the radiation is produced by the induced image charges of the electrons oscillating normal to the surface as the electron beam passes by the rulings. In our experiment, the bombarding electrons are directed normally to the grating surfaces exciting SPO which then radiate. The dispersion relations of these SPO can be obtained in this fashion as illustrated in Fig. 2.

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†A preliminary report was presented at the American Physical Society meeting in New York City, 31 January to 3 February 1967, by one of the authors (YYT).

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FERMI-LIQUID EFFECTS ON PLASMA WAVE PROPAGATION IN ALKALI METALS

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The dispersion of plasma-wave propagation in the alkali metals sodium and potassium has been observed to deviate significantly from that expected for a noninteracting electron gas. Modifications of the dispersion have been calculated using the Landau Fermi-liquid theory and a quantitative evaluation of the first two moments of the interaction function has been made for the case of potassium.

Recently we reported the observation of a new class of wavelike excitations in potassium which propagate perpendicular to a magnetic field near the Azbel'-Kaner cyclotron resonances $n\omega_c = \omega$.[1] The clearest data were obtained with microwave surface currents J flowing parallel to H ($J \parallel H$, "ordinary mode") at field values above the fundamental resonance. The measured dispersion relation (wavelength versus H) for these waves proved to be in quite good agreement with that calculated for a degenerate free-electron gas having an effective mass appropriate to potassium. Later experiments have revealed similar propagation "windows" at each of the first few subharmonics in both the $J \parallel H$ and the $J \perp H$ ("extraordinary mode") polarizations.[2] While the free-electron model is generally successful in accounting for the qualitative existence of all the waves near resonances,[3] it has become apparent that it fails to account quantitatively for the experimental data. The discrepancy is particularly severe for $J \perp H$ near $\omega = \omega_c$. In this Letter we show that the discrepancy is removed if electron correlations are included in the model.

Using the Landau Fermi-liquid theory, we account for the experimental dispersion relation and directly evaluate the first two moments of the Landau interaction function. In principle, precise measurements of the dispersion relation of all the modes would provide a complete quantitative evaluation of all moments of the spin-independent interaction function. The latter is independent of and complementary to the spin-dependent interaction which gives rise to the spin waves recently observed in sodium and potassium.[4,5]

The experimental details remain essential-

ly unchanged[1]: We measure the derivative with respect to magnetic field (dA/dH) of the microwave power absorbed by a plane-parallel slab of highly purified alkali metal at 1.3°K. The linearly polarized surface currents flow on both sample faces in order to enhance sensitivity to any transmission effects. Since the basic unit of length in this experiment is the cyclotron orbit at the fundamental resonance field, $R \propto \omega^{-1}$, the experiments are now performed at two experimental frequencies, $\omega \sim 7.7\times10^{10}$ rad/sec and $\sim 11\times10^{10}$ rad/sec, to check that the structure in the dA/dH curves due to the finite sample thickness, $L \sim 0.10$-0.15 mm, does indeed scale properly with frequency. The sample thickness is directly related to R by separately observing the Gantmakher radio-frequency size effect.

Since these waves penetrate into the bulk of the metallic specimen, it is possible to describe quantitatively their propagation characteristics by neglecting boundary effects and examining their infinite-medium dispersion relation. The dispersion relation for a wave of the form $\vec{E} = \vec{e}e^{i(\omega t - \vec{k}\cdot\vec{r})}$ propagating perpendicular to the static magnetic field pointing in the z direction with $\vec{k}$ along the x axis and $\vec{e}$ the polarization vector in the x-y plane is given by

$$k^2/k_0^2 = \epsilon_{yy} + \epsilon_{xy}^2/\epsilon_{xx}, \tag{1}$$

where $k_0^2 = \omega^2/c^2$. The dielectric tensor $\epsilon_{ij}(k,\omega)$ is simply related to the conductivity tensor,

$$\epsilon_{ij}(k,\omega) = \delta_{ij} + 4\pi\sigma_{ij}(k,\omega)/i\omega. \tag{2}$$

For the parameters involved in these experiments it is sufficient to approximate the dispersion relation Eq. (1) by[6]

$$\sigma_{yy} + (\sigma_{xy})^2/\sigma_{xx} = 0. \tag{3}$$

The physics of the problem lies in the conductivity tensor $\sigma_{ij}(k,\omega)$. For a noninteracting gas of quasiparticles with mass m^*, $\sigma_{ij}(k,\omega)$ may be evaluated from the linearized, Fourier-transformed, Boltzmann equation. A typical element[7] (neglecting dissipative effects) is

$$\sigma_{xy} = -\sigma_{yx} = \frac{3ne^2}{m^*\omega_c}\sum_{n=0}^{\infty}\int_0^{\pi}\frac{a^2}{(a^2-n^2)}\sin^2\theta\,\frac{J_n(b\sin\theta)J_n'(b\sin\theta)d\theta}{b(1+\delta_{n0})}, \tag{4}$$

with $b = kV_F/\omega_c$, $a = \omega/\omega_c$, and $\omega_c = eH/m^*c$. Similar formulas apply to σ_{xx} and σ_{yy}.

A plot of the dispersion relation Eq. (3) utilizing the free-electron conductivity tensor is shown in Fig. 1 along with data for potassium.[8] The agreement is seen to be poor. The reason for this is that in the long wavelength limit ($b \to 0$), and for magnetic fields near the resonance $\omega/\omega_c = 1$, Eq. (3) reduces to

$$1 + b^4/700(\omega/\omega_c - 1) = 0. \tag{5}$$

To order k^2 the free-electron theory predicts no propagation near the first harmonic. To order k^4 there is a weak singular term. In a plot of kR vs ω_c/ω obtained from Eq. (5) the curve stays extremely close to the $\omega_c/\omega = 1$ line, except for very large kR (solid curve in Fig. 1). We can say that a noninteracting electron gas does not "want" to have a propagating mode in the neighborhood of the first cyclotron resonance.[9]

We now show that correlations significant-

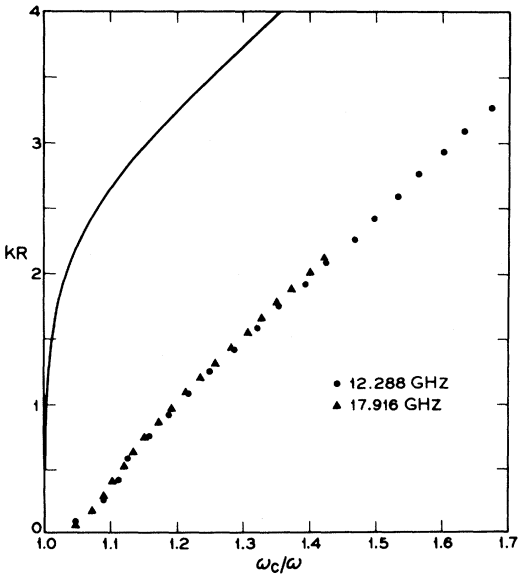

FIG. 1. Comparison of the theoretical noninteracting dispersion curve with the observed extrema of dA/dH taken at two frequencies.

ly modify the dispersion relation in that in the presence of correlations, the dominant term is of order k^2 and not of order k^4.[10] To calculate the conductivity of an interacting electron gas we use the Landau theory of Fermi liquids.[11] The response of the system to slowly varying external disturbances ($\hbar\omega/E_F \ll 1, k/k_F \ll 1$) is completely described by a quasiparticle energy-momentum relationship $E(p, r, t, \sigma)$ and a quasiparticle density matrix $\rho(p, r, t, \sigma)$ or distribution function. In equilibrium, the quasiparticle distribution function $n_0(p)$ is a Fermi distribution and the quasiparticle energy $E^0(p) = p^2/2m^*$.[12] In nonequilibrium, the energy of the quasiparticles is a functional of the distribution function:

$$E(p, r, t, \sigma) = E^0(p) + [1/(2\pi)^3]\,\mathrm{Tr}_{\sigma'} \int d^3p' f(p, \sigma, p', \sigma')\delta\rho(p', r, t, \sigma'), \tag{6}$$

where $\delta\rho \equiv \rho - n_0$. The function f is the basic phenomenological quantity characterizing the Fermi-liquid theory. For an isotropic system, f takes the form

$$f(p, \sigma, p', \sigma') = f(p, p') + \zeta(p, p')\sigma \cdot \sigma'.$$

Since p and p' are fixed on the Fermi surface, f and ζ may be expanded in Legendre polynomials:

$$f(p, p') = \sum_n f_n P_n(\cos\theta)$$

and

$$\zeta(p, p') = \sum_n \zeta_n P_n(\cos\theta).$$

The set of dimensionless quantities

$$A_n = m^* p_F f_n/\pi^2(2n+1),$$

$$B_n = m^* p_F \zeta_n/\pi^2(2n+1)$$

are the parameters which specify the transport properties of the Fermi liquid. Here we are concerned with electrical properties, i.e., the conductivity, and only the A_n enter. For spin waves only the B_n are relevant.[4,5]

The density matrix $\delta\rho$ satisfies a transport equation similar in form to the usual Boltzmann equation. We are interested in the solution of the transport equation in the linear approximation. Defining the quantity g as

$$\mathrm{Tr}_\sigma[\delta\rho] \equiv [\partial n_0/\partial E^0(p)]g,$$

the transport equation for g becomes[11]

$$\frac{\partial g}{\partial t} + \left[\vec{V}\cdot\nabla + \frac{e}{c}(\vec{V}\times\vec{H})\cdot\frac{\partial}{\partial\vec{p}}\right][g + \delta\epsilon_1] = e\vec{E}\cdot\vec{V}, \tag{7}$$

where $|\vec{V}| = p_F/m^*$ and

$$\delta\epsilon_1 = \frac{2}{(2\pi)^3}\int d^3p' f(p, p')g(p')\delta(E_F - E^0(p')).$$

The current j and hence the conductivity tensor are proportional to an integral over g, i.e., $\vec{j} \propto \int \vec{p}\, g\, d\Omega_{\vec{p}}$.

As $k \to 0$, Eq. (7) may be solved analytically. In the neighborhood of the first cyclotron resonance the solution, when inserted into Eq. (3), leads to

$$1 - \frac{b^2}{2(a-\gamma_1)}\frac{1}{(\gamma_1 - 2\gamma_2)} \times\left[\frac{7\gamma_2\gamma_1 + 5\gamma_0\gamma_1 - 2\gamma_2^2 - 10\gamma_0\gamma_2}{15}\right] = 0, \tag{8}$$

where

$$\gamma_n = 1 + A_n.$$

Equation (8) yields a real solution to order k^2. When $\gamma_n \equiv 1$, the coefficient of the k^2 term vanishes and we revert to the uncorrelated case, Eq. (5). As we conjectured, the correlations, even for small A_n, drastically change the character of the resonance near the first harmonic. There now exists a propagating mode to order k^2 whose $k \to 0$ limit is shifted away from $\omega_C/\omega = 1$. The wave first appears at $\omega/\omega_C = 1 + A_1$.[13] The shift of $k \to 0$ intercept is a general conclusion of the theory. The intercepts of the various dispersion curves associated with the nth subharmonics are shifted by the corresponding A_n. That is, the intercepts occur at $\omega/\omega_C = n(1 + A_n)$. In principle, we can therefore determine A_n for $n \geq 1$ by finding the intercepts of the dispersion curves at successive subharmonics.

Since data exist for $0 < kR < 4$, it is desirable to have a solution of Eq. (7) valid for all values of kR. This may be obtained by assuming (for the moment without justification) that A_0 is finite and that $A_n = 0$ for $n \geq 1$. In this case the dispersion relation Eq. (3) is easily com-

puted. We find that

$$\sigma_{yy}^{0}+(\sigma_{xy}^{0})^2/\sigma_{xx}^{0}+[S_1(k,\omega)i\sigma_{xy}^{0}+R_1\sigma_{yy}^{0}]=0, \quad (9)$$

where $\sigma_{ij}^{(0)}$ is the conductivity tensor in the absence of correlation effects and the functions R_1 and S_1 are given by

$$R_1=A_0\sum_{m=0}^{\infty}\frac{m^2}{a^2-m^2}\int_{-1}^{+1}J_m{}^2 d(\cos\theta),$$

$$S_1=A_0\sum_{m=0}^{\infty}\frac{a}{a^2-m^2}\int_{-1}^{+1}J_m J_m{}'\sin\theta d(\cos\theta).$$

The fitting procedure we will adopt for the experimental data near the first harmonic with $J\perp H$ is to neglect all A_n, $n\geq 1$, except for the fact that the finite A_1 shifts the position of the fundamental intercept. This procedure is consistent with all of the available data in both polarizations and is the justification for just including A_0 in an arbitrary kR solution of Eq. (7).

In Fig. 2 we have plotted Eq. (9) with an A_0 of -0.7 and with an intercept which has been shifted to $\omega_c/\omega=1.08$, i.e., $A_1\simeq -0.08$. Typical curves for $A_0=-0.53$ and -0.89 are shown to indicate the sensitivity of the fit to a variation in A_0. In K this fixes A_0 at -0.7 ± 0.1 and A_1 at -0.08 ± 0.02.

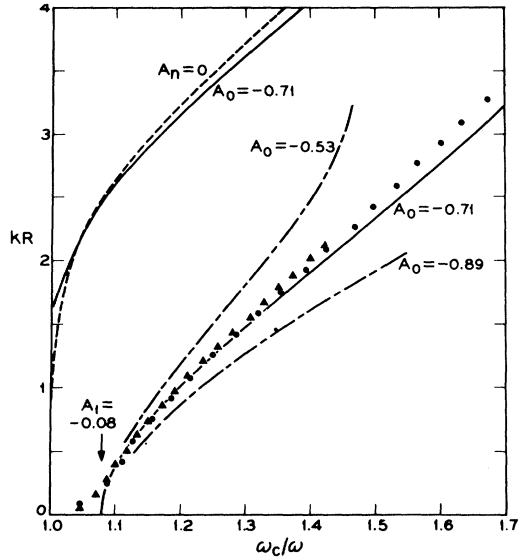

FIG. 2. Comparison of the theoretical dispersion curve, including correlation effects, with the experimental data. The upper branch lying near the noninteracting dispersion curve (dashed) is plotted for a single value of A_0 since it is quite insensitive to that value.

Equation (9) exhibits two branches. The upper branch is close to the correlationless dispersion curve shown dashed in Fig. 2. The lower, longer wavelength branch which we might expect to observe in these experiments fits the data reasonably well (certainly within 10%). There seems to be some peculiar behavior in the region $1.00<\omega_c/\omega<1.1$. This is probably due to interference with oscillations in dA/dH because of the upper branch which still exists in this region. To unravel the details very near the cyclotron resonance, we will ultimately require at least a partial analysis of the actual boundary-value problem. The important point to make here is not that we measure a specific value of A_0 or A_1 (the fitting procedure is certainly open to question), but that correlations qualitatively change the propagation characteristics of these waves. Utilizing these experiments along with the spin-wave experiments,[5] it is now possible, at least in principle, to determine the complete Landau f function for a series of alkali metals.

Similar results have been obtained for Na although they are not as extensive as are the potassium results. We defer discussion of all the available data to a later publication.

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[8]In Ref. 1 the criterion used in assigning the observed oscillations in dA/dH vs H to specific k values was that the net power absorption would be increased when a standing-wave resonance was excited in the slab. Since then we have established that such standing-wave spatial resonances actually decrease the total power absorbed. A more complete discussion of such details will be presented in a future article.

[9]G. E. Smith, L. C. Hebel, and S. J. Buchsbaum, Phys. Rev. 129, 156 (1963), point out [see their Eq. (10)] that in the long wavelength limit, i.e., to the lowest approximation, the electric field at cyclotron resonance is circularly polarized but rotates in the sense opposite to that of the electrons. As a result no resonant absorbtion occurs. This argument can be extended to show that the coefficient of the singular term appears only in order k^4.

[10]In P. M. Platzman and K. C. Jacobs, Phys. Rev. 134, 974 (1964), it is pointed out that in order to see effects on the conductivity tensor due to electron correlation effects, it is necessary to have $kV_F/\omega \sim 1$. These waves are precisely in this regime and we expect a priori to find explicit correlation effects.

[11]L. D. Landau, Zh. Eksperim. i Teor. Fiz. 30, 1058 (1956) [translation: Soviet Phys.—JETP 3, 920 (1965)]. V. P. Silin, Zh. Eksperim. i Teor. Fiz. 33, 495 (1957) [translation: Soviet Phys.—JETP 6, 387 (1958)].

[12]The mass m^* is the effective quasiparticle mass which includes band-structure effects, phonons, and electron-electron effects. It can be shown {see M. Ya. Azbel', Zh. Eksperim. i Teor. Fiz. 34, 766 (1958) [translation: Soviet Phys.—JETP 7, 527 (1958)]} that to a high degree of accuracy, the mass measured in Azbel'-Kaner cyclotron resonance is, in fact, the quasiparticle mass.

[13]The onset of propagation near the fundamental cyclotron resonance occurs at precisely the field value at which one would expect to see resonance in a metal for the uniform excitation [see R. Prange, to be published), Eq. (15)].

DISLOCATION DRAG IN METALS

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When a dislocation moves through a metal, it generates electric fields which produce currents in the conduction-electron gas. We wish to suggest that the energy dissipated by such currents may be the origin of the large, strongly temperature-dependent component of the yield and flow stresses observed in bcc metals.[1-4] The calculation is closely analogous to acoustic attenuation theories[5-8] except that instead of dealing with a single phonon, one must deal with what is essentially a wave packet of phonons, i.e., a moving dislocation. One begins with the Boltzmann equation,

$$\frac{\partial f_{\vec{k}}(\vec{r},t)}{\partial t}+\vec{v}_{\vec{k}}\cdot\frac{\partial f_{\vec{k}}(\vec{r},t)}{\partial \vec{r}}+\frac{e\vec{E}(\vec{r},t)}{m}\cdot\frac{\partial f_{\vec{k}}(\vec{r},t)}{\partial \vec{v}_{\vec{k}}}=-\frac{[f_{\vec{k}}(\vec{r},t)-\bar{f}_{\vec{k}}^{(0)}(\vec{r},t)]}{\tau}, \tag{1}$$

where $f_{\vec{k}}(\vec{r},t)$ is the electron distribution function, giving the probability of finding an electron with momentum $\vec{k}$ at position $\vec{r}$ and time t, $\vec{E}(\vec{r},t)$ is the electric field generated by the moving dislocation, $\vec{v}_{\vec{k}}$ is the electron velocity, and τ is the electron relaxation time. $\bar{f}_{\vec{k}}^{(0)}(\vec{r},t)$ is taken to be the equilibrium electron distribution function in the rest frame of the moving lattice and is expanded in the usual fashion.[6-8]

Putting $f_{\vec{k}}(\vec{r},t)=f_{\vec{k}}^{(0)}+f_{\vec{k}}^{(1)}(\vec{r},t)$, one finds

$$f_{\vec{k}}^{(1)}(\vec{r},t)=-\tau\frac{\partial f_{\vec{k}}^{(0)}}{\partial \epsilon_{\vec{k}}}\sum_{\vec{q},\omega}\frac{\{\vec{v}_{\vec{k}}\cdot[e\vec{E}_{\vec{q}}(\omega)+m\vec{u}_{\vec{q}}(\omega)/\tau]-2E_F\Delta_{\vec{q}}(\omega)/3\tau\}}{(1-i\omega\tau+i\vec{q}\cdot\vec{v}_{\vec{k}}\tau)}e^{i(\vec{q}\cdot\vec{r}-\omega t)}, \tag{2}$$

where $f_{\vec{k}}^{(0)}$ is the ordinary Fermi distribution function, $\epsilon_{\vec{k}}$ and E_F are the unperturbed one-electron and Fermi energies, respectively, and $\vec{E}_{\vec{q}}(\omega)$, $\vec{u}_{\vec{q}}(\omega)$, and $\Delta_{\vec{q}}(\omega)$ are the Fourier transforms of the electric field, lattice velocity, and dilatation, respectively, produced by the moving dislocation. The boundary condition that the dilation, electric field, etc., must have the same values at the space-time point $(\vec{r}+m\vec{b},\ t+m\tau_D)$ as they have at $(\vec{r},t)$ restricts ω to the values

$$\omega=\vec{q}\cdot\vec{b}/\tau_D-2\pi n/\tau_D, \tag{3}$$