}\right)^\alpha=0.73\pm0.15.\quad(5)$$

The relations (2) and (5) can now be used to solve for γ and α giving the values

$$\gamma=0.34\pm0.09,\quad\alpha=0.53\pm0.12.$$

This can now be compared to the Regge parameters determined from asymptotic measurements of $\Delta_{\pi p}$ and $d\sigma/d\Omega$ as analyzed previously,[7]

$$\gamma=0.33\pm0.04,\quad\alpha=0.57\pm0.03.$$

The above results show the self-consistency of the Regge-pole exchange amplitude with both high- and low-energy scattering data.[16]

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[1]J. Hamilton and W. S. Woolcock, Rev. Mod. Phys. 35, 737 (1963); V. K. Samaranayake and W. S. Woolcock, Phys. Rev. Letters 15, 936 (1965); J. Hamilton, Phys. Letters 20, 687 (1966).

[2]There is some disagreement among the results quoted in Ref. 1 as to the size of a_1-a_3. The present author has re-evaluated this quantity and finds $a_1-a_3=0.28\pm0.02$ in better agreement with the results of J. Hamilton.

[3]G. Höhler, J. Baacke, and R. Strauss, Phys. Letters 21, 223 (1966).

[4]K. Igi, Phys. Rev. 130, 820 (1963).

[5]K. Igi and S. Matsuda, Phys. Rev. Letters 18, 625 (1967); A. Logunov, L. Soloviev, and A. Tavkhelidze, Phys. Letters 24B, 181 (1967); D. Horn and C. Schmid, California Institute of Technology Report No. CALT-68-127 (unpublished).

[6]The pion mass is μ, nucleon mass M, $f^2=0.081$, $\hbar^2=0.389$ BeV^2 mb, ω and k are the pion lab energy and momentum in BeV, and q is the c.m. momentum.

[7]The Regge residue has been normalized such that $\Delta_{\pi p}=(8\pi\gamma/Mk)(\omega/\omega_0)^\alpha$; ω_0 is the usual scale factor chosen so that $\omega_0=s_0/2M$ with $s_0=1$ BeV^2. Our conventions are the same as in V. Barger and M. Olsson, Phys. Rev. Letters 18, 294 (1967).

[8]The data for $\Delta_{\pi p}$ are tabulated in M. Focacci and G. Giacomelli, CERN Report No. 66-18, 1966 (unpublished). It should be noted that the error in formula (2) comes predominantly from the 2- to 5-BeV region and reflects to a considerable extent the systematic error in the experiment of A. Citron *et al.*, Phys. Rev. 144, 1101 (1966).

[9]W. Gilbert, Phys. Rev. 108, 1078 (1957).

[10]We use the normalization $\Delta_{\pi p}=(4\pi/k)\,\mathrm{Im}F^{(-)}$.

[11]If a single Regge pole dominates the asymptotic amplitude we can choose

$$F_{\mathrm{asy}}{}^{(-)}=\frac{2\gamma}{M}\left(\frac{\omega}{\omega_0}\right)^\alpha\left(\frac{k}{\omega}\right)[i+\tan(\tfrac{1}{2}\pi\alpha)],$$

the only requirement being that it satisfies the relation

$$\mathrm{Im}F_{\mathrm{asy}}^{(-)}(\omega)=-\frac{2\omega k}{\pi}\int_0^\infty \frac{d\omega'\,\mathrm{Re}F_{\mathrm{asy}}^{(-)}(\omega')}{(\omega'^2-\omega^2)k'}.$$

[12]A generalized form from which both formulas (1) and (3) can be derived as special cases has been recently described by Y. Liu and S. Okubo, Phys. Rev. Letters 19, 190 (1967). For details see M. G. Olsson, University of Wisconsin Report No. COO-881-119 (unpublished).

[13]L. D. Roper and R. M. Wright, Phys. Rev. 138, B921 (1965).

[14]Although between 1.0 and 1.1 BeV/c, $\mathrm{Re}F^{(-)}$ is consistent with zero.

[15]In the (3,3) resonance region a phase-shift analysis (Ref. 13) was found to be useful because of the generally large errors in $d\sigma/d\Omega$ in this region. At higher energies the formula (4) was used. The experimental data for $d\sigma/d\Omega$ below 1 BeV/c are compiled in L. D. Roper, R. M. Wright, and B. T. Feld, Phys. Rev. 138, B190 (1965). Other data used: L. Guerriero, Proc. Roy. Soc. (London) A289, 471 (1966); P. Borgeaud et al., Phys. Letters 10, 134 (1964); A. S. Carroll, Proc. Roy. Soc. (London) A289, 513 (1966); C. Chiu, thesis, University of California, Berkeley [University of California Radiation Laboratory Report No. UCRL-16209 (unpublished)]; W. S. Risk, private communication; P. Falk-Vairant et al., data quoted in Ref. 3.

[16]However, recent results (Olsson, Ref. 12) have shown that there are new sum rules more sensitive to the existence of secondary ρ trajectories.