SCATTERING OF 750-MeV ELECTRONS BY CALCIUM ISOTOPES*

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We report experimental differential cross sections for the scattering of electrons by calcium-40 and by calcium-48 at 750 MeV. We have thereby examined the possible energy dependence of the phenomenological charge distribution in terms of which previous data at 250 MeV have been analyzed,[1] and also have explored larger values of the recoil momentum q than was possible at that energy. We find that while there is good agreement over the range of q measured at 250 MeV, the larger q results are significantly different from those predicted by the phenomenological charge distribution. A method is presented and used for determining directly, from the experimental data at large q, the modification needed by the charge distribution so that it gives agreement at large q while preserving the good fit at smaller q values. What emerges is an oscillating modulation in the charge distribution which resembles qualitatively an effect obtained using the shell model. The effects are surprisingly similar for the two isotopes.

Experimental results are given in Fig. 1 for both Ca^{40} and Ca^{48}. They are compared with the cross sections predicted from the charge distribution $\rho_0(r)$ whose parameters were obtained from the 250-MeV data. (Details are given in the caption to Fig. 1.) An adjustment of +1% has been made in the incident energy, consistent with the possible uncertainty in energy selection that the steering magnets had at the time the data were taken. With no further adjustment except this one, which makes the incident energy 757.5 MeV, the agreement of the 250-MeV prediction with the experimental results is remarkably good out to 35°, the angle at which the recoil momentum $q = 2E_0 \sin\frac{1}{2}\theta$ is the same as at the largest measured angle

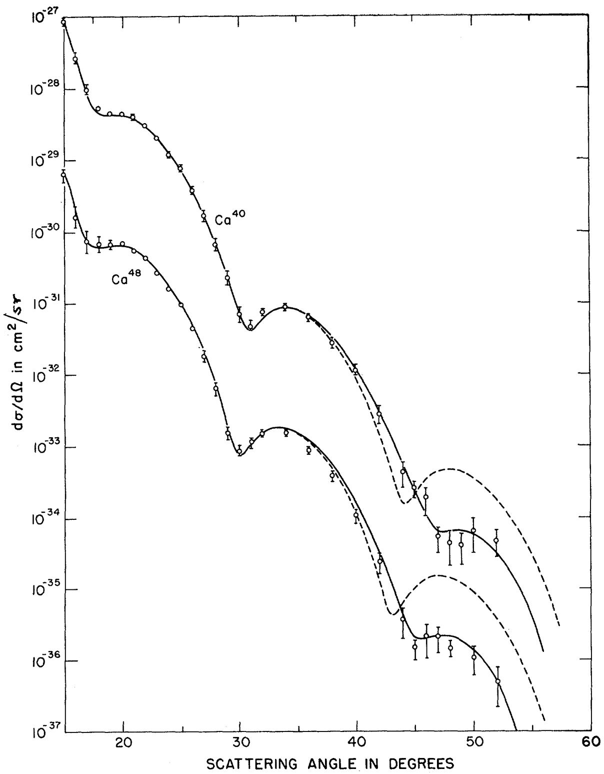

FIG. 1. Experimental and theoretical differential cross sections at 757.5 MeV. The nominal energy was 750 MeV, and a 1% adjustment was made to improve the fit at low q. The dashed curves are the best fits to earlier 250-MeV data. The charge distributions which yield them are parabolic Fermi (three-parameter) shapes [see Eq. (3) of Ref. 1] with the following parameter values: Ca^{40}, $c = 3.6685$ F, $z = 0.5839$ F, $w = -0.1017$; Ca^{48}, $c = 3.7369$ F, $z = 0.5245$ F, $w = -0.0300$. The solid curves, obtained by the method described in this Letter, come from charge distributions with an added $\Delta\rho(r)$, and parameter values $p = 0.5$ F^{-1}, $q_0 = 3.0$ F^{-1}, and $A(Ca^{40}) = 0.5 \times 10^{-3}$, $A(Ca^{48}) = 0.8 \times 10^{-3}$. The cross section for Ca^{40} has been multiplied by 10 and that for Ca^{48} by 10^{-1}.

(125°) at 250 MeV. To an extent limited by the above adjustment, the description of the scattering process in terms of an energy-independent $\rho_0(r)$ is thus consistent with experiment over the energy range 250 to 750 MeV.[2] There is, however, a systematic difference between the 250-MeV prediction and experiment at larger angles, around the third diffraction maximum, and it occurs in a similar way for both isotopes. This may indicate that a basic modification of our present description of scattering, in which the nucleus is described by a static $\rho(r)$, is required. It is possible, however, to avoid such a drastic step, and, by slightly altering $\rho_0(r)$, to regain a good fit over the entire angular range at 750 MeV, as we shall now show. We recognize, of course, that this is not the only possible explanation of the discrepancy, and that further experiments at 900 and 1000 MeV, now in progress, will be helpful in deciding what the correct explanation is.

Although in precise work of the kind reported here the connection between $\rho(r)$ and the differential cross section must always be made by a complete partial-wave analysis, the simple connection given by the Born approximation through the square of the Fourier transform of $\rho(r)$ is qualitatively very useful. The Fourier transform $F_0(q)$ of $\rho_0(r)$, obtained numerically for convenience, is shown in Fig. 2. The diffraction minima correspond to the zeros of $F_0(q)$, and the maxima to either the maxima or minima of $F_0(q)$. The difference between the theoretical curves and the data at large angles could be reduced if the third zero of $F_0(q)$ were shifted to a larger q, and if the subsequent minimum were reduced in magnitude. We can achieve this by adding to $F_0(q)$ the small contribution $\Delta F(q)$ indicated in Fig. 2, or by adding to $\rho_0(r)$ the corresponding (inverse) Fourier transform $\Delta\rho(r)$. For computational convenience we took $\Delta F(q) = A \exp[(q-q_0)^2/p^2]$, so that

$$\Delta\rho(r) = (ZeApq_0^2/2\pi^{\frac{3}{2}})[\sin(q_0r)/q_0r + (p^2/2q_0^2)\cos q_0r]\,e^{(-\frac{1}{4}p^2r^2)}.$$

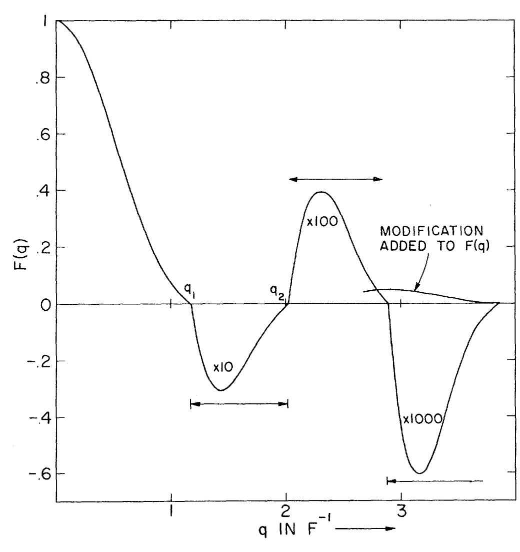

FIG. 2. The form factor $F_0(q)$ of the charge density $\rho_0(r)$ for Ca^{40}. Parameter values are given in the caption of Fig. 1. In order to display this oscillating yet rapidly decreasing function, successive parts have been scaled by the factors indicated. The modification, $\Delta F(q)$, is shown for Ca^{40}.

With the parameters chosen to give the fits illustrated in Fig. 1, the resulting charge densities $\rho_0(r)$, $\Delta\rho(r)$ are displayed in Fig. 3. What has been added to $\rho_0(r)$ is an oscillation. Yet, as the partial-wave calculation which gives the cross sections of Fig. 1 shows, the cross section resulting from $\rho(r) = \rho_0(r) + \Delta\rho(r)$ agrees with that from $\rho_0(r)$ out to about 35°, and from then on does what we wanted. We emphasize that the Fourier transforms have been used only as a guide, and no approximation is involved in obtaining the differential cross sections from the new $\rho(r)$.

The modification involved three additional parameters. The requirements are that the third diffraction minimum be shifted the right amount, that the subsequent maximum be suitably reduced, without affecting $F_0(q)$ appreciably at smaller q, and that $\Delta\rho(r)$ decrease sufficiently rapidly with r that it does not dominate $\rho_0(r)$. These are sufficient to fix the three parameters. There are ambiguities in our $\Delta\rho(r)$ associated with the particular analytic form chosen for $\Delta F(q)$, and with our lack of knowledge of $F(q)$ for even larger q than those observed. The fact that the effect entered at a certain q_1, however, fixes the wavelength of oscillation in $\Delta\rho$ at h/q_1, and the size of the effect there fixes the amplitude. Thus there is less ambiguity than appears at first sight, although this point needs further study.[3]

It is remarkable that the $\Delta\rho$'s needed for Ca^{40} and Ca^{48} are very similar. In the crude fitting reported here they differ only in mag-

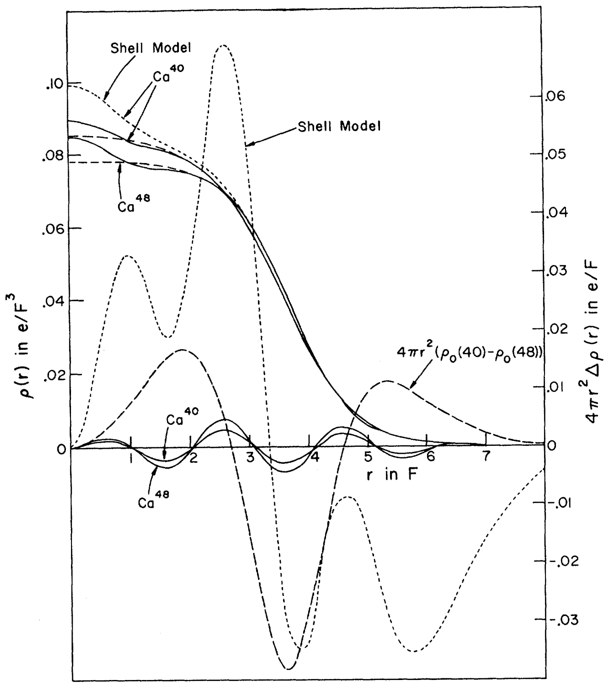

FIG. 3. Charge distributions, plotted in various ways, arising from the present analysis and from previous work. The full lines are the new charge distributions for Ca^{40} and Ca^{48} (referred to the left ordinate) and r^2 times the change $\Delta\rho(r)$ (right ordinate). The dashed curves are the 250-MeV fits whose parameters are given in the caption to Fig. 1, and r^2 times the isotopic difference Ca^{40}-Ca^{48}. The dotted curves are a Ca^{40} charge distribution calculated from the shell model (L. R. Mather, J. M. McKinley, and D. G. Ravenhall, unpublished calculations). The Woods-Saxon potential used had a depth of 55.2 MeV, a radius of $1.269A^{1/3}$ F, a surface thickness of 0.78 F, and a ratio of spin-orbit to central potential characterized by $\lambda = 37.2$. Also displayed is r^2 times the difference between this charge distribution and the 250-MeV fit.

nitude, and it seems clear from the experimental data that the same effect is occurring in both isotopes. For comparison, we give in Fig. 3 also the isotopic difference in charge distributions $(Ca^{40}-Ca^{48}) \times r^2$ obtained from our earlier analysis.[1] The effects on each $\rho(r)$ that we now find are considerably smaller than those isotopic differences, and thus our earlier conclusions are not affected very much.

As regards a possible explanation of this result, we observe that the charge distribution calculated using the nuclear shell model can be made to agree with the phenomenological $\rho_0(r)$ quite closely, but that there always remains a small fluctuation, coming from the oscillations in the radial wave functions of the proton orbitals.[4,5] In Fig. 3 we show a typical case,[4] and it is suggestive that qualitatively the fluctuation is similar in amplitude and wavelength to that we have obtained from the data. We have not been able to adjust the simple shell model so as to give as close a fit to the experimental cross section as the phenomenological $\rho(r)$ does. The fact that the fluctuation given by the shell model is in fact larger than the phenomenological one may indicate that a shell model with a local potential gives too large a fluctuation. We should emphasize that the explanation advanced for the 750-MeV large-angle results is not unique, and it needs corroboration by scattering experiments at higher energies.

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[1]K. J. van Oostrum, R. Hofstadter, G. K. Nöldeke, M. R. Yearian, B. C. Clark, R. Herman, and D. G. Ravenhall, to be published. Preliminary results at 250 MeV appeared in a communication by the same authors, Phys. Rev. Letters 16, 528 (1966).

[2]To some extent this energy independence sets an upper limit on the contribution to the scattering process from virtual nuclear excitation. See, e.g., G. H. Rawitscher, Phys. Rev. 151, 846 (1966). It is not clear to us, however, that the particular mechanism proposed in that reference would produce energy-dependent effects.

[3]An earlier attempt to fit the data [contributed paper by the authors of this Letter, International Conference on High-Energy and Nuclear Physics, Rehovoth, Israel, 1967 (to be published)] involved multiplying $F_0(q)$ by a suitably chosen function of q. That does not shift the diffraction minimum, however, and thus does not give such a close fit to the experimental data. The resulting $\Delta\rho(r)$ is very similar in character and magnitude to the one obtained here, tending to confirm our belief in the present results.

[4]L. R. Mather, J. M. McKinley, and D. G. Ravenhall, unpublished calculations.

[5]Charge distributions resulting from the independent-particle shell model, with a central potential of the Woods-Saxon type, have been reported by several authors. See, for example, L. R. B. Elton and A. Swift, Nucl. Phys. A94, 52 (1967).