TENSOR-FORCE, HARD-CORE, AND THREE-BODY PARAMETERS

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Results are presented for the binding energy of H^3 and the doublet n-d scattering length ($a_{1/2}$) using a rank-four separable potential which simulates the hard core as well as the tensor force. The binding energy is nearly four times the deuteron binding energy, and $a_{1/2}$ is 0.13-0.18 F for the best available ${}^1S_0$ potentials, in close agreement with the latest experimental value.

We present here the results of calculation of the n-d doublet scattering length ($a_{1/2}$), and the triton binding energy (BE), using a rank-four separable potential which includes the tensor force and simulates the effect of a hard core in a "soft manner." Previous calculations of these parameters had been confined successively to s-wave forces in the triplet and singlet N-N states[1] followed by the inclusion of the tensor force for the evaluation of the triton parameters[2] as well as the doublet scattering length.[3,4] While the inclusion of the tensor force had yielded a significant improvement in both the quantities over the corresponding s-wave treatment, it still left (i) a high BE by about 1.5-2.0 MeV and (ii) scope for a flip of sign in $a_{1/2}$, to make it positive as required by experiment. It had thus appeared that the effect of the hard core, as manifest in the characteristic behavior of the phase shifts, should have a significant role to play in the three-body problem, if the very idea of the two-body force made any sense at all.[5] It was with such an objective that a recent formulation of the problem had been given so as to include both the hard core and tensor effects with the help of a rank-four potential[3] but the numerical results with such a program could not be obtained immediately due to computational difficulties. The numerical results we have now obtained seem to bear out rather fully the expectation of the decisive importance of both these effects in three-body investigations, by yielding a binding energy nearly four times that of the deuteron, and a small positive scattering length, in good agreement with its new determination of 0.11 ± 0.07 by Seagrave et al.[6]

As to the essential details, the triplet-even forces used in the calculation are the rank-two Yamaguchi[7] and Naqvi[8] potentials as before.[2,3] The ${}^1S_0$ force, on the other hand, is now a rank-two potential of the form[3]

$$M\langle\vec{p}\,|\,V({}^1S_0)\,|\,\vec{p}'\rangle = -\lambda_{13}[f(p)f(p')-f_1(p)f_1(p')], \qquad (1)$$

where

$$f(p)=(\beta_S{}^2+p^2)^{-1},\ f_1(p)=np^2(\beta_0{}^2+p^2)^{-2}. \qquad (2)$$

For the parameters of the latter we have used two sets of values, one given by Naqvi[9] and a family of recent sets obtained by Gupta.[10,11] The Naqvi and Gupta sets which are tuned, respectively, to $r_{0S}=2.355$ and 2.7 F, are listed in Table I. Of the Gupta sets, it is probably sufficient to record that the best fits to the ${}^1S_0$ phase shifts up to about 340 MeV are obtained with $\beta_S=5.5\alpha$ and $\beta_0=10.0\alpha$, in units of the deuteron binding parameter α, while the quality of the ${}^1S_0$ fits gets progressively poorer as β_S is increased from 5.5α and β_0 correspondingly decreased from 10.0α. The results of evaluation of BE and $a_{1/2}$ using different sets of potentials in conjunction with the triplet forc-

Table I. Parameters of the ${}^1S_0$ potential, together with a_S and r_{0S}. (For notation, see text.[a])

Set	β_S/α	β_0/α	N^2	$\lambda_{13}\alpha^{-3}$	$-a_S$ (F)	r_{0S} (F)
N	8.1	8.0	4.986	62.40	23.7	2.355
G_1	5.5	10.0	31.1	16.92	18	2.7
G_1'	6.7	7.8	7.508	18.949	18	2.7
G_1''	5.8	7.3	5.334	20.078	18	2.7
G_2	6.0	6.8	3.783	22.56	18	2.7
G_2'	6.0	6.8	3.928	22.85	20	2.7
G_3	6.2	6.6	3.30	25.32	18	2.7

[a]N and G stand, respectively, for the Naqvi and Gupta sets.

Table II. Values of $a_{1/2}$ in Fermi and BE in MeV with different combination of triplet and $^1S_0$ potentials. (For notation, see text.)

	Binding energy (MeV)		Doublet scattering length (F)	
$^1S_0$ Set	$(C+T)_Y$	$(C+T)_N$	$(C+T)_Y$	$(C+T)_N$
N	9.25	9.79	0.135	−0.262
G_1	9.21	9.74	0.180	−0.216
G_1'	9.01	9.50	0.371	−0.101
G_1''	8.92	9.45	0.452	+0.075
G_2	8.79	9.23	0.606	0.235
G_2'	8.88	9.34	0.561	0.212
G_3	8.66	9.10	0.716	0.383

es $(C+T)_Y$ and $(C+T)_N$, in the notation of Ref. 2, are shown in Table II. The general trend of the BE shows that the Naqvi triplet potential (which is incomplete to the extent that an $\vec{L}\cdot\vec{S}$ term has not been included) is somewhat stronger than the Yamaguchi triplet. The trend of $a_{1/2}$ also shows the same feature, when one remembers that the more negative $a_{1/2}$ is, the more it corresponds to stronger attraction.[12] Physically, therefore, it appears that the Yamaguchi set is somewhat preferred over Naqvi, contrary to our earlier conclusions with rank-one $^1S_0$ potentials.[13]

As for the actual values, the potentials N and G_1 which give the best fits to the $^1S_0$ phase shifts seem to give somewhat overbinding, though of course the discrepancy from experiment (~0.7 MeV) is much less than was the case without hard-core effects, when $(C+T)_Y$ had yielded[2] 10.40 MeV. This is in accord with a general expectation that the hard core should provide a 10-15% decrease in the binding energy. While the binding energy is still in excess by ~0.7 MeV, and would be enhanced to ~1 MeV by the inclusion of relativistic corrections,[14] this magnitude seems to be well within the bounds of neglected effects like the $^1D_2$, ^3D $(\vec{L}\cdot\vec{S})$, etc., terms, and perhaps also to some extent, the shape dependence of the potentials. From this point of view we feel rather reluctant to attach too much significance to the better results obtained, e.g., with G_2 or G_3 at the cost of a less satisfactory fit to the $^1S_0$ phase shifts.

As for $a_{1/2}$, the greater sensitivity of this parameter to the input potentials produces a spectrum of values ranging from 0.135 F for N to 0.716 F for G_3. While the "best" Gupta $^1S_0$ potential G_1 gives 0.180 F, rather close to N, the other Gupta sets produce appreciably higher values. In this respect, the experimental situation which was stable for so many years at $a_{1/2}\approx 0.7\pm 0.3$ F seems to have suddenly changed to an appreciably lower value, viz. 0.11 ± 0.07 F, found from the measurements of Seagrave et al.[6] It is rather amusing to note that the "best" potentials N and G_1 give a striking agreement with this new value, though they leave a gap of 0.7-0.8 MeV in the BE. On the other hand, the "old" value of 0.7 F seems to be reproduced by the potentials G_2 or G_3 which simultaneously give appreciably better values for BE.

We would like to summarize the situation in the following way: From the point of view of using potentials which are better tuned to the two-body data, we consider the results of $(C+T)_Y$ in conjunction with $^1S_0$ (N or G_1) as somewhat more significant than those with G_2 or G_3 (which compromise on two-body fits). This still calls for further theoretical efforts to bridge a gap in the BE to a maximum extent of ~1 MeV, so as not to cause much variation in the determination of $a_{1/2}$ (assuming the new value to be more reliable). Such efforts could be in the direction of (1) changing the triplet parameters to give a different D-state deuteron probability from the 4% value[7] with $(C+T)_Y$, (2) considering somewhat different shape parameters which would cause variations in the off-shell extensions, and (3) calculating the effects of the neglected potential terms, at least in a perturbative manner. In any case, the magnitudes of the discrepancies at this stage are such as to warrant the conclusion that these are essentially matters of finer details. The calculations presented here should leave little doubt not only about the correctness of the two-body force as the basic mechanism, but of the decisively quantitative role of the tensor and hand-core effects[15] in the three-body problem.

We add a few final remarks. As a comparison of results with N and G_1 shows, the sensitivity of BE and $a_{1/2}$ to r_{0S} is much less in the background of the hard core than when the latter is not considered.[2-4] Comparison of G_2 with G_2' also shows little variation with a_S. Secondly, the quartet scattering length, which has a repulsive kernel, would be little affected by the hard-core and tensor effects. Thirdly, the reduction of 10-15% in the BE due to the hard core encourages the expectation that (1) the Coulomb energy which had been found to be ~10% higher than experiment would decrease to the desired value,[16] and (2) the electromagnetic radii of H^3 and He^3, which were earlier found about 10% smaller than experiment,[17] would also increase to the desired levels.

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[1]A. N. Mitra and V. S. Bhasin, Phys. Rev. 131, 1265 (1963); A. G. Sitenko and V. F. Karchenko, Nucl. Phys. 49, 15 (1963); R. Aaron, R. Amado, and Y. Yam, Phys. Rev. Letters 13, 574 (1964); V. S. Bhasin, G. L. Schrenk, and A. N. Mitra, Phys. Rev. 137, B398 (1965); A. C. Phillips, Phys. Rev. 142, 984 (1966).

[2]B. S. Bhakar and A. N. Mitra, Phys. Rev. Letters 14, 143 (1965).

[3]A. N. Mitra, G. L. Schrenk, and V. S. Bhasin, Ann. Phys. (N.Y.) 40, 357 (1966).

[4]A. G. Sitenko, V. F. Kharchenko, and N. Petrov, Phys. Letters 21, 54 (1966).

[5]In Ref. 4 the authors also examined the variation of $a_{1/2}$ with the pp effective range as well as the form factor of the assumed $^1S_0$ potential of rank-one. While they found that the value 0.7 F for $a_{1/2}$ was compatible with a very strong momentum dependence of the $^1S_0$ potential, the latter form is incapable of reproducing the energy dependence of the $^1S_0$ phase shifts.

[6]J. Seagrave et al., Symposium on Light Nuclei, Brela, Yugoslavia, June-July 1967 (to be published).

[7]Y. Yamaguchi, Phys. Rev. 95, 1635 (1954).

[8]J. H. Naqvi, Nucl. Phys. 36, 578 (1962).

[9]J. H. Naqvi, Nucl. Phys. 58, 289 (1964).

[10]V. K. Gupta, thesis, Delhi University, 1966 (unpublished).

[11]We have not used the Tabakin potentials [Ann. Phys. (N.Y.) 30, 51 (1964)] mainly for calculational convenience; see Ref. 3.

[12]Indeed, the earlier history (Ref. 3) of $a_{1/2}$ shows that the largest negative values occurred with effective s-wave forces, and these magnitudes decreased as such forces were made less attractive by making r_{0S} larger and/or including the tensor force.

[13]The reason for this apparent discrepancy from our earlier conclusion of weaker attraction with the Naqvi potential (Ref. 2) can be traced to the magnitude of the triplet strength parameter λ_{31}. In Ref. 8 this value was given as $22.9\alpha^3$, in the presence of a spin-orbit term, and was used as such in Ref. 2. However, in Ref. 3 and in the present work, we have considered an effective value $23.59\alpha^3$ adjusted to fit the deuteron BE just with $(C+T)_N$. The actual Naqvi value, viz., $22.9\alpha^3$, has been checked to give 8.90 and 8.88 MeV with N and G_1, respectively, but the nature of the computer program makes it impossible to check the effect on $a_{1/2}$ so easily.

[14]V. K. Gupta, B. S. Bhakar, and A. N. Mitra, Phys. Rev. Letters 15, 974 (1965). The estimated correction in this paper was +0.5 MeV, but this was on the basis of a smaller nucleus which would give somewhat larger values of $\langle\Delta T\rangle$ and $\langle\Delta V\rangle$. The present inclusion of the hard core would tend to decrease these quantities and hence their net contribution to something less than 0.5 MeV.

[15]We are using the word "hard core" in a rather loose sense, viz., as a potential which gives the desired energy behavior of the $^1S_0$ phase shifts and a quality fit up to about 400 MeV. As a matter of fact, the form (2) of the function $f_1(p)$ suggests a structure which looks more like a "hard shell" [F. Tabakin, Phys. Rev. 137, B65 (1965); J. Dabrowski et al., Phys. Letters 24B, 125 (1967)], since it shows a peaking, not around $p=0$ but at $p\approx\beta_0$.

[16]V. K. Gupta and A. N. Mitra, Phys. Letters 24B, 27 (1967).

[17]V. K. Gupta et al., Phys. Rev. 153, 1114 (1967).